



UNIVERSITI PUTRA MALAYSIA

**DEVELOPMENT AND APPLICATION OF A FINITE ELEMENT
DISTRIBUTED RAINFALL RUNOFF SIMULATION MODEL**

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DISTRIBUTED RAINFALL RUNOFF SIMULATION MODEL**

By

HUANG YUK FENG

**Thesis Submitted in Fulfilment of the Requirement for
the Degree of Master of Science in the Faculty of Engineering
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December 2000



Dedicated To
My Beloved Grandmother, Mother, Sisters, Brothers,
And In Memory of My Beloved Father



Abstract of thesis presented to the Senate of Universiti Putra Malaysia
in fulfilment of the requirement for the degree of Master of Science

**DEVELOPMENT AND APPLICATION OF A FINITE ELEMENT
DISTRIBUTED RAINFALL-RUNOFF SIMULATION MODEL**

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Supervisor : Ir. Dr. Lee Teang Shui

Faculty : Engineering

A deterministic model to simulate rainfall runoff from pervious and impervious surfaces is presented. With precipitation excess as input, the surface runoff model is based on a one-dimensional, variable width, kinematic wave approximation to the St Venant's equation and Manning's equation, and was used to mathematically route overland and channel flow using the finite element method. The Galerkin's residual finite element formulation utilizing linear and quadratic one-dimensional Lagrangian elements is presented for the spatial discretization of the nonlinear kinematic runoff equations. Temporal excess rainfall discretization using linear transition over two time steps was used to eliminate abrupt discontinuities in excess rainfall intensities. The system of nonlinear equations was solved using successive substitutions employing Thomas algorithm and Gaussian elimination. The whole formulation was set up using the MapBasic and MapInfo Geographical Information System.

A laboratory rainfall runoff physical model was set up for observation data in order to test the numerical model. Parameters considered include, surface roughness, plane slope, constant or changing rainfall intensities. Maximum infiltration, overland flow discharge, and overland with channel flow discharge were observed for model verification. Finite element simulations have been shown to compare favourably with observed laboratory data. Linear element simulation was found to give results as accurate as the quadratic element simulation. The increased number of elements employed in the model to simulate runoff from a homogenous surface did not give any obvious added advantage. While maximum time step increment for computation is given by the Courant Criterion, it is however, always recommended that as small a time increment is to be used to eliminate any oscillatory instability. Time increment for channel flow routing was found to be always smaller when compared to lateral overland flow. Thus, the chosen time step increment for channel flow routing must be a common factor of that of lateral overland flow in order to satisfy the linear interpolation of overland outflow hydrograph as input into the channel. For laboratory scale catchments, problems of big physical elemental interface roughness differences (eg. 0.033 for bare soil surface upstream and 0.300 for grass surface downstream) can result in small wavy oscillatory at the rising limb. On the contrary, when the upstream roughness is larger than the downstream roughness, such discrepancies do not appear in the simulation. Differences in elemental interface slope (eg. 5% and 10% bare soil surfaces) can be catered for rather well in the model.

A hypothetical watershed and imaginary tropical rainstorm was also studied to verify the capability of the model in larger runoff catchments. Channels, which

are initially dry or with existing flows can be simulated incorporating additional rainfall. Large catchments with large physical elemental roughness and slope differences can be well simulated by the model to give typical hydrograph characteristics, without oscillations, evident in laboratory scale tests.

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**PEMBANGUNAN DAN PENGGUNAAN MODEL UNSUR TERHINGGA
TERAGIH PENYELAKUAN HUJAN-AIR LARIAN**

Oleh

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Sebuah model berketentuan demi untuk menyelaku air larian hujan dari permukaan telap dan tak telap telah dikemukakan. Dengan kelebihan hujan sebagai data masukan ke dalam model, model air larian permukaan adalah berdasarkan pendekatan ombak-kinematik kepada persamaan St Venant yang berdimensi satu, lebar bolehubah dan persamaan Manning. Ianya digunakan untuk mengira secara matematik, aliran permukaan dan aliran saluran dengan kaedah unsur terhingga. Formulasi unsur terhingga berbaki Galerkin berdasar unsur-unsur Lagrangian satu dimensi lurus dan kuadratik dipersembahkan demi untuk pembahagian secara ruang persamaan air larian kinematik tak lurus. Pembahagian kelebihan hujan semasa dengan menggunakan peralihan lurus diantara dua langkah masa berturutan digunakan untuk menghapuskan dadakan yang disebabkan tak berterusan dalam kelebihan hujan. Sistem persamaan tak lurus diselesaikan dengan menggunakan penggantian berturut-turut berasas algoritma Thomas



dan penghapusan Gaussian. Pembentukan persamaan-persamaan keseluruhannya diaturcarakan dengan MapBasic dan Sistem Matlumat Geografi, MapInfo.

Sebuah model air larian hujan fizikal makmal dibentukkan demi untuk menguji kebolehan model. Parameter-parameter yang dipertimbangkan termasuk kekasaran permukaan, kecerunan, keamatan hujan tetap atau berubah-ubah. Penyusupan maxima, hasil larian air permukaan dan hasil larian air saluran serta permukaan dicerapkan untuk pengesahan model. Kaedah penyelakuan unsur terhingga telah menunjukkan perbandingan yang baik dengan data makmal. Penyelakuan unsur lurus menghasilkan keputusan setepat penyelakuan unsur kuadratik. Pertambahan bilangan unsur yang digunakan dalam model tidak menunjukkan sebarang kelebihan. Sementara nilai langkah masa tokokan untuk pengiraan dalam model boleh dikirakan mengikut Kriteria Courant, ianya disyorkan langkah masa yang lebih kecil boleh digunakan demi untuk menghapuskan ketakstabilan ayunan. Nilai tokokan masa untuk aliran saluran adalah sentiasa lebih kecil dibandingkan dengan nilai yang digunakan pada permukaan sisi. Oleh sebab itu, pilihan tokokan masa untuk saluran mestilah merupakan factor sepunya kepada yang digunakan untuk aliran permukaan sisi, demi untuk memenuhi kehendak interpolasi lurus bagi graphidro sebagai data masukan ke dalam saluran. Bagi tadahan berskalar makmal, masalah penggunaan dua nilai kekasaran yang berlainan untuk unsur-unsur berdekatan (contohnya, 0.300 untuk permukaan rumput dan 0.033 untuk permukaan tanah terdedah) ialah berlakunya dadakan pada lengkung menaik graphidro. Sebaliknya, perbezaan kecerunan permukaan diantara dua unsur (contohnya, 5% dan 10% kecerunan untuk permukaan tanah terdedah) boleh diatasi dengan baik dalam model.

Sebuah kawasan tadahan hipotesis dan andaian hujan tropikal juga dikaji untuk membuktikan kebolehan model untuk kawasan yang sewajah. Saluran-saluran yang pada awalnya dalam keadaan yang kering atau berair boleh dipenyelakukan bersamaan dengan hujan tambahan yang masuk padanya. Tadahan besar yang berair, kekasaran unsur dan kecerunan fizikal yang perbezaannya besar boleh diramalkan dengan model dan menghasilkan sebuah graphidro yang khusus tanpa dadakan yang sentiasa wujud dalam kajian makmal.



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NOTATION

Q_{fc}	Channel Flow
Q_{fo}	Overland Flow
Q	Discharge
A	Wetted Area
P	Channel Wetted Area Perimeter
w	Overland Element Width
R	Hydraulic Radius
l	Element Length
ERF	Excess Rainfall
V	Flow Velocity
N	Interpolation Function
q	Lateral Inflow
S	Plane Slope
n	Roughness Coefficient
n	Number of Nodes In An Element
Δx	The Smallest Element Length
Δt	Time Increment
Δt_c	Courant Time Increment
t_c	Time of Concentration
i_{max}	Maximum Rainfall Intensity
i	Rainfall Intensity

CHAPTER 1

INTRODUCTION

Background of Study

The objective of a hydrologic system analysis is to study the system operations and predict its output, for example, an outflow hydrograph. In general, a hydrologic system model is an approximation of the actual system, in which its inputs and outputs are measurable hydrologic variables and are linked by a set of equations. The hydrologic system's synthesis and prediction are the most essential activities in the design of water resources systems.

The derivation of relationship between the rainfall over a catchment area and the resulting flow in the river system is a fundamental problem in any hydrologic study. Estimating runoff or discharge from rainfall measurements is needed for the assessment of water resources and flood prediction. The noticeable increase in the frequency and magnitude of floods in recent years has further



prompted the examination of the hydrologic impact of land-use and land-use changes in a watershed.

Flood forecasting is an important issue in engineering hydrology. The information obtained from the forecasting is primarily needed for the design of hydraulic structures. The quantity of runoff resulting from a given rainfall event depends on a number of factors such as antecedent moisture content of soil, land-use, slope of catchment, as well as intensity, distribution, and duration of the rainfall. Thus, linking of the geomorphologic parameters with the hydrological characteristics of the basin can provide a simple way to understand the hydrologic behavior of the watershed.

The advent of the high-speed digital computer with a large capacity of data storage has stimulated research in many disciplines, including the hydrological process simulation in hydrologic studies. The principal techniques of hydrological modeling make use of the two powerful facilities of the digital computer, the ability to carry out vast numbers of iterative calculations and the ability to answer the specifically designed interrogations. Complex theories describing hydrologic processes are now applied using computer simulations and, vast quantities of observed data are reduced to summary statistic for better understanding of hydrologic phenomena and for establishing hydrologic design level. More sophisticated approaches are now feasible, thus enabling the hydrological processes to be simulated more precisely.



The relatively recent introduction of Geographic Information System (GIS) into hydrologic modeling has provided the means of an efficient use of spatial data. The development in GIS and microcomputer technology has enhanced the capabilities to handle large database describing the detailed spatial configuration of land surface. With these facilities of technology, a watershed can be divided into a number of elements, and a single value of a land surface parameter is assigned to each element. In other words, it provides a digital representation of watershed characteristics used in hydrologic modeling. GIS also provides the storage and preprocessing capabilities for the data needed in analysis and displays the results in the tabular and maps form for analysis purpose and visual representation.

Statement of Problem

Mathematical modeling to predict the hydrologic response of a watershed system has been the subject of many researches in recent years. Many of these hydrologic models are lumped parameter models in which the flow is calculated as a function of time alone and at a particular location. The flow of water through the soil and stream channels of a watershed is a distributed process, since the flow rate, velocity, and depths usually show temporal and spatial variation throughout the watershed. Therefore, estimates of flow rate or water discharge at any location in the channel system can be obtained using a distributed model. By using this type of model, the flow rate can be computed as a function of space and time, rather than of time alone as in a lumped model.



The distributed model can be used to describe the transformation of storm rainfall into runoff over a watershed to produce a flow hydrograph for the watershed outlet, and then to take this hydrograph as lateral input into a river system and route it to the downstream end. By using this type of model, it provides for more reliable temporal and spatial data, and the effects of land-use changes as can be reflected in the runoff hydrograph.

Most of the hydraulic models require a large number of input data and might produce a large set of output data. Using GIS technology as an underlying platform for hydraulic models provides an effective integration mechanism for performing large-area surface water and drainage management studies. A complex, large-area, multi-basin drainage study requires significant effort in terms of data organization, development of models, and presentation of results. To overcome these problems and difficulties, a GIS system can be used to organize, store, and display spatial (maps) and non-spatial (characteristic) data for the study.

Although a lot of advanced and useful hydrograph models have been developed for this purpose in other countries, but almost none of them can be applied directly in our country without major modification or proper calibration and verification. Some of the models are developed specifically for their own applications of a particular geomorphology and climate, and they are also not economical to purchase. Due to the different climate and topography in our humid and tropical country compare to the foreign country, a lot of the developed models cannot be applied directly and it is suggested that we should develop our own model, which is suited for our country's environment.