



UNIVERSITI PUTRA MALAYSIA

***SORET AND DUFOUR EFFECTS ON CONVECTIVE HEAT AND MASS
TRANSFER OVER A STRETCHING / SHRINKING SURFACE***

SHAHANAZ PARVIN

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By

SHAHANAZ PARVIN

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia
in Fulfilment of the Requirements for the Degree of Doctor of Philosophy**

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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March 2022

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Mathematical Modelling of convective flow, heat, and mass transfer over a stretching or shrinking surface is studied to show the effects of Soret-Dufour by considering different types of fluids (Newtonian, non-Newtonian and Hybrid nanofluid). The mathematical models that have been investigated are as follows: Two-dimensional model of double-diffusive MHD Casson and Maxwell fluid flow over an exponentially permeable shrinking sheet, two-dimensional model of hybrid nanofluid flow past an exponentially permeable shrinking or stretching sheet, three-dimensional model of double-diffusive MHD Newtonian fluid flow and heat transfer over an exponentially stretching or shrinking sheet and the two-dimensional model of triple diffusive Sodium Chloride and Sucrose water over a nonlinear permeable shrinking sheet. The mathematical model is formed by a set of partial differential equations such as continuity, momentum, energy, and concentration. Similarity transformation is applied to transform the partial differential equations into ordinary differential equations. The MATLAB bvp4c program is the main mathematical program that is used to obtain the final numerical solutions for the reduced ordinary differential equations. The numerical results for the skin friction coefficient, local Nusselt number, local Sherwood number, and the profiles of velocity, temperature, and concentration are presented via plots to analyze the impact of governing parameters (buoyancy ratio, shrinking/ stretching, suction, mixed convection, magnetic field, Brownian motion, thermophoresis parameter, radiation parameter, Prandtl number, Soret number, Dufour number, Schmidt number, Deborah number, Eckert number, Lewis number) in the model. The MATLAB bvp4c program is also implemented to develop stability analysis when dual numerical solutions exist. Positive eigenvalue shows that the solution is stable and physically reliable. On the other hand, the negative eigenvalue represents the unstable solution and is rejected. In the presence of dual solutions, the first solution is accepted as the stable solution and the second solution is unstable. It is found that the temperature of the fluid increases with the increment of the Dufour number while fluids concentration is inclined with increased Soret number. Besides, all the governed parameters affected the variations of the fluid flow, heat transfer, mass transfer, and the profiles of velocity, temperature, and concentration.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

KESAN SORET DAN DUFOUR DALAM PEMINDAHAN HABA DAN JISIM TERHADAP PERMUKAAN MEREANG / MENGECEUT

Oleh

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Pemodelan matematik bagi aliran pemindahan haba dan jisim terhadap lapisan meregang atau mengecut dikaji untuk menunjukkan kesan Soret-Dufour dengan mempertimbangkan pelbagai jenis cecair (bendalir Newtonian, bendalir bukan-Newtonian, nanobendalir hibrid). Model matematik yang dikaji adalah: Model dua dimensi MHD dwiresapan aliran bendalir Casson dan Maxwell terhadap lapisan telap mengecut secara eksponen, model dua dimensi aliran nanobendalir hibrid merentasi lapisan telap meregang atau mengecut secara eksponen, model tiga dimensi MHD dwiresapan aliran bendalir Newtonian dan pemindahan haba terhadap lapisan telap meregang atau mengecut secara eksponen dan model dua dimensi triresapan Natrium klorida dan cecair Sukrosa merentasi lapisan telap mengecut secara tak linear. Model matematik dibentuk daripada set persamaan pembezaan separa iaitu keselantaran, momentum, tenaga, dan kepekatan. Penjelmaan keserupaan diaplikasikan untuk mengubah persamaan pembezaan separa menjadi persamaan pembezaan biasa. Dalam kajian ini, program MATLAB bvp4c adalah program matematik utama yang digunakan untuk mendapatkan penyelesaian berangka bagi persamaan pembezaan biasa yang diturunkan. Keputusan berangka untuk pekali geseran kulit, nombor Nusselt setempat, nombor Sherwood setempat, dan profil halaju, suhu dan kepekatan dibentangkan melalui graf untuk menganalisis kesan parameter (nisbah daya apungan, regangan/kecutan, sedutan, olakan campuran, medan magnet, gerakan Brown, parameter termoforesis, parameter radiasi, nombor Prandtl, nombor Soret, nombor Dufour, nombor Schmidt, nombor Deborah, nombor Eckert, nombor Lewis) yang terdapat di dalam model. Program MATLAB bvp4c juga dilaksanakan untuk membangunkan analisis kestabilan apabila dua penyelesaian berangka wujud. Nilai eigen positif menunjukkan bahawa penyelesaiannya stabil dan boleh dipercayai secara fizikal. Sebaliknya, nilai eigen negatif mewakili penyelesaian yang tidak stabil dan ditolak. Dengan kehadiran dual penyelesaian, penyelesaian pertama diterima sebagai penyelesaian stabil dan penyelesaian kedua tidak stabil. Didapati bahawa suhu bendalir meningkat dengan penambahan nombor Dufour manakala kepekatan cecair meningkat dengan peningkatan

nombor Soret. Selain itu, semua parameter yang menaakluk mempengaruhi variasi aliran bendalir, pemindahan haba, pemindahan jisim, dan profil halaju, suhu, dan kepekatan.



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LIST OF ABBREVIATIONS

Roman Letters

| | |
|----------------------|--|
| a, A | Positive constant |
| Al_2O_3 | Alumina |
| B_0 | Uniform strength of magnetic field |
| C | Concentration of the fluid |
| C_0 | Reference concentration of shrinking/stretching sheet |
| $C_{12}H_{22}O_{11}$ | Sucrose |
| C_f | Skin friction coefficient |
| C_{fx}, C_{fy} | Skin friction coefficient along x and y -direction |
| C_{fl} | Specific heat coefficient of fluid |
| C_p | Specific heat at constant pressure |
| C_s | Concentration susceptibility |
| C_w | Concentration of the fluid near to the plate |
| C_∞ | Concentration of the fluid in the free stream |
| Cu | Copper |
| D | Mass diffusivity |
| Db | Dufour number |
| D_B | Brownian motion coefficient |
| D_{CT} | Soret type of diffusivity |

| | |
|----------|--|
| D_T | Thermophoretic diffusion coefficient |
| D_{TC} | Dufour type of diffusivity |
| Ec | Ecart number |
| f, h | Non-dimensional stream function |
| g | Acceleration due to gravity |
| Gr_x | Thermal Grashof number |
| H | Magnetic field parameter |
| k | Thermal conductivity |
| k^* | Coefficient of mean absorption |
| K | Permeability constant |
| K_T | Thermal diffusion ratio |
| L | Characteristic length of the plate |
| Le | Lewis number |
| N | Buoyancy ratio |
| $NaCl$ | Sodium Chloride |
| Nb | Brownian motion parameter |
| Nt | Thermophoresis parameter |
| Nu_x | Local Nusselt number |
| Pr | Prandtl number |
| q_r | Radiative heat flux |
| q_w | Heat transfer rate at the surface flux at the wall |
| q_m | Mass transfer rate at the surface flux at the wall |

| | |
|-------------------|---|
| Ra_x | Rayleigh's number |
| Rd | Radiation parameter |
| Re_x, Re_y | Reynolds number along x and y -direction |
| Ri | Mixed convection parameter |
| Ri_x, Ri_y | Mixed convection parameter along x and y -direction |
| S | Suction parameter |
| Sc | Schmidt number |
| Sh_x | Local Sherwood number |
| Sr | Soret number |
| t | Time |
| T | Temperature of the fluid |
| T_0 | Reference temperature of shrinking/stretching sheet |
| T_m | Mean fluid temperature |
| T_w | Temperature of the fluid near to the plate |
| T_∞ | Temperature of the fluid in the free stream |
| $u_w(x)$ | Velocity of the sheet |
| $u_w(x, y)$ | Fluid velocity along the x -axis. |
| $v_w(x), W(x, y)$ | Wall mass suction velocity |
| U_0, V_0 | Constant value |
| u, v, w | Velocity components along x , y , and z -axis |
| x, y, z | Cartesian coordinate system |

X Dimensionless coordinate along the plate parameter

Greek Symbols

α Thermal diffusivity

β Deborah number

β_c Coefficient of solutal expansions

β_T Coefficient of thermal expansion

γ Smallest eigenvalue

δ Porosity parameter

ϕ Dimensionless concentration

ϕ_1 Volume fraction of Alumina nanoparticles

ϕ_2 Volume fraction of Copper nanoparticles

ψ Stream function

η Boundary layer thickness

μ Fluid viscosity

ν Kinematic viscosity

τ Dimensionless time variable

τ_w Shear rate at the surface

τ_{zx}, τ_{zy} Wall shear stress along x and y -direction

λ Shrinking/Stretching parameter

λ_r Fluid relaxation time

θ Dimensionless temperature

| | |
|---------------|--|
| θ_w | Temperature ratio parameter |
| ρ | Fluid density |
| ρ_∞ | Density of the fluid far from the wall |
| ρC_p | Volumetric heat capacity |
| σ | Electrical conductivity |
| σ^* | Stefan-Boltzmann constant |
| ε | The ratio of effective heat capacity |
| χ | Inclination angle from the y-axis |
| ω | Casson parameter |
| ζ | Inclination angle from the sheet |

Subscripts

| | |
|----------|--|
| w | Location at the shrinking sheet |
| ∞ | Location far from the shrinking sheet |
| f | Fluid |
| nf | Nanofluid |
| hnf | Hybrid nanofluid |
| $n1$ | Solid component for the first nanofluid |
| $n2$ | Solid component for the second nanofluid |
| c | Critical point |
| 0 | Reference value |
| 1 | Sodium Chloride |
| 2 | Sucrose |

CHAPTER 1

INTRODUCTION

1.1 Fluid Dynamics

As stated by the American Heritage Dictionary, fluid dynamics is “a branch of practical science concerned with the flow of liquids and gases”, and it is extended as a sub-discipline of fluid mechanics. Mechanics is the eldest physical science that studies the interactions of forces on both inactive and moving objects. Statics is the subdivision of mechanics concerned with bodies at rest, while dynamics is concerned with bodies in motion. Fluid mechanics is a branch of science and it studies the behavior of fluids at rest and in motion, as well as their connections with solids or other fluids at the boundaries. Rouse and Ince (1957) and Tokaty (1971) provide excellent histories of fluid dynamics. By applying the particular case, motion with zero velocity that refers to fluids at rest, fluid mechanics can be referred to as fluid dynamics.

Fluid dynamics are separated into various branches as well. Among them, hydrodynamics is the study of the movement of essentially incompressible fluids (for example, liquids, particularly water, and gases moving at low speeds). Some other focused branches are hydraulics, Gas dynamics, aerodynamics, meteorology, oceanography, and hydrology. Hydraulics explains the liquid flows inside pipes and exposed channels while the fluids flow with large density changes. For instance, gases flowing through nozzles at high speeds are dealt with gas dynamics. On the other hand, aerodynamics is related to the flow of gases (particularly air) above rockets, automobiles, and, aircraft, etc. at low or high speeds, and meteorology, hydrology, and oceanography are involved with natural flows of fluid (Cengel, and Cimbala, 2010).

1.2 Boundary Layer Flow

Boundary layer flow talks about a thin layer of viscous fluid adjoining with a solid moving surface where the velocity differs from zero (at the wall) to the velocity at the boundary. In 1904, Ludwig Prandtl established the notion of boundary layer flow, which revolutionized fluid dynamics knowledge and analysis. The core idea recommended by Prandtl is that fluid flow can be separated into two portions: (a) inviscid flow which occurs in the main area, (b) the thin layer next to the solid surface named as the boundary layer. Following the above idea of Prandtl, by applying the boundary layer conception the Navier-Stokes equations can be simplified. He stated that the impact of viscosity on fluid flow is more significant for higher Reynolds numbers, $Re \gg 1$.

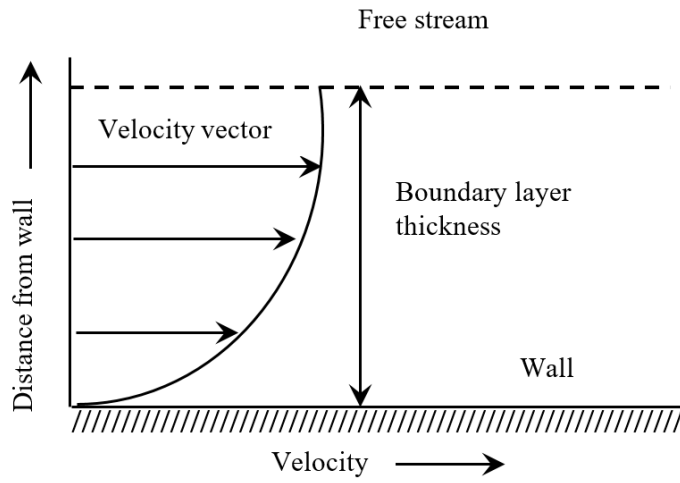


Figure 1.1 : Profile of a boundary layer
 (<https://history.nasa.gov/SP-4103/app-f.htm>)

1.3 Heat Transfer

Heat transfer is the movement of energy between a boundary layer system and its surroundings as a result of a temperature differential. Heat transportation from higher to lower temperature happen until it reaches the same temperature which is referred to as thermal equilibrium. Following Fourier's law, heat flux can be defined as

$$q = -k \frac{\partial T}{\partial n}, \text{ where}$$

k = Thermal conductivity Coefficient,
 n = Normal to the solid surface,

q = Heat flux,
 T = Temperature.

The negative indication implies that the heat flow is causing the temperature to drop. Conduction, convection, and radiation are the three ways of heat transport (Figure 1.2).

- **Conduction:** The transfer of heat between two solid bodies by conduction, also known as diffusion, occurs when they come into direct touch. The heat burners on a stove, for example, will transmit heat to the bottom of a pot, and the pot will then conduct heat to its substances.
- **Convection:** The transmission of heat between the liquid and the solid surface is known as convection. Convection can happen in three ways such as natural, forced, and mixed convection.

- **Natural Convection:** Natural or free convection arises when fluid motion is affected by buoyancy forces that happen due to disparities of temperature in the fluid. The fluid's motion isn't controlled by external or internal factors but is instead influenced only by density changes. Fluid density differences may stem from temperature differences, concentrations, or composition (Rudramoorthy and Mayilsamy, 2006).
- **Forced Convection:** When an outside source forces fluid to flow over a surface to form an artificially-induced convection current, the phenomenon is known as forced convection. An external source can be of different types, including a pump, fan, or suction appliance (Rathore and Kapuno, 2011). The external sources will result in a fluid with higher speed, and hence lower thermal resistance. It is well known that forced convection is more successful than natural convection. Forced convection will be more likely to transport significant amounts of heat energy more efficiently and produce quicker results.
- **Mixed Convection:** There is always some natural convection with forced convection in the presence of gravitational force. This applies to every forced convection scenario (Dawood et al., 2015), Natural convection, which happens spontaneously and transfers heat along with forced convection, is often called mixed convection. The interaction of buoyant and pressure forces is also defined as mixed convection. Temperature, flow, shape, and direction all influence the quantity of convective form that contributes to heat transfer. Mixed convection is widely utilized in high-power output devices that run at extremely high temperatures and when forced convection is insufficient to disperse the requisite heat.
- **Radiation:** Radiation refers to a heat transfer system between two bodies that are at different temperatures and have a distance between them. Electromagnetic waves in the atmosphere create heat transportation in radiation (Siegel and Howell, 1992). A common phenomenon of radiation is the earth gets heated by the sun's energy.

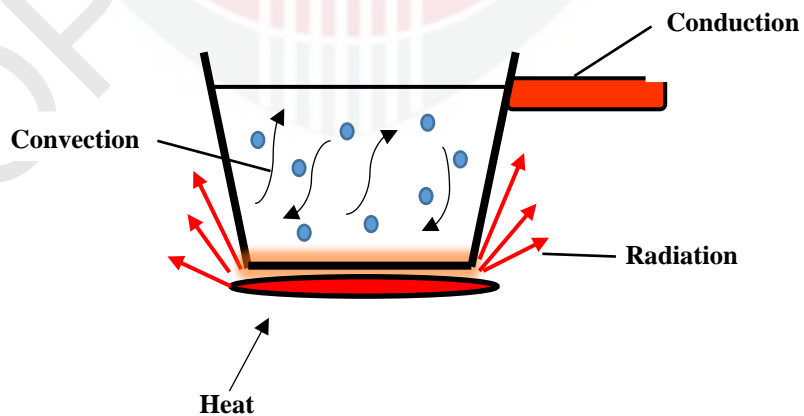


Figure 1.2 : Profile of heat transfer

(<https://byjus.com/physics/heat-transfer-conduction-convection-and-radiation>)

1.4 Mass Transfer

A net movement of mass from one area to another is referred to as mass transfer. Mass transfer, also defined as diffusive and convective transportation of chemical species inside boundary layer flow. Diffusion is the term used to describe mass transfer that occurs as a result of random molecule mobility in a laminar-flow fluid. Convective mass transfer refers to mass transfer that occurs as a result of a difference in concentration between species at the surface and the fluid above the surface. The simplest form of mass transfer can occur in a resting medium, where the force is caused by concentration changes in adjacent parts of the medium, which is known as molecular diffusion. The mass transfer will diffuse from a greater concentration to a lesser concentration caused by mass flow. The evaporation of water from a river to the atmosphere, as well as the purifying of blood in the liver and kidneys, are all examples of mass transfer procedures in nature.

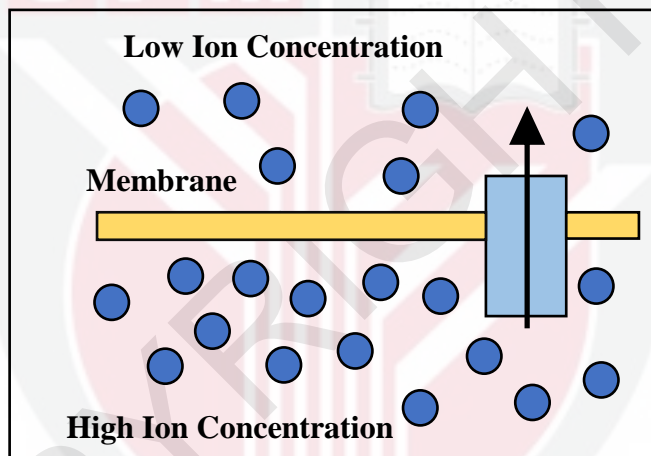


Figure 1.3 : Profile of mass transfer
(<http://ecoursesonline.iasri.res.in/mod/page/view.php?id=2367>)

1.5 Double Diffusive Convection

Convective motions develop into a fluid while only one parameter, such as temperature variations, impacts the density. When gradients of two or more diffusing properties are significant, a completely new set of phenomena may occur, and perception based on fundamental thermal convection may be incorrect. In several methods of interest, for example, instabilities can emerge even as the net density falls vertically, and the system would thus be regarded hydrostatically steady in a single-component fluid. In the event of a double or multiple component fluid, diffusion, which is a stabilizing impact in a fluid containing a single solute, might act to liberate potential energy in the component that is weightiest at the top. According to Huppert and Turner (1981), the fluid must have two or more components with distinct molecular diffusivities, and selected components must contribute opposing behavior to the vertical density gradient for double-diffusive

convection to occur. In oceanography, salt and heat concentrations have different gradients and diffuse at separate rates, which is an example of double-diffusive convection. The entry of cold water from the iceberg has an impact on both of these variables.

1.6 Triple Diffusive Convection

Multicomponent convection is the phenomenon where the density depends on three or additional agencies having diverse molecular diffusivities. Particularly if the density depends on three components with different molecular diffusivities is referred to as triple diffusive convection. A triple diffusive scheme behaves another way compare to double-diffusive systems. This is because adding a slower diffusing feature to the bottom layer of a double-diffusive system that would otherwise yield a finger interface could result in the formation of a diffusive interface. Likewise, adding the same feature to the topmost layer of a different system could alter the interface from diffusive to salt finger (Shivakumara and Naveen Kumar, 2014).

1.7 Stream Function

Simple flows can be used to explain the stream function. If no friction exists, the flow between two parallel plates must be dispersed uniformly throughout the gap between them. Because there is no friction to slow the fluid near the walls, the velocity will be the same all the way across. As a result, we can estimate that one-fourth of the total flow occurs in the first quarter of the channel's width, as illustrated in Figure 1.4. Imagine floats or dye streams introduced at regular intervals across the channel to designate out pathways, with an equal flow between each pair of lines. The steam function describes how a streamline interacts with a reference streamline. This line remains constant along a streamline, distinguishing it from other lines (Thompson, 2013).

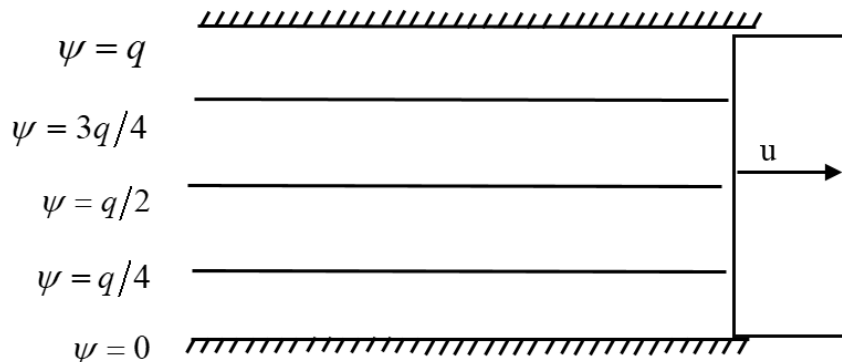


Figure 1.4 : Stream function in a uniform flow

1.8 Different Types of Fluid

1.8.1 Newtonian Fluid

The linear relationship between shear stress and deformation rate is assumed in Newtonian fluids theory. At zero shear stress, a Newtonian fluid has a constant viscosity with a zero shear rate, meaning that shear rate and shear stress are proportional to each other. Water, organic solvents, inorganic salt solutions with low molecular weight, thin motor oil, glycerin, molten metals, light-hydrocarbon oils, and air are just a few examples of this type of fluid. If τ is shear stress, μ is viscosity and k is shear rate then, $\tau = k\mu$ (Islam and Hossain, 2020).

1.8.2 Non-Newtonian Fluid

If the strain rate does not have a linear relation with the shear stress, the fluid is considered non-Newtonian. The shear stress is measured as the proportion of the force acting on the area in the direction of the forces perpendicular to the area. It is defined as $\tau_{xy} = F/A$.

where, x = direction of the shear stress, and y = the direction of the area. For instance, synthetic lubricants, paints, sugar syrup, clay coating, certain oils, and drilling muds are usual examples of non-Newtonian fluids (Rehman et al., 2019). The differential, integral, and rate types are used to classify non-Newtonian fluid models in general. The differential and rate types have been researched in greater depth out of these. In the present thesis, Casson and Maxwell non-Newtonian fluid are considered.

- a. **Casson Fluid:** Casson fluid is a shear-thinning liquid with an infinite viscosity at zero rates of shear, yield stress below which no flow occurs, and a viscosity of zero at an infinite rate of shear. When shear stress exceeds yield stress, it deforms; in the opposite case, it solidifies. Jelly, tomato sauce, honey, chocolate, and concentrated fruit juice are the most well-known Casson fluids (Ibrahim et al., 2017).
- b. **Maxwell Fluid:** A Maxwell substance is a viscoelastic material that has both viscosity and elasticity. Maxwell model has been proposed by Maxwell and it is also known as a Maxwell fluid. He was the first person to model a non-Newtonian fluid. The Maxwell model illustrates a substantial with a linear Hookean spring attached in series with a Newtonian dashpot (Kumar et al., 2012). It is also identified as an iso-stress model since two elements (spring and dashpot) are both dependent on the same stress ($\sigma = \sigma_s = \sigma_d$). The model is depicted in Figure 1.5.

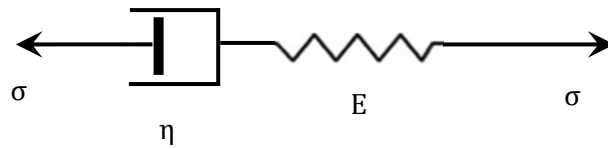


Figure 1.5: The Maxwell model

(<https://www.sciencedirect.com/topics/engineering/maxwell-model>)

The total strain (ε_t) is equal to the sum of elastic and viscous strains ($\varepsilon_d, \varepsilon_s$):

$$\varepsilon_t = \varepsilon_d + \varepsilon_s$$

It is easier to differentiate the strain equation and then write the spring and dashpot strain rates in terms of the stress to find a single equation that relates the stress to the strain:

$$\dot{\varepsilon}_t = \dot{\varepsilon}_d + \dot{\varepsilon}_s = \frac{\dot{\sigma}}{\eta} + \frac{\dot{\sigma}}{E}$$

1.8.3 Nanofluid

A nanofluid is made up of nanometer or micrometer-sized particles whose diameter ranges from 0 to 100 nanometers. The most common nanoparticles mixed in nanofluids include metals, carbides, oxides, and carbon nanotubes, while the most popular base fluids are water, oil, and ethylene glycol. Choi proposed the term nanofluids in 1995 and claimed that adding a tiny amount of nanoparticles to the base fluid (<1% volume percentage) doubles the thermal conductivity. This is because nanometer-sized nanoparticles operate like fluid molecules, according to Khanafer et al., (2003). Das et al., (2006) explained that the physical and chemical features of nanoparticles increase the thermal conductivity of the fluid as well as the heat transfer rate. Nanofluids' unique qualities allow them to be used in a variety of engineering systems, ranging from improved nuclear reactors to drug delivery systems.

1.8.4 Hybrid Nanofluid

According to Ali, (2020), hybrid nanofluids are a relatively new type of nanofluid that contains two or more nanoparticles. It has a high thermal conductivity outer layer and a phase change material inner core with thermal storage capabilities because of latent heat received or released during phase shift. These hybrid nanoparticles can be employed in a heat transfer gel, sensors, thermal interface materials, and biological applications (Mohapatra et al., 2011). In comparison to the constituent polymer or inorganic nanoparticles, the hybrid nanoparticles may have better physical or chemical properties. Encapsulating inorganic nanoparticles with polymers, for example, can progress their chemical stability and disposability, mechanical properties and improve the thermal

stability of the polymers, make it easier to modify the inorganic nanoparticles by applying commoners with reactive groups, and give common polymers extra functions like magnetism and fluorescence (Qi et al., 2014). Depending on the type of nanoparticles utilized to create the hybrid nanofluids, they can be classified into three groups:

1. metal composites such as $\text{Al}_2\text{O}_3/\text{Cu}$, MgO/Fe , $\text{Al}_2\text{O}_3/\text{Ni}$, and Al/CNT ;
2. ceramic compounds such as $\text{Al}_2\text{O}_3/\text{Cu}$, MgO/Fe , $\text{Al}_2\text{O}_3/\text{Ni}$, and Al/CNT ; and
3. polymer mixtures such as polymer/ TiO_2 and polymer/ CNT .

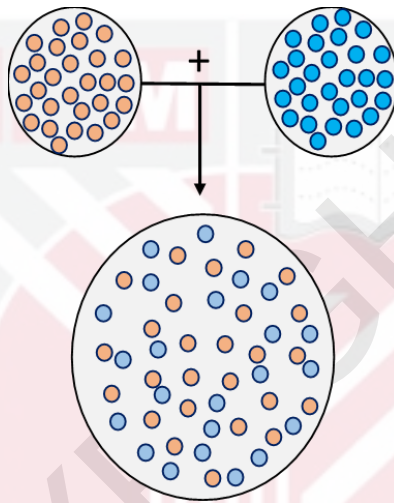


Figure 1.6: Hybrid nanofluid
(Water 2020, 12, 1723; doi:10.3390/w12061723)

1.9 Classification of Fluid Flow

1.9.1 Viscous Fluid Flow

A viscous fluid prevents movement and item passage through it. These fluids may flow slowly or not at all, depending on their viscosity. The elements that make up a fluid determine the viscosity of the fluid (or resistance). The viscosity of a substance is determined by its temperature, such as liquid's viscosity become low and gases viscosity become high as the temperature rises. A fluid with extraordinarily high viscosity may have qualities that make it behave more like a solid than a liquid. Butter is a common example of viscous fluid. Glass is a liquid that cools and hardens into a solid-like condition.

1.9.2 Compressible and Incompressible Flow

Depending on the amount of density variation throughout the flow, a flow can be characterized as compressible or incompressible. If the density of a flow stays virtually constant throughout, it is assumed to be incompressible. The volume of each part of the fluid remains constant along its trip when the flow (or the fluid) is incompressible. Compressible flow is a field of fluid mechanics that deals with fluids that change density drastically in reply to pressure changes. High-speed airplanes, gas pipelines, jet engines, commercial uses like abrasive blasting, and a variety of other disciplines all benefit from compressible flow research (Zohuri, 2017).

1.9.3 Magnetohydrodynamics (MHD) Flow

Magnetohydrodynamics (MHD) is the study of how magnetic fields interact with flowing, conducting fluids. Many natural and artificial flows are affected by magnetic fields. In industrial, they are frequently used to pump, stir, heat, and levitate liquid metals. The earthly magnetic field is continued by the motion of the fluid in the earth's core, the solar magnetic field causes solar flares and sunspots, and the galactic magnetic field is hypothesized to impact star formation from interstellar clouds. Magnetohydrodynamics is the scientific study of these types of fluid flows (MHD). The laws of Faraday and Ampere play a part in the mutual interaction of the magnetic and velocity field, as does the Lorentz force experienced by a current-carrying body (Soward, 2002). Hannes Alfvén was the first to discover magnetohydrodynamics, for which he was awarded the Nobel Prize in Physics (Dessler et al., 1970).

1.9.4 Steady and Unsteady Flow

The state of fluid characteristics can be categorized as steady or unsteady at any point in a fluid flow. The term steady denotes that there is no change over time at a given position. The fluid characteristics within a device can fluctuate during steady flow, but they remain constant at any fixed point. Steady flows can be characterized arithmetically by $\partial P / \partial t = 0$ where $P = P(x, y, z)$ represent the fluid properties as velocity, density, and pressure. Turbines, boilers, condensers, pumps, and refrigeration systems, for example, can closely approach steady flow conditions. The flow is unsteady or non-steady if its properties (velocity, density, and pressure) change over time or are simply time-dependent.

1.9.5 Stretching and Shrinking Sheet Flow

When an elastic sheet in an incompressible fluid is stretched by applying stress, stretching sheet flow is induced. This sheet has a behavior named elasticity, which refers to a sheet's ability to whether distorting force and then reappearance to its original shape and size after the stress is removed. The stretching or shrinking sheet moves at a different

velocity according to how far it is from a fixed point. Regardless, a shrinking sheet is the polar opposite of a stretched sheet; the sheet is compressed, which affects fluid flow and heat transmission rate. Wang (1990) was the first to observe the flow of fluid towards a shrinking surface. One of two criteria must be met for a contracting sheet to flow: either a stagnation flow is assumed to maintain the shrinking sheet's velocity in the boundary layer, or sufficient suction is provided on the boundary (Miklavcic and Wang, 2006). In extrusion operations such as polymer extrusion, metal sheet extrusion, and other industrial processes, boundary layer flow caused by a stretching surface is essential. Shrinking is relevant to environmental management methods, capillary impacts in smaller pores, shrink-swell behavior, and the hydraulic properties of farming clay soils, all of which are vital for agricultural enlargement (Batool and Ashraf, 2013).

1.10 Dimensionless number

- **Soret Number:** The ratio of temperature difference to concentration is called the Soret number (thermal diffusion factor). As a result, a higher Soret number indicates a greater temperature differential and steeper gradient. Thus the increased thermal diffusion factor increases the fluid velocity. The Soret number effect, for example, has been applied for isotope separation and in combination between gases with a very light and medium molecular weight (Srinivasacharya et al., 2015).
- **Dufour Number:** The contribution of concentration gradients to the thermal energy flux in a flow is denoted by the Dufour number. It is clear that as the Dufour number rises, the temperature rises as well (Srinivasacharya et al., 2015). Dufour number effects are an important issue because it has an extensive range of applications, including moisture migration via air trapped in fiber insulation, chemical contaminants spreading into the soil, grain storage insulations, and drug diffusion in blood veins (Hayat et al., 2012).
- **Prandtl Number:** A number that approximates the proportion of momentum diffusivity to heat diffusivity is identified as the Prandtl number (Olson, 2003). The Prandtl number, along with velocity and pressure, is a fundamental number of fluid dynamics. The number is essential because it is effective in determining the thermal conductivity of gases at high temperatures. It can be formulated as:

$$Pr = \frac{\mu C_p}{k} = \frac{\nu}{\alpha}$$

μ = The dynamic viscosity

C_p = specific heat at constant pressure

k = Thermal conductivity

ν = Fluid's kinematic viscosity

α = Thermal diffusivity

The range of Prandtl number for different fluids are as: gases, $0.7 < Pr < 1$; water $1 < Pr < 10$; liquid metals $0.001 < Pr < 0.03$ etc.

- **Schmidt Number:** Incropera et al. (2011) define the Schmidt number as a measure of the relative efficiency of momentum and mass transmission by diffusion in the velocity and concentration boundary layers. It can be defined as:

$$Sc = \frac{\text{Momentum diffusivity}}{\text{Mass diffusivity}} = \frac{\nu}{D}$$

- **Deborah Number:** The proportion between the timescale and the observation process of the phenomenon is specified as:

$$D = \frac{\text{The timescale of the observed}}{\text{The timescale of the observer}} = \frac{\text{Relaxation time}}{\text{observation time}}$$

The Deborah number is a well-known number in rheology, particularly viscoelasticity (Moura-Ramos, and Correia; 2001).

- **Ecarter Number:** The measurement of the kinetic energy of the flow compared to the enthalpy differential through the thermal boundary layer, named after Ernst R. G. Eckert. In high-speed flows with strong viscous dissipation, the Eckert number plays a significant role (Bergman and Incropera, 2011).
- **Lewis Number:** The Lewis number is a measure of the comparative temperature and concentration boundary layer thickness that is well-defined after Warren K. Lewis (1882-1975). It's used to describe fluid movements that involve both heat and mass transport (Cohen, 2007). The ratio of the thermal diffusivity to the mass diffusivity is applied to compute the Lewis number. It can also be articulated as the ratio of Schmidt number, Sc and Prandtl number Pr as follows:

$$Le = \frac{\text{Thermal diffusivity}}{\text{Mass diffusivity}} = \frac{Sc}{Pr} = \frac{\alpha}{D}$$

1.11 Parameters Associated with Mathematical Formulation

- **Suction Parameter:** Suction is a boundary layer control strategy that aims to reduce drag on bodies in an external flow or to reduce energy losses in channels. The primary goal of suction through a fluid's bounding surface is to modify the rate of heat transfer from the surface and radically alter the flow field. Thus, physical suction increases the heat transfer rate as well as skin friction coefficients. Ludwig Prandtl first proposed the suction technique in a circular cylinder in 1904 (Berg, 2012).

- **Magnetic Field Parameter:** The forces generated by magnetic fields, can control fluid motion. In 1861, the macroscopic formulation for the electromagnetic field known as Maxwell's equation was introduced by James Clerk Maxwell to replace the microscopic version named Lorentz force. Maxwell's equation has widely been used in solving the real field problem, which involves electricity and magnetism.
- **Buoyancy Ratio:** The buoyancy ratio is defined as the ratio of the fluid's specific weight to the object's specific weight. Alternatively, the weight of the ejected fluid minus the weight of the object.
- **Mixed Convection Parameter:** The mixed convection parameter, Ri measured by the ratio between the Grashof number, Gr , and the square of the Reynolds number, Re (Sparrow et al., 1959). It is expressed as the following:

$$Gr = \frac{\text{buoyancy force}}{\text{viscous force}} = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2}$$

$$Re = \frac{\text{inertial force}}{\text{viscous force}} = \frac{\rho u L}{\mu} = \frac{u L}{\nu}$$

$$Ri = \frac{Gr}{(Re)^2} = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} \times \frac{\nu^2}{u^2 L^2} = \frac{g\beta(T_w - T_\infty)L}{u^2}$$

Here g , L , β , ν , T_w , T_∞ , ρ , μ , u are defined as gravitational acceleration, vertical length, thermal expansion coefficient, kinematic viscosity, the temperature of the surface, bulk temperature, the density of the fluid, dynamic viscosity, and velocity respectively.

- **Thermophoresis Parameter:** Thermophoresis, also known as the Soret effect, occurs in suspended particle and fluid mixes. The movement of fluid molecules in the hot area, as well as high energy levels in this area, causes the nanoparticles to be displaced toward the cool region, which is induced by the temperature differential. Moving heated particles from the hot region to the cold region speeds up the heat transfer process. This force is only significant at extremely low fluid velocities, notably in natural convection (Ali, 2020), due to the small size of the fluid molecules.
- **Radiation Parameter:** The conduction heat transfer contribution to thermal radiation transfer is determined by the radiation parameter. It is self-evident that raising the radiation parameter raises the temperature within the boundary layer (Reddy, 2012).

- **Brownian Motion Parameter:** According to Jang and Choi (2004), Brownian motion is the unsystematic movement of particles floating in a fluid (liquid or gas) as a result of collisions with fast-moving molecules or atoms in the fluid. The faster the rate of collision amongst the atoms or molecules in a fluid, the more suspended components there are. As a result, Brownian motion can help nanofluids transmit heat more efficiently.
- **Porosity Parameter:** Porosity is an essential parameter for determining the microstructure of a material. Concerning the total volume of the material, it refers to the volume of spaces that may hold fluid. The pore system, which consists of a network of communicative or non-communicating pores and small-diameter channels, can be divided into many porosity classes. The section of the pore system made up of occluded pores that have no communication with the remainder of the pore spaces or the outside is referred to as residual porosity or closed porosity. Although this form of porosity does not affect mass transfer, it does affect material strength. Additionally, communicative pores produce effective or open porosity, which is the porosity that accounts for the volume of connected or accessible pores. The pore size distribution can be used to refine the description of the pore system (Issaadi et al., 2018).

1.12 Problem Statement

Soret and Dufour's effect contribute a vital role in geoscience, chemical engineering, in the situation of density variances in flow, and a variety of other domains. However, a review of the literature reveals that there have been few attempts to analyze the magnetic field effect with Soret-Dufour impact. To address this issue, our research suggests the development of a new mathematical model that includes induced effects. (magnetic field, suction, Soret-Dufour, Prandtl number, stretching or shrinking). Then, a numerical program called the bvp4c MATLAB is applied to solve these governing equations subject to the boundary conditions. Also, stability analysis is performed to regulate the stability and significance of the solutions when dual solution occurs. Following that, the necessary numerical computations are carried out, followed by a comparison of past results.

1.13 Objectives of Research

The intention of this thesis is to explore the Soret and Dufour impacts on convective heat and mass transfer flow of different types of fluid in different areas configurations and subjected to various source terms and boundary conditions. The following five problems are:

1. The inclined factors of magnetic field and shrinking sheet in Casson fluid flow, heat, and mass transfer.

2. Double-diffusive convection flow of MHD Maxwell nanofluid in the presence of inclined shrinking sheet.
3. Boundary layer flow and heat transfer of water-based hybrid nanofluid with nano-particles of alumina and copper over a permeable exponentially stretching and shrinking surface.
4. Three-dimensional model of Newtonian fluid flow and heat transfer over an exponentially stretching or shrinking plane in the presence of magnetohydrodynamics.
5. The two-dimensional triple diffusive model of boundary layer flow, heat, and mass transfer over an inclined shrinking sheet.

The objectives of the current research are to:

1. construct and derive the mathematical model for the various non-linear problems,
2. develop an algorithm to solve the mathematical model numerically via bvp4c solver in MATLAB software and conduct the validation tests for the current research in comparison with the numerical results in the literature,
3. provides the formulation and conduct the stability analysis for the dual solutions obtained to determine which of the solutions represent a stable flow, and
4. analyzes the influence of the considered parameters on the characteristics of the fluid flow, heat and mass transfer along with Soret and Dufour effect.

1.14 Scope of Research

The scope of the study is limited to convective heat and mass transfer over a stretching or shrinking surface. Two or three-dimensional, Newtonian or Non-Newtonian (Casson, Maxwell, Hybrid nano-fluid) fluids with double or triple diffusive natural or mixed convection are considered in the study. Although the study mainly focuses on Soret and Dufour effect, other parameters, for example, suction, magnetic field, Brownian motion, buoyancy ratio, mixed convection, Deborah number, Lewis number, thermophoresis parameter, Eckert number, radiation parameter, also analyzed.

1.15 Outline of the Thesis

This thesis contains nine chapters, including the present chapter. The current chapter started with the basic discussion of fluid dynamics along with different types of fluid, classification of fluid flow. In the last part of this chapter objective, scope, and outline of the thesis are described.

In Chapter 2, the literature review includes descriptions of prior studies conducted by various researchers that are relevant to the study's topic. Since the study is linked to the convective heat and mass transfer of different types of fluid (Newtonian, Casson, Maxwell, Hybrid nanofluid) in the presence of the Soret and Dufour effect, related research is focused. This chapter is subdivided according to different types of fluid. In the last part literature review of stability analysis is described.

Chapter 3 will explain the methodology with numerical computation, which is used in the considered five problems. Initially, it started with the introduction, and then two and three-dimensional double-diffusive Newtonian fluid, and two-dimensional triple diffusive model of boundary layer flow, heat, and mass transfer are explained. These mathematical models are associated with governing equations, boundary conditions, similarity transformation, physical parameters (skin friction coefficient, local Nusselt number, local Sherwood number), and stability analysis (for dual solutions) chronologically and stepped by step. Lastly, an algorithm for the `bvp4c` program is shown with stability analysis.

Chapter 4 to 6 has experimented with two-dimensional Casson, Maxwell, and Hybrid nanofluid's ($\text{Cu-Al}_2\text{O}_3/\text{H}_2\text{O}$) flow, heat, and mass transfer, respectively. In three cases, stability analysis is performed because of the existence of deal solutions. Every chapter contains five sections. All the chapters started with the introduction of the correlated problem, the method of the numerical solution is modified from Chapter 3, then stability analysis, results, and discussion, and the last section is the conclusion. Tables and figures are used to present the findings. The results are presented graphically in the form of the velocity, temperature, concentration, skin friction coefficient, local Nusselt number, and local Sherwood number profile. In the conclusion part, the consequences of connected factors in the system of all the problems are listed.

The fourth issue of this study, namely the three-dimensional model of Newtonian fluid flow and heat transfer over an exponentially stretching plane with the Soret- Dufour effect, is narrated in detail in Chapter 7. This chapter begins with an introduction, followed by problem formulation (methodology described in Chapter 3), then results and discussion, and finally a conclusion. A comparison table is made to check the validity of our study. The effects of Soret-Dufour number, mixed convection parameter, buoyancy ratio, stretching parameter, magnetic field parameter, Schmidt and Prandtl number are demonstrated graphically.

The final problem with two-dimensional triple diffusive natural convection flow over an inclined shrinking plate is explained in Chapter 8. Two distinct components namely, Sodium chloride (NaCl) and Sucrose ($\text{C}_{12}\text{H}_{22}\text{O}_{11}$) with differing concentrations are taken to do the study. The effect of buoyancy ratio, radiation parameter, thermophoresis parameter with Soret-Dufour impact is observed.

The final chapter, Chapter 9, includes a summary of the current study as well as some recommendations for future research.



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