



UNIVERSITI PUTRA MALAYSIA

REAL-TIME THERMAL FIELD THEORY AND ITS APPLICATIONS

MOHAMMED ABDULMALEK ABDULRAHEEM AHMED

IPM 2022 7



REAL-TIME THERMAL FIELD THEORY AND ITS APPLICATIONS

By

MOHAMMED ABDULMALEK ABDULRAHEEM AHMED

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,
in Fulfillment of the Requirements for the Degree of Doctor of Philosophy**

June 2022

COPYRIGHT

All material contained within the thesis, including without limitation text, logos, icons, photographs and all other artwork, is copyright material of Universiti Putra Malaysia unless otherwise stated. Use may be made of any material contained within the thesis for non-commercial purposes from the copyright holder. Commercial use of material may only be made with the express, prior, written permission of Universiti Putra Malaysia.

Copyright © Universiti Putra Malaysia



DEDICATIONS

*I would like to dedicate this thesis
...to the soul of my beloved mother who had dreamed about this day
&
...to my beloved father, my wonderful wife,
my daughter Lojain and my son Abdulmalek
with my love*



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

REAL-TIME THERMAL FIELD THEORY AND ITS APPLICATIONS

By

MOHAMMED ABDULMALEK ABDULRAHEEM AHMED

June 2022

Chairman: Hishamuddin Bin Zainuddin, PhD
Institute: Mathematical Research

The many-body physics in physics require the use of Thermal Field Theory (TFT) or statistical field theory methods. TFT can describe a large ensemble of interacting particles in a thermodynamic environment. However, it will involve the use of path integral techniques, the ability to calculate for abelian interactions like quantum electrodynamics (QED) and non-abelian interactions like quantum chromodynamics (QCD). The first success of TFT was an understanding and quantitative study of phase transitions (or phase changes) of the matter within quantum field theory (QFT). There are two formalisms of TFT or QFT at finite temperature in Real Time. Real Time Formalism (RTF) is fully consistent with some suitable changes in the structure of the Bogoliubov transformation (BT). In the complex t plane, there is a one-parameter family of paths within RTF. This parameter is σ , which manifests in the Feynman rules for two popular choices of propagators, and they are $\sigma = 0, 1/2$. The first choice for the path is a closed contour when $\sigma = 0$, which explains why the associated formalism is known as closed-time path (CTP) formalism. The second choice for path parameter σ reproduces the Feynman rules of an operatorial approach to quantum thermal field theory known as ThermoField Dynamics (TFD) in which this parameter equals to $1/2$. The propagator was given in the momentum space, but it is also appropriate to work in the mixed coordinates where the Green's functions are defined as functions of the time coordinate and the spatial momentum. The propagator in mixed space showed that there is an unexpected simple relation between any temperature dependent $T \neq 0$ graph and its temperature independent $T = 0$ counterpart, through a multiplicative scalar operator which carried the entire temperature dependence. In RTF, we usually use Boltzmann-Gibbs (BG) statistics to study the condensed matter phenomena like QED plasma and QCD plasmas, the early universe including other many-body physics phenomena. In this thesis, we have shown systematically that an operator description for a theory defined in RTF with an arbitrary σ does indeed exist. Therefore, some works gathering those formalisms are

given through two popular choices for contour parameter σ , i.e. $\sigma = 0$ and $1/2$ for both CTP and TFD, respectively. By analogy, the viewpoint suggests modifying Transformation Matrix by adding a new parameter λ and use the new transformation in the definition of the thermal propagator, which leads to the fermion field propagator as well to the scalar field propagator, for specific values of λ . Then, we compute the scalar propagator, and tadpole self-energies within scalar QED. However, we want to treat them in a systematic and comprehensive way when one considers these components of propagator in different bases. Thereafter, we use those components to compute the tadpole self-energies in momentum space and the mixed space (momentum-time) within RTF. Hence, we have studied explicitly the photon-photon interaction within RTF (for any value of the arbitrary parameter σ), which allows for a path integral description. Indeed, the photon interaction has an application with the cosmic microwave background (CMB) radiation. Note that the photon self-energy is the same for both approaches in RTF, in which one considers the relevant effective $2n$ -photon vertex in a thermal photon gas where $n \geq 2$ at low energy within QED in both spaces. In addition, we constructed the propagator in TFD with Tsallis statistics. Next we consider two applications: the thermal photon-photon interaction within QED at low energy for scalar Tsallis thermal propagator, and the phase of e^+e^- annihilation into hadrons is considered through a single photon exchange within QCD at high temperature for fermion Tsallis thermal propagator. In addition, we have observed that a non-extensive MIT bag equation of state obtained with the help of the Tsallis distributions given. Our study is relevant to the Deconfinement Phase Transition from a Hadronic Gas (HG) to a Quark-Gluon Plasma (QGP) or Partonic Plasma (PP). We investigate the behavior of some thermodynamic quantities of the system, such as the energy density, the pressure, the interaction measure and entropy.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

TEORI-MEDAN TERIMA MASA NYATA DAN APLIKASINYA

Oleh

MOHAMMED ABDULMALEK ABDULRAHEEM AHMED

Jun 2022

Pengerusi: Hishamuddin Bin Zainuddin, PhD
Institut: Penyelidikan Matematik

Masalah berbilang-jasad dalam fizik memerlukan penggunaan Teori Medan Terma (TMT) atau kaedah teori medan statistik. TMT dapat memerihail suatu ensembel besar zarah-zarah bersaling tindak dalam persekitaran termodinamik. Namun, ini melibatkan penggunaan teknik kamiran lintasan dan kebolehan untuk mengira saling tindakan abelan seperti teori elektrodinamik kuantum (EDK) dan saling tindakan tak-abelan seperti kromodinamik kuantum (KDK). Kejayaan pertama TMT adalah kefahaman dan kajian kuantitatif peralihan fasa (atau perubahan fasa) bagi jirim dalam teori medan kuantum (TMK). Terdapat dua formalisme TMT atau TMK pada suhu terhingga bagi masa nyata. Formalisme Masa Nyata (FMN) adalah konsisten sepenuhnya dengan beberapa perubahan dalam struktur matriks transformasi Bogoliubov (BT) yang dibuat. Dalam satah t kompleks, terdapat famili lintasan satu parameter dalam FMN. Parameter ini adalah σ yang tampak dalam petua Feynman bagi dua pilihan popular dalam perambat iaitu $\sigma = 0, 1/2$. Pilihan pertama lintasan adalah kontur tertutup apabila $\sigma = 0$, yang menjelaskan kenapa formalisme berkaitan dipanggil formalisme lintasan masa-tertutup (LMT). Pilihan kedua bagi parameter lintasan σ menerbit semula petua Feynman bagi pendekatan operator kepada teori medan terma yang dikenali sebagai Dinamik MedanTermo (DMT), apabila parameter bersamaan dengan $1/2$. Perambat ini diberi dalam ruang momentum, sementara itu ia turut berlaku dalam koordinat campuran apabila fungsi Green ditakrif sebagai fungsi koordinat masa dan momentum spatial. Perambat dalam ruang campuran ini menunjukkan bahawa terdapat hubungan mudah antara graf bersandar keada suhu $T \neq 0$ dengan padanan $T = 0$ yang tak bersandar kepada suhu, melalui suatu operator scalar pendaraban yang membawa keseluruhan persandaran suhunya. Dalam FMN, lazimnya kita menggunakan statistik Boltzmann-Gibbs (BG) untuk mengkaji fenomena jirim terkondensasi seperti plasma EDQ dan plasma KDK, alam semesta awal serta pelbagai fizik berbilang jasad. Dakan tesis ini, kita dapat tunjukkan secara sistematik bahawa perihalan operator bagi teori tertakrif dalam FMN dengan

sebarangan σ turut wujud. Dengan itu, beberapa kajian mengumpulkan formalisme-formalisme tersebut melalui dua pilihan popular parameter kontur σ , iaitu $\sigma = 0$ dan $1/2$ bagi masing-masing LMT dan DMT dibuat. Secara analogi, pandangan yang disarankan adalah untuk mengubah Matriks Transformasi dengan menambah suatu parameter baharu λ dan menggunakan transformasi baharu ini dalam takrifan perambat terma yang membawa kepada perambat medan fermion dan juga perambat medan skalar bagi beberapa nilai tentu λ . Kemudian, kami kirakan perambat skalar, dan swa-tenaga gelung tunggal dalam EDK scalar, namun, kita berkehendakkan suatu pengolagan yang sistematik dan komprehensif apabila mempertimbangkan komponen-komponen kuantiti tersebut dalam asa berlainan. Sejurus itu, kita gunakan komponen tersebut untuk mengira swa-tenaga gelung tunggal dalam ruang momentum dan ruang campuran (momentum-masa) dalam FMN. Dengan itu, kita kaji secara eksplisit, saling tindakan foton-foton dalam RTF (bagi setiap nilai parameter sebarangan σ) yang membolehkan perihalan kamiran lintasan. Malahan, saling tindakan foton dalam sinaran mikrogelombang latarbelakang (SML) telah diambil sebagai suatu aplikasi. Perhatikan bahawa swa-tenaga foton adalah sama dalam kedua-dua pendekatan dalam RTF, yang mana verteks $2n$ -foton relevan dalam gas foton pada tenaga rendah dalam EDK bagi kedua-dua ruang. Sebagai tambahan, kita bangunkan perambat dalam FMN bagi statistik TG. Kemudian, kita pertimbang dua aplikasi: saling tindakan foton-foton terma dalam EDK pada tenaga rendah bagi perambat terma Tsallis skalar, dan pertimbangan fasa pemusnahan e^+e^- ke hadron melalui tukar-ganti foton tunggal dalam KDK pada suhu tinggi bagi perambat terma fermion Tsallis. Sebagai tambahan, kita turut menjumpai persamaan keadaan beg MIT tak-ekstensif, diperolehi dengan bantuan taburan Tsallis. Kajian kita adalah relevan kepada Peralihan Fasa Nyahkurungan dari gas hadronic (HG) ke Plasma Quark-Gluon (PQG) atau Plasma Parton (PP). Kita turut kaji perlakuan beberapa kuantiti termodinamik sistem, seperti ketumpatan tenaga, tekanan, ukuran saling tindakan dan entropi.

ACKNOWLEDGEMENTS

First and foremost, I want to thank *Allah Subhanahu Wa Taala* for providing me with the strength, direction, and patience I needed to finish this thesis. Every stage of my life, I thank Allah for His enormous mercy and bounty. May Allah bless and keep Prophet Mohammad *Sallallahu Alaihi Wasallam*, who was sent to bring mercy to our world.

I would like to express my sincere gratitude to my supervisor, Associate Professor Dr Hishammudin Zainuddin, for giving me a chance to study and work with him. His encouragement, advice, guidance and technical support have not only making this research possible but also helping me to be a better person. I would also like to thank my supervisory committees, Dr Nurisya Shah, Associate Professor Dr Chen kien, Professor Dr Madjid Ladrem and Professor Dr Faqir Khanna, for their valuable supports and discussions during this period of study.

My time as a PhD student at UPM has been a journey for me, both professionally and personally. This journey would not have been possible without the support of many individuals. My best thank and appreciation are extended to all my friends and colleagues, who have helped me in countless ways, bring cheers and good wishes to me during my journey. Last but not least, I wish to express my gratitude to my family, especially my lovely wife Houida Ahmed, for her support and understanding.

Finally, I'd want to thank everyone who contributed to the successful completion of this thesis, and I apologize for not mentioning everyone individually.

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

Hishamuddin Bin Zainuddin, PhD

Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Chairman)

Nurisyah binti Mohd Shah, PhD

Senior
Faculty of Science
Universiti Putra Malaysia
(Member)

Chen Soo Kien, PhD

Associate Professor
Faculty of Science
Universiti Putra Malaysia
(Member)

Faqir Khanna, PhD

Professor Emeritus
Faculty of Science
University of Alberta, Canada
(Member)

Madjid Ladrem, PhD

Professor
Faculty of Science
Taibah University, Saudi Arabia
(Member)

ZALILAH MOHD SHARIFF, PhD

Professor and Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 8 September 2022

TABLE OF CONTENTS

	Page
ABSTRACT	i
ABSTRAK	iii
ACKNOWLEDGEMENTS	v
APPROVAL	vi
DECLARATION	viii
LIST OF TABLES	xiii
LIST OF FIGURES	xiv
CHAPTER	
1 INTRODUCTION	1
1.1 Brief Introduction to thermal field theory	1
1.2 Problem Statement	2
1.3 Objectives of Research	3
1.4 Thesis Organization	4
2 LITERATURE REVIEW	5
2.1 Introduction	5
2.2 Statistical mechanics	9
2.2.1 Boltzmann-Gibbs statistical mechanics	11
2.2.2 Tsallis statistical mechanics	13
2.3 Quantum field theory	15
2.3.1 Quantum Chromodynamic	17
2.3.1.1 MIT-Model in QCD	19
2.3.1.2 Phase transition in QCD	20
2.3.1.3 Cross section at finite temperature	21
2.3.2 Quantum Electrodynamics	22
2.3.2.1 Effective Field Theory	22
3 THEORY AND METHODOLOGY	24
3.1 Introduction	24
3.2 The significance of Thermal Field Theory	24
3.2.1 Quantum Field Theory applied to functional contours	25
3.2.2 Imaginary-Time Contour	25
3.3 Green's Functions in thermal field theory	26
3.4 Thermal equilibrium: Real-Time Contour	29
3.4.1 Closed-Time Path formalism	29
3.4.2 ThermoField Dynamics formalism	35
3.4.2.1 Dirac field in ThermoField Dynamics	37
3.4.2.2 Real scalar field in ThermoField Dynamics	40
3.4.2.3 Tilde conjugation rules	42
3.4.2.4 Thermal propagator in TFD	43

4	GENERALIZATION OF THE TRANSFORMATION MATRIX IN THE REAL-TIME THERMAL FIELD THEORY	48
4.1	Introduction	48
4.2	Transformation Matrix in Real Time Formalism	48
4.2.1	Thermal propagator of scalar field with a chemical potential	51
4.2.2	Thermal propagator of fermion field with a chemical potential	53
4.3	Real Time Formalism Representation	56
4.4	Conclusions	58
5	REAL-TIME THERMAL SELF-ENERGIES: IN THE VARIATIONAL BASES AND SPACES	60
5.1	Introduction	60
5.2	The scalar propagator of an arbitrary path in the $1/2$ basis	60
5.2.1	The scalar propagator in the momentum space	60
5.2.2	The scalar propagator in the mixed space	62
5.3	The scalar propagator of an arbitrary path in the new basis	64
5.3.1	The scalar propagator in the momentum space	64
5.3.2	The scalar propagator in the mixed space	65
5.4	The self-energies in the $1/2$ basis	67
5.4.1	The self-energies for scalar tadpole in the momentum space	69
5.4.2	The self-energies for scalar tadpole in the mixed space	71
5.5	The self-energies in the new basis	72
5.5.1	The self-energies for scalar tadpole in the momentum space	72
5.5.2	The self-energies for scalar tadpole in the mixed space	74
5.6	Conclusions	75
6	REAL TIME THERMAL PHOTON-PHOTON INTERACTIONS: IN THE VARIATIONAL SPACE	77
6.1	Introduction	77
6.2	Effective theory of Euler-Heisenberg in Quantum Electrodynamics	77
6.3	The thermal propagator of boson field in the momentum space as well as the mixed space	82
6.4	The thermal self-energy of photons in the momentum space	83
6.5	The thermal self-energy of photons in the mixed space	85
6.6	Conclusion	92
7	THERMOFIELD DYNAMIC OF THE TSALLIS STATISTICS: AN APPLICATION	94
7.1	Introduction	94

7.2	Tsallis statistics in ThermoField Dynamics	95
7.3	Applications: scalar field and fermion field at finite temperature	101
7.3.1	The boson application	101
7.3.2	The fermion application	105
7.4	Conclusions	108
8	NON-EXTENSIVE ON PHASE TRANSITION FROM HADRONIC GAS TO PARTONIC PLASMA	112
8.1	Introduction	112
8.2	Partition Function of system	113
8.3	Finite Size QCD Thermodynamics and Equation Of State	115
8.3.1	The Importance of the EoS in the Hydrodynamical Description	115
8.3.2	Thermal Response Functions	116
8.4	Conclusion	126
9	CONCLUSION AND SUGGESTIONS	128
9.1	Conclusion	128
9.2	Suggestion for Improvement and Future Works	130
	REFERENCES	131
	APPENDICES	141
	BIODATA OF STUDENT	162
	LIST OF PUBLICATIONS	163

LIST OF TABLES

Table	Page
5.1 $D^{(\sigma,1/2)}$ for CTP and TFD formalisms in the momentum space.	63
5.2 $D^{(\sigma,1/2)}$ for CTP and TFD formalisms in the mixed space.	63
5.3 $\tilde{D}^{(\sigma,1/2)}$ for CTP and TFD formalisms in the momentum space.	66
5.4 $\tilde{D}^{(\sigma,1/2)}$ for CTP and TFD formalisms in the mixed space.	67
6.1 The thermal photon self-energy for each component in the momentum space.	84
6.2 The thermal photon self-energy for each component in the mixed space	86
8.1 $T_0(V)$ is the transition point temperature for different Tsallis parameter values with volume.	117

LIST OF FIGURES

Figure	Page
2.1 The diagram for Both Imaginary Time Formalism and Real Time Formalism	7
3.1 The closed time path contour that forms the basis of the in-in formalism.	31
3.2 The general time path contour in the complex t -plane with $0 \leq \beta \leq 1$, β denotes the inverse temperature in units of the Boltzmann constant. .	32
5.1 A flow chart of the thermal propagator in two different bases as well as various spaces.	68
5.2 The tadpole diagram: The green blob denotes the effective vertex.	68
5.3 The tadpole self energies $\Pi(\sigma, (12, 21))$ for RTF in the momentum space.	70
5.4 The tadpole self energies. (a) $\Pi(\sigma=0, (11, 12, 21, 22))$ for CTP. (b) $\Pi(\sigma=\frac{1}{2}, (11, 12, 21, 22))$ for TFD.	70
5.5 The tadpole self energies $\tilde{\Pi}(\sigma, (11, 22))$ for RTF in the mixed space.	72
5.6 The tadpole self energies. (a) $\tilde{\Pi}(\sigma=0, 22)$ for CTP. (b) $\tilde{\Pi}(\sigma=\frac{1}{2}, 11, 22)$ for TFD .	75
6.1 $e^-e^+ \rightarrow$ photon-photon scattering process in Quantum Electrodynamics.	79
6.2 Tadpole diagram: The red blob denotes the effective photon vertex (retarded vertice)	80
6.3 (a) The 3-Dim plot of $\omega(t; T)$ versus temperature and time. (b) The 3-Dim plot of $\Omega(t; T)$ versus temperature and time.	87
6.4 (a) The plot of $\omega(t; T)$ versus temperature and when $t = (0, 1, 2.5, 5, 10, 12) \times 10^{-2}$. (b) The plot of $\omega(t; T)$ versus temperature and when $t = (0.5, 1, 1.5, 2, 2.5, 3)$	88

- 6.5 The plot of $t_{min}(T)$ versus temperature with the fit $t_{min}(T) = 5.73973 T^{-1}$. 88
- 7.1 (a). The graphs of the thermal function $\omega_q(T)$ versus T and q .
 (b). The graphs of the thermal function ω_q versus T and when $q = (1.1, 1.15, 1.2 \text{ and } 1.4)$. 103
- 7.2 The graphs of the thermal function $\Omega_q(T)$ versus T and when $q = (1.1, 1.15, 1.2 \text{ and } 1.4)$. 104
- 7.3 The graphs of the thermal function $R_q(T)$ versus temperature T and Tsallis parameter q : when $m_\gamma = \mu = 0$. (a) $\sqrt{s} = 0.8 \text{ GeV}$. (b) $\sqrt{s} = 1.2 \text{ GeV}$. (c) $\sqrt{s} = 1.6 \text{ GeV}$ and (d) $\sqrt{s} = 2 \text{ GeV}$. 110
- 7.4 The graphs of the thermal function $R_q(T)$ versus temperature T and Tsallis parameter q : when $m_\gamma = 0$ and $\mu = 400 \text{ MeV}$. (a) $\sqrt{s} = 0.8 \text{ GeV}$. (b) $\sqrt{s} = 1.2 \text{ GeV}$. (c) $\sqrt{s} = 1.6 \text{ GeV}$ and (d) $\sqrt{s} = 2 \text{ GeV}$. 111
- 8.1 $H_q(V = 100 \text{ fm}^3, T)$ is the order parameter of the system as a function of temperature. 117
- 8.2 (a) The order parameter versus temperature, when $q = 1.01$ with volumes $V = (0.1, 0.25, 0.5, 1)(10 \text{ fm})^3$. (b) The order parameter versus temperature, when $V = (10 \text{ fm})^3$ with set Tsallis parameter $q = (1.01, 1.04, 1.15, 1.8, 1.9)$. 118
- 8.3 3-Dim normalized energy density $\frac{\epsilon_q(T)}{T^4}$ of the system as a function of temperature, when volume $V = 100 \text{ fm}^3$. 118
- 8.4 (a) The energy density versus temperature T , when $q = 1.01$ with set volumes $V = (0.1, 0.25, 0.5, 1)(10 \text{ fm})^3$. (b) The energy density versus temperature, when $V = (10 \text{ fm})^3$ with Tsallis parameter $q = (1.01, 1.04, 1.07, 1.1)$. 119
- 8.5 3-Dim normalized pressure $\frac{p_q(T)}{T^4}$ of the system as a function of (T) , when the volume $V = 100 \text{ fm}^3$. 120
- 8.6 (a) The normalized pressure versus temperature, when $q = 1.01$ with set volumes $V = (0.25, 0.5, 0.75, 1)(10 \text{ fm})^3$. (b) The normalized pressure versus temperature, when $V = (10 \text{ fm})^3$ with Tsallis parameter $q = (1.01, 1.04, 1.07, 1.1)$. 120

- 8.7 3-Dim normalized Entropy density $\frac{s_q(T)}{T^3}$ of the system as a function of temperature, when volume $V = 100fm^3$. 121
- 8.8 (a) The Entropy density versus temperature, when $q = 1.01$ with set volumes $V = (1, 0.25, 0.5, 1)(10fm)^3$. (b) The Entropy density versus temperature, when $V = (10fm)^3$ with Tsallis parameter $q = (1.01, 1.04, 1.07, 1.1)$. 122
- 8.9 3-Dim trace anomaly $\frac{\theta_q(T)}{T^4}$ of the system as a function of (T, V) , when the volume $V = 100fm^3$. 123
- 8.10 (a) The trace anomaly versus temperature, when $q = 1.01$ with set volumes $V = (0.1, 0.5, 0.75, 1)(10fm)^3$. (b) The trace anomaly versus temperature, when $V = (10fm)^3$ with Tsallis parameter $q = (1.01, 1.04, 1.07, 1.1)$. 124
- 8.11 3-Dim Plot of $\frac{p_q}{\epsilon_q}(T; V)$ of the system as a function of $(V; T)$, when the volume $V = (10fm)^3$. 124
- 8.12 (a) Plot of $\frac{p_q}{\epsilon_q}(V; T)$ versus temperature, when $q = 1.01$ with set volumes $V = (0.25, 0.5, 0.75, 1)(10fm)^3$. (b) The Plot of $\frac{p_q}{\epsilon_q}(V; T)$ versus temperature, when $V = (10fm)^3$ with Tsallis parameter $q = (1.01, 1.04, 1.07, 1.1)$. 125
- 8.13 Sound velocity $c_s^2(V = (10fm)^3; T)$ vs temperature for different values of q . 126

CHAPTER 1

INTRODUCTION

1.1 Brief Introduction to thermal field theory

In Quantum Field Theory (QFT) at zero temperature is often assumed to ignore its computations in this level of energy. This is because, under typical conditions, thermal energy is too small to excite virtual particles; for example, at ambient temperature, $T = 273K$, thermal energy is just 25×10^{-3} eV, despite the fact that an electron's rest energy is 511 keV. However, given the incredibly high temperatures that dominate the early cosmos, their contributions cannot be ignored. This will be critical for the mechanism of particle creation. Thermal field theory will be used to investigate these contributions. To describe a Thermal Field Theory (TFT), there are two commonly used real time formalisms (RTF). The first is the Schwinger-Keldysh formalism (also known as the closed time path formalism (CTP)). Secondly, an operator formalism, called ThermoField Dynamics (TFD), was suggested by Takahashi and Umezawa. Both approaches are tailor-made for calculating Green functions (Schwinger (1961); Keldysh (1965); Takahashi and Umezawa (1996)). While the complete theory can be reconstructed from Green functions in principle, for practical and theoretical reasons, an operator formulation of *TFT* may be useful. In particular, it could illustrate why it is important to double the degrees of freedom for the real time formalism, and the standard Green's function is replaced by four Green's functions with arguments on two different parts of the corresponding integration contour represented by 2×2 matrix in both formalisms CTP and TFD. However, in contrast to TFD, there are no mutually commuting representations of the basic operator algebra. The RTF represents the propagators of fermionic and scalar bosonic fields in matrix form. So, a close comparison between the CTP and TFD can be made. We find that TFD and CTP are in many ways the same in form; in particular, the two approaches are identical in stationary situations. However, TFD and CTP are quite different in time-dependent out of equilibrium situations. The main source of this difference is that the time evolution of the density matrix itself is ignored in CTP while in TFD it is replaced by a time-dependent Bogoliubov transformation (BT). The BT constructed which connects the doubling of field degrees of freedom in the RTF of the non thermal vacuum state to the thermal vacuum in this case, leads to Green's function at finite temperature "propagator" as an expectation value in the thermal vacuum. The RTF can be applied to describe the systems in thermal equilibrium, and also on an extension to describe nonequilibrium situations. As well in RTF, the number of independent fields doubles as mentioned earlier. Instead of a single scalar field ϕ , we encounter two scalar fields, ϕ_1 and ϕ_2 , called type-1 and type-2 fields, respectively. In other words, we have two types of vertex now, where each type has only fields that emerge from it and has its usual value, e.g. the four-particle vertex, fields of type-2 are not mixed with fields of type-1. Because of the anti-time ordering, the vertex involving fields of type-2 has a relative minus sign. The propagator at finite temperature is given in many works for the momentum space. In

TFT, we can use Fourier transformed to represent the propagator in the mixed space at finite temperature. Fourier transforming these elements of the propagator in the energy variable, this transformation leads to obtaining the propagator for the general contour, in the mixed space. The transformed propagator is a function of time and the spatial component of momentum. Such a (mixed space) representation is quite helpful to study some graphs at finite temperature.

In RTF, the elements of the propagator matrix are usually given in the momentum space. It is simple to check that the basic finite temperature propagator factorizes again by Fourier transforming this to mixed space. What is more impressive is that although the propagator is a matrix, each element of the matrix can be factorised by the same thermal operator (Das (2006); Das et al. (2018)). In this regard, the first element of the propagator consists of the sum of two parts; the first one is a non-thermal part, and the second is the thermal part, and the integration is taken over a real, continuous, energy p_0 with the presence of the delta function. The finite temperature contribution is trivial to obtain, thus suggesting the possibility of a “real time” perturbation theory (Kapusta and Gale (2006)). It is interesting that perturbation theory at finite temperature can be formulated directly in both RTF and ITF. The RTF will be adopted in this thesis. In RTF, we usually use the Boltzmann-Gibbs (BG) statistics to study condensed matter phenomena like hot and dense plasmas, then early universe, many-body physics and so on. As we know, the Gaussian distribution is signature of the BG statistical mechanics (Tsallis (1988)). There are, however, quasi-stationary states related to the q -Gaussian distributions (Tsallis (2009)). Analogous to Gaussian distribution optimize BG-equilibrium states, q -Gaussians can play roughly the same role in Tsallis statistics. Moreover, we can obtain BG statistics from Tsallis statistics, when the parameter q goes to the unity, so we can say BG statistics is a particular instance of Tsallis statistics.

1.2 Problem Statement

One of the main issues of real time thermal field theory is the many-body physics. The problem here is to determine the thermal propagator of the system. There are various approaches to study the thermal effect. Some researchers compute the one-loop in scalar QED for the photon in the momentum space and the $1/2$ basis. On the other side, it is not easy to study the scalar propagator and the fermion propagator at finite temperature for Tsallis statistics. This motivates us to find a generalization of the thermal matrix, study the photon self-energy in the mixed space in a new basis, and construct ThermoField Dynamic with Tsallis statistics and introduce some applications. In many-body physics, the TFT has been used to describe response functions, namely the mean value of physical observables such as energy, free energy, entropy, and so on, which will be completely parallel to the TFT at zero temperature. This necessitates the development of a method for calculating physical mean values. As a result, we should employ propagators at finite temperature in RTF. In the Real Time Thermal Field Theory, there is a one-parameter family of paths in the complex

t. This parameter σ has two popular choices appearing in the Feynman rules for the elements of the propagator in momentum space (Schwinger (1961); Keldysh (1965); Takahashi and Umezawa (1996); Gozzi and Penco (2011); Das and Kalauni (2016)). To construct the thermal propagator in the mixed space we use Fourier transformed to represent the thermal propagator in the mixed Das (2006); Das et al. (2018), which is of our interest here. In our research, we are interested in the transformations matrix at finite temperature for the general path (Xu (1996)). We introduce a very convenient representation, which is constructed from linear combinations of the components of the RTF, for both scalar and fermion fields. The results are represented the propagators with the chemical potential in both spaces momentum and mixed (Lundberg and Pasechnik (2021)). We next construct the thermal scalar propagator and a tadpole self-energy in two different bases in the momentum space (Chou et al. (1985); Landsman and van Weert (1987); Smilga (1997); Ghiglieria et al. (2020)) as well as the mixed space (Das (2006); Das et al. (2018)). Then, we proceed to study the photon self-energies for effective photon vertex, e.g. the tadpole diagram of photons for the four-photon vertex within QED in one basis. In addition, there are some research has emerged that studied the TFT of the Tsallis statistics (Rahaman et al. (2021)) in real time. This is our technique to investigate the thermodynamic variables of a hot and dense system in TFD with Tsallis statistics, for which we will construct the propagator in TFD with Tsallis statistics to study the thermal photon-photon interaction within QED. We will also study the phase of e^+e^- annihilation into hadrons within QCD (Thoma (2000b); Chekerker et al. (2011)). This is relevant to the Deconfinement Phase Transition from a hadronic matter to a QGP (Bhattacharyya and Mukherjee (2020)).

1.3 Objectives of Research

This work aims to understand better the thermal effect in many-body physics governed by quantum electrodynamics and quantum chromodynamics. The real time thermal field theory is used to calculate some physical quantities describing these systems at a finite temperature, unifying in equilibrium and out of equilibrium processes due to the doubling of the degrees of freedom. This method allows us to study the interactions of scalar and fermion fields in the momentum space as well as the mixed space. Overall, the objectives of this research are as follows:

1. To generalize the thermal matrix transformation in RTF.
2. To study the thermal self energies in the momentum space as well as the mixed space for two different bases in RTF.
3. To compute the thermal self-energy of photons in the momentum space as well as the mixed space in the new basis.
4. To construct the propagator in TFD with Tsallis statistics.
5. To use TFD with TS to study the phase of e^+e^- annihilation into hadrons in QCD, and Deconfinement Phase Transition from a HG to a QGP.

1.4 Thesis Organization

The thesis comprises of nine chapters, and we briefly mention the layout of this thesis as follows.

Chapter 1 gives a brief introduction, the problem statement, and the objectives of the current research.

Chapter 2 includes an overview of relevant literature

Chapter 3 contains some definitions and relevant background information related to the research conducted and the method applied.

Chapter 4 gives transformations matrix at finite-temperature in RTF with an arbitrary parameter σ . The thermal propagator is presented for both scalar and fermion fields with the chemical potential in both momentum and mixed spaces.

Chapter 5 introduces a systematic study for studying the scalar propagator and tadpole self-energy by considering an arbitrary parameter σ that allows for a path integral description in RTF. Next, the scalar propagator and a tadpole self-energy are constructed in two different bases in the momentum space and the mixed space.

Chapter 6 we used the effective Lagrangian at a low temperature to study photon-photon interaction in RTF with an arbitrary path σ within the new basis. The thermal scalar propagator in the mixed space without chemical potential is introduced. We then calculate some electromagnetic properties, such as dielectric tensor and velocity of light from photon self-energy in the mixed space.

Chapter 7 presents the thermal propagator in ThermoField Dynamic with Tsallis statistics for scalar field and fermion field. We then study two applications: the first one in QED at low temperature and the second in QCD at high temperature.

Chapter 8 we give the distribution of particle in ThermoField Dynamic of the Tsallis statistics for scalar field and fermion field to study the Deconfinement Phase Transition from a HG to a QGP. We investigate the behaviour of some thermodynamic quantities of the system, such as the energy density, the pressure and the interaction measure, etc.

Chapter 9 provides the conclusion of this work. We also make a few suggestions here for future works.

REFERENCES

- Alberico, W. M., Lavagno, A., and Quarati, P. (2000). Non-extensive statistics, fluctuations and correlations in high-energy nuclear collisions. *The European Physical Journal C*, 12(3):499–506.
- Amelino-Camelia, G., Ellis, J., Mavromatos, N., Nanopoulos, D., and Sarkar, S. (1998). Tests of quantum gravity from observations of gamma-ray bursts. *Nature*, 393:763–765.
- Aurenche, P. and Becherrawy, T. (1992). A comparison of the real-time and the imaginary-time formalisms of finite-temperature field theory for 2, 3 and 4-point green functions. *Nuclear Physics B*, 379(1):259–303.
- Azmi, M. and Cleymans, J. (2015). The Tsallis distribution at large transverse momenta. *The European Physical Journal C*, 75(9):1–5.
- Baier, R., Pire, B., and Schiff, D. (1988a). Dilepton production at finite temperature: Perturbative treatment at order α_s . *Physical Review D*, 38(9):2814.
- Baier, R., Pire, B., and Schiff, D. (1988b). Perturbative aspects of dilepton production at finite temperature. *Zeitschrift für Physik C*, 38(1):265–267.
- Balian, R. (2007). *From Microphysics to Macrophysics: Methods and Applications of Statistical Physics*. Theoretical and Mathematical Physics. Springer-Verlag Berlin Heidelberg.
- Barter, C., Blaschke, D., and Voss, H. (1992). Thermodynamics of quark matter with saturated confinement interactions. *Physics Letter B*, 293:423–429.
- Barton, G. (1990). Faster-than-c light between parallel mirrors. the scharnhorst effect rederived. *Physics Letters B*, 237(3):559–562.
- Beck, C. (2000). Non-extensive statistical mechanics and particle spectra in elementary interactions. *Physica A: Statistical Mechanics and its Applications*, 286(1):164–180.
- Bediaga, I., Curado, E., and de Miranda, J. (2000). A non-extensive thermodynamical equilibrium approach in $e^+e^- \rightarrow$ hadrons. *Physica A: Statistical Mechanics and its Applications*, 286(1):156–163.
- Ben-Menahem, S. (1990). Causality between conducting plates. *Physics Letters B*, 250(1):133–138.
- Bermúdez Manjarres, A., Kelkar, N., and Nowakowski, M. (2017). Electric fields at finite temperature. *Annals of Physics*, 386:58–75.
- Bethke, S. (1998). Jet physics at lep and world summary of α_s . *arXiv preprint hep-ex/9812026*.
- Bethke, S. (2000). Standard model physics at lep. In *Particle Production Spanning MeV and TeV Energies*, pages 385–428. Springer.

- Bezzerides, B. and DuBois, D. F. (1968). Many-body theory for quantum kinetic equations. *Physical Review Journals Archive.*, 168:233–248.
- Bhattacharyya, T. and Mukherjee, A. (2020). Propagation of non-linear waves in hot, ideal, and non-extensive quark–gluon plasma. *The European Physical Journal C*, 80(7):1–9.
- Blasone, M., Jizba, P., and Luciano, G. (2018). Unified formalism for thermal quantum field theories: a geometric viewpoint. *Annals of Physics.*, 397:213–233.
- Boltzmann, L. (1877). Sitzungsberichte der kaiserlichen akademie der wissenschaften in wien. *Mathematisch-Naturwissenschaftliche Classe*, 76:373–435.
- Bose, S. (1924). Planck’s law and light quantum hypothesis. *Zeitschrift für Physik*, 26:178–181.
- Brambilla, N., Eidelman, S., Hanhart, C., Nefediev, A., Shen, C. P., Thomas, C. E., Vairo, A., and Yuan, C. Z. (2020). The xyz states: experimental and theoretical status and perspectives. *Physics Reports*, 873:1–154.
- Brandt, F. T., Das, A., Espinosa, O., Frenkel, J., and Perez, S. (2005). Thermal operator representation of finite temperature graphs. *Physical Review. D*, 72:085006.
- Brandt, F. T., Das, A., Espinosa, O., Frenkel, J., and Perez, S. (2006). Thermal operator representation of finite temperature graphs. II. *Physical Review. D*, 73:065010–1–065010–13.
- Büyükkiliç, F. and Demirham, D. (1993). A fractal approach to entropy and distribution functions. *Physics Letters A*, 181(1):24–28.
- C. E. Lee, Y. W. Yang, W. F. T. (1984). Hadronization in the e^+e^- annihilation: The model. *Chinese Journal of Physics*, 22(3):19–30.
- Capolupo, A., De Martino, I., Lambiase, G., and Stabile, A. (2019). Axion-photon mixing in quantum field theory and vacuum energy. *Physics Letters B*, 790:427–435.
- Carignano, S., Manuel, C., and Soto, J. (2018). Power corrections to the HTL effective Lagrangian of QED. *Physics Letters B*, 780:308–312.
- Carrington, M. E., Defu, H., and Thoma, M. H. (1999). Equilibrium and non-equilibrium hard thermal loop resummation in the real time formalism. *The European Physical Journal C*, 7:347–354.
- Cerqueti, R., Rotundo, G., and Ausloos, M. (2020). Tsallis entropy for cross-shareholding network configurations. *Entropy*, 22(6).
- Chekerker, M., Ladrem, M., Khanna, F., and Santana, A. (2011). Thermofield dynamics and e^+e^- reactions. *International Journal of Modern Physics A*, 26:2881–2897.

- Cheng, M., Christ, N. H., Datta, S., van der Heide, J., Jung, C., Karsch, F., Kaczmarek, O., Laermann, E., Mawhinney, R. D., Miao, C., Petreczky, P., Petrov, K., Schmidt, C., Soeldner, W., and Umeda, T. (2008). QCD equation of state with almost physical quark masses. *Physical Review D*, 77:014511.
- Cherif, S., Ladrem, M. L. H., Alfull, Z. Z. M., Alharbi, R. M., and Ahmed, M. A. A. (2021). Finite-size effects and finite-size scaling in time evolution during a colorless confining phase transition. *Physica Scripta*, 96(10):105302.
- Chodos, A., Jaffe, R., Johnson, K., and Thorn, C. B. (1974a). Baryon structure in the bag theory. *Physical Review D*, 10(8):2599.
- Chodos, A., Jaffe, R., Johnson, K., Thorn, C. B., and Weisskopf, V. (1974b). New extended model of hadrons. *Physical Review D*, 9(12):3471.
- Chou, K., Su, Z., Hao, B., and Yu, L. (1985). Equilibrium and nonequilibrium formalisms made unified. *Physics Reports*, 118(1):1–131.
- Cleymans, J., Gavai, R., and Suhonen, E. (1986). Quarks and gluons at high energies and temperatures. *Physics Reports*, 130:217–292.
- Cleymans, J. and Worku, D. (2012). The Tsallis distribution in proton-proton collisions at $\sqrt{s}=0.9$ TeV at the LHC. *Journal of Physics G: Nuclear and Particle Physics*, 39(2):025006.
- Cleymans, J. and Worku, D. (2012). Relativistic thermodynamics: Transverse momentum distributions in high-energy physics. *The European Physical Journal A*, 48(11):160.
- Cohen-Tannoudji, C. (2015). *Atomic and molecular physics*. Leçons inaugurales. Collège de France.
- Colangelo, P. and Khodjamirian, A. (2001). QCD sum rules, a modern perspective. In *The Frontier of Particle Physics: Handbook of QCD (in 3 Volumes)*, pages 1495–1576. World Scientific.
- Conroy, J., Miller, H., and Plastino, A. (2010). Thermodynamic consistency of the q-deformed Fermi–Dirac distribution in non-extensive thermostatics. *Physics Letters A*, 374(45):4581–4584.
- Conroy, J. M. and Miller, H. (2008). Color superconductivity and Tsallis statistics. *Physical Review D*, 78(5):054010.
- Cruz, C. N. and da Silva, F. A. (2018). Variation of the speed of light and a minimum speed in the scenario of an inflationary universe with accelerated expansion. *Physics of the Dark Universe*, 22:127–136.
- Daniels, R. and Shore, G. (1994). 'Faster than light' photons and charged black holes. *Nuclear Physics B*, 425(3):634–650.
- Das, A. (1997). *Finite Temperature Field Theory*. World Scientific.

- Das, A. (2006). Thermal operator representation of Feynman graphs. *Brazilian journal of physics*, 36(4A):1130–1136.
- Das, A., Deshamukhya, A., Kalauni, P., and Panda, S. (2018). Bogoliubov transformation and the thermal operator representation in the real time formalism. *Physical Review D*, 97(4):045015.
- Das, A. and Kalauni, P. (2016). Operator description for thermal quantum field theories on an arbitrary path in the real time formalism. *Physical Review. D*, 93:125028.
- De, B. (2014). Non-extensive statistics and understanding particle production and kinetic freeze-out process from p T-spectra at 2.76 TeV. *The European Physical Journal A*, 50(9):1–11.
- DeGrand, T., Jaffe, R., Johnson, K., and Kiskis, J. (1975). Masses and other parameters of the light hadrons. *Physical Review D*, 12(7):2060.
- Deppman, A. (2012). Self-consistency in non-extensive thermodynamics of highly excited hadronic states. *Physica A: Statistical Mechanics and its Applications*, 391(24):6380–6385.
- Dicus, D. A., Kao, C., and Repko, W. W. (1998). Effective lagrangians and low energy photon-photon scattering. *Physical Review. D*, 57:2443–2447.
- Dirac, P. A. M. (1926). On the theory of quantum mechanics. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 112:661–671.
- Dittrich, W. (2019). The role of Riemann’s Zeta function in mathematics and physics. *Universe*, 5.
- Drummond, I. T. and Hathrell, S. J. (1980). QED vacuum polarization in a background gravitational field and its effect on the velocity of photons. *Physical. Review. D*, 22:343–355.
- Dunne, G. V. (2012). The Heisenberg-Euler effective action: 75 years on. *International Journal of Modern Physics A*, 27:1260004.
- Eidelman, S. I. (2003). Recent results from $e^+e^- \rightarrow$ hadrons. *Nuclear Physics B - Proceedings Supplements*, 123:186–192. Proceedings of the Seventh International Workshop on Tau Lepton Physics.
- Einstein, A. (1924). Quantum theory of the monatomic ideal gas. *Preußische Akademie der Wissenschaften*, 14:261–267.
- Ejiri, S., Karsch, F., Laermann, E., and Schmidt, C. (2006). Isentropic equation of state of 2-flavor QCD. *Physical Review D*, 73:054506.
- Elze, H.-T. and Heinz, U. (1982). *Quark-gluon transport theory*. World Scientific.
- Emelyanov, S. (2017). Effective photon mass from black-hole formation. *Nuclear Physics B*, 919:110–122.

- Erdélyi, A., Magnus, W., Oberhettinger, F., and Tricomi, F. (1955). *Higher Transcendental Functions*. McGraw-Hill.
- Espinosa, O. and Stockmeyer, E. (2004). Operator representation for Matsubara sums. *Physical Review D*, 69(6):065004.
- Euler, H. and Kockel, B. (1935). The scattering of light by light in Dirac's theory. *Naturwiss.*, 23(15):246–247.
- Evans, T. and Steer, D. (1996). Wick's theorem at finite temperature. *Nuclear Physics B*, 474(2):481–496.
- Fazarinc, Z. (2015). Fermi–Dirac, Bose–Einstein, Maxwell–Boltzmann, and computers. *Computer Applications in Engineering Education*, 23(5):746–759.
- Feinberg, E. (1976). Direct production of photons and dileptons in thermodynamical models of multiple hadron production. *Il Nuovo Cimento A*, 34(3):391–412.
- Fermi, E. (1926). Zur quantelung des idealen einatomigen gases. *Zeitschrift für Physik*, 36:902–912.
- Fetter, A. and Walecka, J. (1971). *Quantum Theory of Many-Particle Systems*. McGraw-Hill.
- Feynman, R. P. (1982). *Statistical Mechanics: A Set of Lectures*. Frontiers in physics. Benjamin Cummings.
- Fixsen, D. J. (2009). The temperature of the cosmic microwave background. *The Astrophysical Journal*, 707:916–920.
- Gasser, J. and Leutwyler, H. (1984). Chiral perturbation theory to one loop. *Annals of Physics*, 158(1):142–210.
- Gasser, J. and Leutwyler, H. (1985). Chiral perturbation theory: expansions in the mass of the strange quark. *Nuclear Physics B*, 250(1-4):465–516.
- Gawiser, E. and Silk, J. (2000). The cosmic microwave background radiation. *Physics Reports*, 333:245–267.
- Ghiglieria, J., Kurkelab, A., Strickland, M., and Vuorinen, A. (2020). Perturbative thermal QCD: Formalism and applications. *Physics Reports*, 880:1–73.
- Ghosh, S. K., Mukherjee, T. K., Mustafa, M. G., and Ray, R. (2006). Susceptibilities and speed of sound from the Polyakov–Nambu–Jona-Lasinio model. *Physical Review D*, 73:114007.
- Gibbs, J. W. (2010). *Elementary Principles in Statistical Mechanics: Developed with Especial Reference to the Rational Foundation of Thermodynamics*. Cambridge University Press.
- Gozzi, E. and Penco, R. (2011). Three approaches to classical thermal field theory. *Annals of Physics*, 326:876–910.

- Greiner, W. and Reinhardt, J. (2008). *Quantum Electrodynamics*. Springer Publishing Company.
- Gross, D. J. and Wilczek, F. (1973). Ultraviolet behavior of non-abelian gauge theories. *Physical Review Letter*, 30:1343–1346.
- Grozin, A. (2020). Effective field theories. *Particles*, 3(2):245–271.
- Halter, J. (1993). An effective lagrangian for photons. *Physics Letters B*, 316:155–157.
- Hasegawa, H. (2009). Bose-Einstein and Fermi-Dirac distributions in non-extensive quantum statistics: Exact and interpolation approaches. *Physical Review E*, 80(1):011126.
- Hatsuda, T. and Kunihiro, T. (1994). QCD phenomenology based on a chiral effective lagrangian. *Physics Reports*, 247(5-6):221–367.
- Heisenberg, W. and Euler, H. (1936). Folgerungen aus der Diracschen theorie des positrons. *Zeitschrift für Physik*, 98:714–73.
- Henning, P. A. and Umezawa, H. (1994). Diagonalization of propagators in thermo field dynamics for relativistic quantum fields. *Nuclear Physics B*, 417:463–505.
- Hou, S. Q., He, J. J., Parikh, A., Kahl, D., Bertulani, C. A., Kajino, T., Mathews, G. J., and Zhao, G. (2017). Non-extensive statistics to the cosmological lithium problem. *The Astrophysical Journal*, 834(2):165.
- Itzykson, C. and Zuber, J. B. (1980). *Quantum Field Theory*. International Series In Pure and Applied Physics. McGraw-Hill, New York.
- Jaffe, R. (1977). Multiquark hadrons. I. Phenomenology of $Q^2\bar{Q}^2$ mesons. *Physical Review D*, 15(1):267.
- Jenkins, E., Manohar, A., and Stoffer, P. (2018). Low-energy effective field theory below the electroweak scale: Anomalous dimensions. *Journal of High Energy Physics*, 2018:84.
- Kapusta, J. I. and Gale, C. (2006). *Finite-Temperature Field Theory: Principles and Applications*. Cambridge University Press, 2nd edition.
- Keldysh, L. (1965). Diagram technique for nonequilibrium processes. *Journal of Experimental and Theoretical Physics*, 20:1018–1026.
- Khanna, F. C., Malbouisson, A. P., Malbouisson, J. M., and Santana, A. E. (2014). Quantum field theory on toroidal topology: Algebraic structure and applications. *Physics Reports*, 539:135–224.
- Khanna, F. C., Malbouisson, A. P. C., Malbouisson, J. M. C., and Santana, A. E. (2009). *Thermal Quantum Field Theory*. World Scientific.
- Khinchin, A. (1949). *Mathematical Foundations of Statistical Mechanics*. Dover.

- Khuntia, A., Sahoo, P., Garg, P., Sahoo, R., and Cleymans, J. (2016). Speed of sound in hadronic matter using non-extensive tsallis statistics. *The European Physical Journal A*, 52(9):292.
- Kobes, R. L. and Semenoff, G. W. (1985). Discontinuities of green functions in field theory at finite temperature and density. *Nuclear Physics B*, 260(3):714–746.
- Kondo, K.-i. and Yoshida, K. (1995). Finite-temperature and finite-density QED: the Schwinger-Dyson equation in the real-time formalism. *International Journal of Modern Physics A*, 10(02):199–232.
- Ladrem, M. L. H., Ahmed, M. A. A., Cherif, S., Alfull, Z. Z. M., and Almarashi, M. M. (2019). Detailed study of the QCD equation of state of a colorless partonic plasma in finite volume. *International Journal of Modern Physics A*, 34(09):1950051.
- Landsman, N. (1986). Consistent real-time propagators for any spin, mass, temperature and density. *Physics Letters B*, 172(1):46–48.
- Landsman, N. and van Weert, C. (1987). Real- and imaginary-time field theory at finite temperature and density. *Physics Reports*, 145(3):141–249.
- Lavagno, A. and Quarati, P. (2001). Solar reaction rates, non-extensivity and quantum uncertainty. *Physics Letters B*, 498(1-2):47–52.
- Le Bellac, M. (1996). *Thermal Field Theory*. Cambridge Monographs on Mathematical Physics. Cambridge University Press.
- Lee, T.-D. and Yang, C.-N. (1952). Statistical theory of equations of state and phase transitions. II. Lattice gas and Ising model. *Physical Review Journals Archive*, 87(3):410.
- Lima, J. A. S. and Deppman, A. (2020). Tsallis meets boltzmann: q -index for a finite ideal gas and its thermodynamic limit. *Physical Review E*, 101(4):040102.
- Lundberg, T. and Pasechnik, R. (2021). Thermal field theory in real-time formalism: concepts and applications for particle decays. *The European Physical Journal A*, 57:1–57.
- Majhi, B. R. and Vagenas, E. C. (2013). Modified dispersion relation, photon's velocity, and Unruh effect. *Physics Letters B*, 725(4):477–480.
- Marques, L., Andrade-II, E., and Deppman, A. (2013). Nonextensivity of hadronic systems. *Physical Review D*, 87(11):114022.
- Marques, L., Cleymans, J., and Deppman, A. (2015). Description of high-energy pp collisions using Tsallis thermodynamics: Transverse momentum and rapidity distributions. *Physical Review D*, 91(5):054025.
- Matone, M., Pasti, P., Shadchin, S., and Volpato, R. (2006). Compactified strings as quantum statistical partition function on the Jacobian torus. *Physical Review Letter*, 97:261601.

- Matrasulov, D., Butanov, K. T., Rakhimov, K., Khanna, F., et al. (2009). Spectra of heavy-light mesons at finite temperature. *arXiv preprint arXiv:0909.2399*.
- Matsubara, T. (1955). A new approach to quantum-statistical mechanics. *Progress of theoretical physics*, 14:351–378.
- Matsumoto, H., Nakano, Y., and Umezawa, H. (1984). An equivalence class of quantum field theories at finite temperature. *Journal of Mathematical Physics*, 25:3076–3085.
- Mayer, J. E. (1937). The statistical mechanics of condensing systems. I. *The Journal of chemical physics*, 5(1):67–73.
- Mayer, J. E. and Ackermann, P. G. (1937). The statistical mechanics of condensing systems. II. *The Journal of Chemical Physics*, 5(1):74–83.
- Mayer, J. E. and Harrison, S. F. (1938). Statistical mechanics of condensing systems. III. *The Journal of Chemical Physics*, 6(2):87–100.
- Mayer, J. E. and Mayer, M. G. (1940). *Statistical Mechanics*. John Wiley and Sons.
- McLerran, L. D. and Toimela, T. (1985). Photon and dilepton emission from the quark-gluon plasma: Some general considerations. *Physical Review D*, 31:545–563.
- Mitra, S. (2018). Thermodynamics and relativistic kinetic theory for q-generalized Bose–Einstein and Fermi–Dirac systems. *The European Physical Journal C*, 78(1):1–15.
- Nambu, Y. and Jona-Lasinio, G. (1961a). Dynamical model of elementary particles based on an analogy with superconductivity. i. *Physical Review*, 122:345–358.
- Nambu, Y. and Jona-Lasinio, G. (1961b). Dynamical model of elementary particles based on an analogy with superconductivity. I. *Physical Review Journals Archive*, 122:345–358.
- Nambu, Y. and Jona-Lasinio, G. (1961c). Dynamical model of elementary particles based on an analogy with superconductivity. II. *Physical Review Journals Archive*, 124:246–254.
- Neumann, J. v. (1927). Mathematische begründung der quantenmechanik. *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse*, 1927:1–57.
- Niemi, A. and Semenoff, G. (1984). Finite-temperature quantum field theory in Minkowski space. *Annals of Physics*, 152(1):105–129.
- Pasechnik, R. and Šumbera, M. (2017). Phenomenological review on quark-gluon plasma: Concepts vs. observations. *Universe*, 3(1).
- Pennini, F., Plastino, A., and Plastino, A. (1995). Tsallis entropy and quantal distribution functions. *Physics Letters A*, 208(4-6):309–314.

- Peskin, M. E. and Schroeder, D. V. (1995). *An Introduction to quantum field theory*. Addison-Wesley.
- Petrov, A. A. and Blechman, A. E. (2016). *Effective Field Theories*. World Scientific.
- Petzold, A. (2008). Hadronic final states in $e^+ e^-$ annihilation at BaBar. *Journal of Physics: Conference Series*, 110(2):022041.
- Politzer, H. D. (1973). Reliable perturbative results for strong interactions? *Physical Review Letter*, 30:1346–1349.
- Prudêncio, T., Filho, T. M. R., and Santana, A. E. (2020). Corrigendum: Quantum teleportation of thermofields. *Physica Scripta*, 95:069501.
- Rahaman, M., Bhattacharyya, T., and Alam, J.-e. (2021). Phenomenological tsallis distribution from thermal field theory. *International Journal of Modern Physics A*, 36(20):2150154.
- Rajagopal, A. K., Mendes, R. S., and Lenzi, E. K. (1998). Quantum statistical mechanics for nonextensive systems: Prediction for possible experimental tests. *Physical Review Letter*, 80(18):3907–3910.
- Rajagopal, K. and Wilczek, F. (2001). The condensed matter physics of QCD. *At The Frontier of Particle Physics*, pages 2061–2151.
- Rakhimov, A. and Khanna, F. (2001). Finite temperature amplitudes and reaction rates in thermofield dynamics. *Physical Review C*, 64(6):064907.
- Ramezani, Z. and Pourdarvish, A. (2021). Transfer learning using Tsallis entropy: An application to Gravity Spy. *Physica A: Statistical Mechanics and its Applications*, 561(C).
- Ruelle, D. (1999). *Statistical mechanics: Rigorous results*. World Scientific.
- Rybczynski, M. and Włodarczyk, Z. (2014). Tsallis statistics approach to the transverse momentum distributions in p-p collisions. *The European Physical Journal C*, 74:2785.
- Saito, K., Maruyama, T., and Soutome, K. (1989). Collective modes in hot and dense matter. *Physical Review C*, 40:407–431.
- Santos, A. F. and Khanna, F. C. (2016). Quantized gravitoelectromagnetism theory at finite temperature. *International Journal of Modern Physics A*, 31(20n21):1650122.
- Satz, H., Specht, H. J., and Stock, R. (1988). *Quark Matter*. Springer.
- Schaefer, B.-J., Wagner, M., and Wambach, J. (2010). Thermodynamics of (2+1)-flavor QCD: Confronting models with lattice studies. *Physical Review D*, 81:074013.
- Scharnhorst, K. (1990). On propagation of light in the vacuum between plates. *Physics Letters B*, 236(3):354–359.

- Scharnhorst, K. (2019). Photon-photon scattering and related phenomena. experimental and theoretical approaches: The early period. *arXiv preprint arXiv:1711.05194v3*.
- Schrodinger, E. (1952). *Statistical Thermodynamics: A Course of Seminar Lectures*. Cambridge University Press.
- Schwinger, J. (1951). On gauge invariance and vacuum polarization. *Physical Review Journals Archive*, 82:664–679.
- Schwinger, J. (1961). Brownian motion of a quantum oscillator. *Journal of Mathematical Physics*, 2:407–432.
- Sena, I. and Deppman, A. (2013). Systematic analysis of p_T -distributions in p+ p collisions. *The European Physical Journal A*, 49(2):1–5.
- Shalaby, A. G., Oikonomou, V. K., and Nashed, G. G. (2021). Non-extensive thermodynamics effects in the cosmology of f (t) gravity. *Symmetry*, 13(1):75.
- Shannon, C. E. (1948). A mathematical theory of communication. *The Bell system technical journal*, 27(3):379–423.
- Shifman, M. A., Vainshtein, A. I., and Zakharov, V. I. (1979). QCD and resonance physics. Theoretical foundations. *Nuclear Physics B*, 147:385–447.
- Shore, G. (1996). 'faster than light' photons in gravitational fields- causality, anomalies and horizons. *Nuclear Physics B*, 460(2):379–394.
- Smilga, A. (1997). Physics of thermal QCD. *Physics Reports*, 291:1–106.
- Spieles, C., Stöcker, H., and Greiner, C. (1998). Phase transition of a finite quark-gluon plasma. *Physical Review C*, 57:908–915.
- Takahashi, Y. and Umezawa, H. (1996). Thermo field dynamics. *International journal of modern Physics B*, 10:1755–1805.
- Tarrach, R. (1983). Thermal effects on the speed of light. *Physics Letters B*, 133:259–261.
- Teweldeberhan, A., Plastino, A., and Miller, H. (2005). On the cut-off prescriptions associated with power-law generalized thermostatics. *Physics Letters A*, 343(1-3):71–78.
- Thoma, M. H. (2000a). New developments and applications of thermal field theory. *arXiv: High Energy Physics - Phenomenology*.
- Thoma, M. H. (2000b). Photon-photon interaction in a photon gas. *Europhysics Letters*, 52:498.
- Tirnakli, U. and Borges, E. P. (2016). The standard map: From Boltzmann-Gibbs statistics to Tsallis statistics. *Scientific Reports*, 6.
- Tsallis, C. (1988). Possible generalization of Boltzmann-Gibbs statistics. *Journal of statistical physics*, 52(1):479–487.

- Tsallis, C. (2009). Nonadditive entropy: The concept and its use. *The European Physical Journal A*, 40(3):257–266.
- Tsallis, C., Mendes, R., and Plastino, A. R. (1998). The role of constraints within generalized non-extensive statistics. *Physica A: Statistical Mechanics and its Applications*, 261(3-4):534–554.
- Umezawa, H. (1993). *Advanced field theory: Micro, macro, and thermal physics*.
- Umezawa, H., Matsumoto, H., and Tachiki, M. (1982). *Thermo field dynamics and condensed states*. North-Holland.
- Van Eijck, M. A., Kobes, R., and van Weert, C. G. (1994). Transformations of real-time finite-temperature Feynman rules. *Physical Review D*, 50:4097–4109.
- Van Hove, L. (1949). Quelques propriétés générales de l'intégrale de configuration d'un système de particules avec interaction. *Physica*, 15(11-12):951–961.
- Von Neumann, J. (1929). Beweis des ergodensatzes und desh-theorems in der neuen mechanik. *Zeitschrift für Physik*, 57:30–70.
- Wachs-Lopes, G., Santos, R., Saito, N., and Rodrigues, P. (2020). Recent nature-Inspired algorithms for medical image segmentation based on Tsallis statistics. *Communications in Nonlinear Science and Numerical Simulation*, 88:105256.
- Weinberg, S. (1979). Phenomenological lagrangians. *Physica A: Statistical Mechanics and its Applications*, 96(1-2):327–340.
- Weldon, H. A. (1982). Covariant calculations at finite temperature: The relativistic plasma. *Physical Review D*, 26:1394–1407.
- Wilk, G. and Wlodarczyk, Z. (2000). Interpretation of the nonextensivity parameter q in some applications of Tsallis statistics and Lévy distributions. *Physical Review Letters*, 84(13):2770.
- Wilson, K. G. (1974). Confinement of quarks. *Physical Review D*, 10:2445–2459.
- Wong, C.-Y. and Wilk, G. (2013). Tsallis fits to p_T spectra and multiple hard scattering in pp collisions at the LHC. *Physical Review D*, 87:114007.
- Wong, C.-Y., Wilk, G., Cirto, L. J. L., and Tsallis, C. (2015). From QCD-based hard-scattering to non-extensive statistical mechanical descriptions of transverse momentum spectra in high-energy pp and $p\bar{p}$ collisions. *Physical Review D*, 91:114027.
- Xu, H. H. (1996). Thermodynamic potential in real-time formalisms of thermal field theory. *Communications in Theoretical Physics*, 25:443.
- Yang, C.-N. and Lee, T.-D. (1952). Statistical theory of equations of state. Theory of condensation. *Physical Review Journals Archive*, 87(3):404.
- Zhao, Y.-P. (2020). Thermodynamic properties and transport coefficients of QCD matter within the non-extensive Polyakov–Nambu–Jona-Lasinio model. *Physical Review D*, 101(9):096006.