UNIVERSITI PUTRA MALAYSIA

FRACTAL CODING OF BIO-METRIC IMAGE FOR FACE AUTHENTICATION

MD AHADULLAH

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FRACTAL CODING OF BIO-METRIC IMAGE FOR FACE AUTHENTICATION

By

MD AHADULLAH

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Doctor of Philosophy

August 2021

DEDICATIONS

To my wife
Runi Rahman
and
My Children MD ARVID ULLAH and MD ARIYAN ULLAH.

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Doctor of Philosophy

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Chairman : Associate Professor Siti Hasana Binti Sapar, PhD
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Fractal objects have patterns, convergence, determinism, and reduction in dimensionality. Fractal geometry is looking for self-similarity, self-similarity resonance, and self-similarity convergence. As a result, Fractal Geometry has evolved into a scientific discipline with high predictability. Benoit Mandelbrot established the concept of the natural fractal object, characterized by merging the basics of self-similarity, scaling correlation, and statistical components. Because of these self-features, the fractal image coding method is a more dependable alternative to choose in image coding schemes. Due to this adoption, fractal image coding has already established numerous significant applications in image biometrics, picture compression, image signature, image watermarking, image extraction, and even image texture segmentation.

Despite this, the fractal encoder's popularity with the partition iterated function system rapidly drops due to its long encoding period. However, the existing strategy, which comprises two major phases and is based on the partition iterated function system, can be altered. The first stage involves encoding a statistically self-similar input image into the fixed point of an IFS, and the second involves decoding the IFS data to obtain the fractal image. Unfortunately, in existing methods, the collection of IFS data in the first step synchronizes badly, which results in lousy image quality in the decoding step. So both time complexity and image quality are not suitable for biometric face authentication. However, with these difficulties, fractal coding does not apply to personal biometric authentication unless it is resolved at a certain optimized level. This information loss in image and extended encoding times was mitigated by proposing an appropriate fractal coding technique for a biometric image.

This thesis examines how fractal image coding is used in biometric cryptography for personal face authentication. This thesis implements the algorithms for the Methods of CPM, BPBM fractal coding, and its application. This thesis proposes a novel way of integrating Fractal coding into a digital smart card with embedded cryptographic encryption and decryption and guard against all assaults, according to cryptoanalysis research. The thesis gains information on the time required to encode the biometric image by implementing techniques of CPM and BPBM and measures their impact on the encoding duration. The thesis also compares the results of enough images of various sizes generated by the proposed algorithms with the results of other fractal coding methods to confirm the algorithms' clarity, reliability and validity. Finally, this thesis will apply BPBM methods in biometric face authentication.

The thesis finds that there are core gaps and plans accordingly based on the literature review. The CPM and BPBM have been proposed, with two main streams: encoding and decoding of both. In the encoding, the fractal function converges to its self-similarity as IFS. The inverse function calls IFS back to create a corresponding fractal object in the decoding stream. The first method (CPM) blueprints blocking dimension, pooling factor, method, and block matching. The second method designs (BPBM) *Pixel Binarization*. Both methods, CPM and BPBM, are clarified, justified and validated with experiments and Benchmarks. Finally, BPBM is considered implementing biometric face authentication.

In CPM, the odd size pixel dimension of blocks, odd size pooling and max pooling scheme for smoothing, and the entire domain blocks search by only a single central pixel are better redeemable aspects throughout the encoding phase by adjusting to the previous idea. The key implication of these principles is that for both blocks, the symmetrical central pixel is used to search the relevant domain block rather than the entire neighborhood of the block. This study determined that this symmetrical central pixel will conform to the best-fit domain block by matching the same range block. In BPBM, the thesis contributes to search space reduction by converting eight-bit pixels to two-bit pixels using the central pixel value of blocks and the related eight-neighbors. As a result, the thesis contributes that excess block space is shortened before the search begins.

Because of the limited bits of domain blocks, we achieve faster coding by reducing run-time compared to any current exhaustive block hunting approach. This additional investigation encourages the improvement of encoding speed and afterward reveals enhanced results employed in personal authentication.

We assess the study results in three essential areas: encoding time complexity in second, fractal object (image) quality in PSNR, SSIM, FSIM (seventeen quality features have been used), dimensionality reduction (Compression Ratio). In all aspects, CPM and BPBM confirm the superiority.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

PENGKODAN FRAKTAL IMEJ BIO-METRIK UNTUK PENGECAMAN WAJAH

Oleh

MD AHADULLAH

Ogos 2021

Pengerusi : Profesor Madya Siti Hasana Binti Sapar, PhD
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Objek fraktal mempunyai corak, penumpuan, determinisme dan pengurangan kematraan. Geometri fraktal melihat kepada keserupaan diri, resonans keserupaan diri dan penumpuan keserupaan diri. Akibatnya, geometri fraktal telah berkembang menjadi disiplin saintifik dengan ramalan yang tinggi. Benoit Mandelbrot memperkukuhkan konsep objek fraktal semulajadi, mencirikan dengan menggabungkan asas kesamaan kendiri, korelasi skala dan unsur berstatistik. Disebabkan oleh ciri-ciri ini, kaedah pengekodan imej fraktal adalah pilihan yang lebih dipercayai untuk dipilih dari skema pengekodan imej. Oleh kerana ini, pengekodan imej fraktal telah memperkukuhkan pelbagai aplikasi yang signifikan dalam biometrik imej, pemampatan imej, tandatangan imej, tanda air imej, pengekstrakan imej, dan juga bahagian tekstur imej.

Walau bagaimanapun, populariti pengekod fraktal dengan sistem fungsi pemetakan terlelar, menurun dengan ketara kerana tempoh pengekodannya yang panjang. Walau bagaimanapun, strategi sedia ada, yang terdiri daripada dua peringkat utama dan berdasarkan sistem fungsi pemetakan terlelar boleh diubah. Fasa pertama melibatkan pengekodan imej input yang serupa secara berstatistik ke titik IFS yang ditetapkan, dan yang kedua melibatkan penyahkodan data IFS untuk mendapatkan imej fraktal. Malangnya, dalam kaedah yang sedia ada, pengumpulan data IFS pada langkah pantama berlaku dengan kurang baik mengakibatkan kualiti imej yang buruk dalam langkah penyahkodan. Oleh itu, kerumitan masa dan kualiti imej tidak sesuai untuk pengecaman wajah biometrik. Walau bagaimanapun, dengan kesukaran ini, pengekodan fraktal tidak terpakai kepada pengesahan biometrik peribadi melainkan ia ditangani pada tahap tertentu yang dioptimumkan. Kehilangan data ini semasa imej dan pengekodan lanjutan telah dikurangkan dengan

mencadangkan teknik pengekodan fraktal yang sesuai untuk imej biometrik.

Tesis ini mengkaji bagaimana pengekodan imej fraktal digunakan dalam kriptografi biometrik untuk pengecaman wajah peribadi. Tesis ini melaksanakan algoritma untuk Kaedah CPM, pengekodan fraktal BPBM, dan aplikasinya. Tesis ini mencadangkan cara baru untuk mengintegrasikan pengekodan Fractal ke dalam kad pintar digital dengan penyulitan kriptografi terbenam dan penyahsulitan dan berjaga-jaga terhadap semua serangan, menurut penyelidikan kriptoanalisis. Tesis memperolehi maklumat mengenai masa yang diperlukan untuk mengekod imej bio-metrik dengan melaksanakan teknik CPM dan BPBM, dan mengukur kesannya terhadap tempoh pengekodan. Tesis juga membandingkan hasil imej yang cukup dari pelbagai saiz yang dihasilkan oleh algoritma yang dicadangkan dengan hasil kaedah pengekodan fraktal lain untuk mengesahkan kebolehpercayaan dan kesahihan algoritma. Akhirnya, tesis ini akan menggunakan kaedah BPBM dalam pengecaman wajah biometrik.

Tesis mendapati bahawa terdapat jurang utama dan rancangan sewajarnya berdasarkan kajian kesusasteraan. CPM dan BPBM telah dicadangkan, dengan dua aliran utama: pengekodan dan penyahkodan kedua-duanya. Dalam pengekodan, fungsi fraktal menumpu kepada kesamaan kendiri sebagai IFS. Fungsi songsang memanggil IFS kembali untuk mencipta objek fraktal yang sepadan dalam strim penyahkodan.Pelan tindakan kaedah pertama (CPM) menyekat dimensi, faktor pengumpulan, kaedah, dan pemadanan blok. Kaedah kedua mereka bentuk (BPBM) *Pixel Binarization*. Kedua-dua kaedah, CPM dan BPBM, diperjelaskan, dibenarkan, disahkan dengan eksperimen dan Penanda Aras. Akhirnya, BPBM dipertimbangkan melaksanakan pengecaman wajah biometrik.

Dalam CPM, dimensi piksel saiz ganjil bagi blok, pengumpulan saiz ganjil dan skim pengumpulan maksimum untuk pelicinan, dan keseluruhan carian blok domain dengan hanya satu piksel pusat adalah aspek yang lebih baik boleh ditebus sepanjang fasa pengekodan dengan menyesuaikan diri dengan idea sebelumnya. Implikasi utama prinsip-prinsip ini ialah untuk kedua-dua blok, piksel pusat simetri digunakan untuk mencari blok domain yang berkaitan dan bukannya keseluruhan kejiranan blok. Kajian ini menentukan bahawa piksel pusat simetri ini akan mematuhi blok domain yang paling sesuai dengan memadankan blok julat yang sama. Dalam BPBM, tesis ini menyumbang kepada pengurangan ruang carian dengan menukar piksel lapan bit kepada piksel dua bit menggunakan nilai piksel pusat blok dan lapan jiran yang berkaitan. Akibatnya, tesis menyumbang bahawa ruang blok yang berlebihan dipendekkan sebelum carian bermula.

Oleh kerana blok domain yang terhad, kami mencapai pengekodan yang lebih cepat dengan mengurangkan masa berjalan berbanding dengan pendekatan memburu blok lengkap semasa. Penyiasatan tambahan ini menggalakkan peningkatan bagi kelajuan pengekodan dan selepas itu mendedahkan hasil yang dipertingkatkan yang digunakan dalam pengesahan peribadi.

Kami menilai hasil kajian dalam tiga bidang penting: kerumitan masa pengekodan dalam kedua, kualiti objek fraktal (imej) dalam PSNR, SSIM, FSIM (tujuh belas ciri kualiti telah digunakan), pengurangan dimensi (Nisbah Mampatan). Dalam semua aspek, CPM dan BPBM mengesahkan keunggulan.

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This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee were as follows:

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Declaration by Members of Supervisory Committee

This is to confirm that:

- the research conducted and the writing of this thesis was under our supervision;
- supervision responsibilities as stated in the Universiti Putra Malaysia (Graduate Studies) Rules 2003 (Revision 2012-2013) are adhered to.

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LIST OF ABBREVIATIONS

FG	Fractal Geometry
PIFS	Partition Iterated Function System
CP	Central Pixel
CPM	Central Pixel Method
BLM	Base Line Method
BFIC	Baseline Fractal Image Compression
BPA	Biometric Personal Authentication
FIE	Fractal Image Encoding
FID	Fractal Image Decoding
IFS	Iterative Function System
PIFS	Partition Iterated Function System
IQAT	Image Quality Assessment Tools
FSSBP	Fixed Square Size Block Partition
MSE	Mean Square Error
SNR	Signal to Noise Ratio
PSNR	Peak Signal to Noise Ratio
WPSNR	Weighted PSNR
SSIM	Structural Similarity Index Metric

MDSIM	Mean Deviation Similarity Index
FSIM	Features Similarity Index Metric
ESSIM	Edge Based Similarity Index Metric
MSSIM	Multi-scale Structural Similarity Index Metric
SRSIM	Spectral Similarity Index Metric
NSSIM	No Reference Structural Similarity Index Metric
CPPD	Contrast Per Pixel Deviation
GMSDV	Gradient Magnitude similarity Deviation
ED	Entropy Difference
NLSE	Normalized Least Square Error
IFM	Image Fidelity Metric
SFF	Sparse Feature Fidelity
FAR	False Acceptance Rate
FRR	False Rejection Rate
FP	False positive
FN	False Negative
TN	True Negative
TP	True Positive
RR	Recognition Rate

TER	Total Error Rate
FR	Face Recognition

CHAPTER 1

INTRODUCTION

1.1 Background

The fractal theory is an important branch of nonlinear dynamics since 1970s. In 1977, Mandelbrot proposed the Fractal Geometry of Nature . According to Benoit Mandelbrot, Euclidian geometry is incapable of explaining the shape of trees, fog, hills, and coastlines, whereas fractal geometry can determine the properties of a shape. The introduction of Fractal Geometry has begun with this lesson. As a result, Fractal Geometry (FG) is now widely used in astrophysics, biological sciences, computer graphics, quantum mechanics, architecture, medicine, steganography, satellite imaging, engineering, biometrics and cryptography, distribution of earthquake patterns, financial stock market to understand and forecast patterns, and encryption techniques. In addition, Agarwal (2017a,b, 2020) demonstrated image encryption algorithms based on fractal functions. Tian et al. (2020) examine the drawbacks of public-key cryptography in smart card environments. However, fractal coding has already been employed in a biometric cryptosystem using a smart card in a few studies.

Then, Hutchinson (1981) presented a fundamental framework of self-similar fractals with the compact set K in a real plane to enhance Mandelbrot's fractal theory. Hutchinson's theoretical framework consisted of a finite set $S = [s_i | 0 < s_i < 1]$ of contraction maps on K, all of which must result in K. The equation (1.1) demonstrates his substantial contribution to the fractal attractor's existence and uniqueness.

$$K = \bigcup_{i=1}^{N} s_i K \tag{1.1}$$

Demko et al. (1985) was the first who proposed computer graphics using iterated function approaches to model complex structures. Barnsley and Demko (1985) coined the term iterated function system to describe the collection of $(X, \rho, T_1, ..., T_n)$, where X is a complete metric space with metric ρ, and $T_1, ..., T_n$ are the collection of contractive maps. Barnsley et al. (1986) proposed algorithms to solve the inverse problem named collage theory. Shortly later, Barnsley et al. (1987) expanded on the ideas and advocated using fractal coding to model natural scenes. The scheme of Barnsley had not been popular due to failing automation. However, the inverse problem had become known for finding good fractal algorithms and parameterizing them well by Jacquin (1989), the first published automated scheme to solve the inverse problem. Jacquin et al. partitioned two identical images into domain and range blocks and then categorized the domain and range blocks by block geometry (Jacquin (1990a,b)). The image coding method used by Jacquin et al. (1992) was based on the approach of Barnsley by changing to the block-wise iteration. As a result, the researchers produced a succession of block-images from a domain pool using block-wise self-transformability, approximating the range block, and collecting all data using the Partitioned Iterated Function Method during the encoding system. By the decoding technique from any beginning image, PIFS-Data approximates an original image as a fractal image.

However, the complexity of encoding time was encountered due to a wide variety of harmonizing with a selection of large domain blocks for each range block. Several scientists have already implemented strategies that minimize the search complexity of the domain pool to overcome this most fundamental impediment to fractal image coding. This encoding time of fractal image is not suitable for the method shown in Chapter 5. However, according to the National Institute of Standards and Technology (NIST), biometric match-on-card authentication will take less than 2.5 seconds (Ibjaoun et al. (2016)).

In the application stage of fractal coding for biometric image recognition, Al-Saidi and Said (2014) take five training images of fingerprints for encoding. Al-Saidi et al. then calculate the average of average the five fractal vector features of each image as IFS and use it to recognize the test image. Mokni and Kherallah (2016) suggested an approach for personal identification based on palm-print features extracted using the various methods of fractal theory. Tang et al. (2017) uses fractal theory to recognize face biometric modality by compressing faces and distinguishing facial features to speed up compression. Then, using fractal neighbor distance, they were able to detect similarities within images.

Besides image recognition, biometric protocols are used to verify an individual's identity with an official authority and to encrypt or sign a message. These methods are based on universal and distinctive biometric features. There are numerous approaches to implementing biometric identification and a biometric smart card. However, A microprocessor-based smart card with on-board arithmetic enables Public Key Infrastructure (PKI) functions (Strhársky (2016)). Because of the public parameters (keys) stored on the card, a biometric card can conduct authentication, encryption, and data signing (Ibjaoun et al. (2016)).

Motivated by current concerns, the authors contribute to responding to the following questions.

Do pixel value, sub-image block design, pooling factor, number of domain blocks, and pooling mechanism affect encoding time, image compression, or image quality? Does biometric facial authentication work with fractal codes and their images if the encoding time is really not feasible?

1.2 Problem Statement

As a fractal image encoding method, its disadvantages are a long encoding time and poor decoded image quality. Many academics offered rapid fractal encoding methods to make it more realistic, and the encoding process has been substantially shortened. Furthermore, other scholars are interested in enhancing the decoding image quality and the compression ratio. A few studies have also discovered that the distribution of collage errors is not uniform. The range blocks with the most substantial matching faults are the leading cause of decoded image degradation, (Zou et al. (2015)).

On the other hand, the Internet's growth and development have been enormous in recent years. Nonetheless, it is a problematic downside in terms of security and authentication. There are numerous methods for creating passwords and sensitive cards in the literature to provide network connection security. However, traditional authentication techniques have several drawbacks. Altogether, we noticed a handful of real problems, which are listed below:

1. In the scientific literature, using the Base Line Method of fractal image coding with an improved technique that balances time complexity and image quality is uncommon [Jacquin (1989), Beaumont (1990), Oien et al. (1991), Jacquin et al. (1992), Fisher et al. (1992), Monro (1993), Saupe (1996), Thao (1996), Chang et al. (1997), Conci and Aquino (1999), Wu et al. (2003), Truong et al. (2004), Chung and Hsu (2006), Koli and Ali (2006), Wang et al. (2014), Bhattacharya et al. (2015), Nandi (2019), and Kumar and Manimegalai (2020)].

2. Fractal image quality, with the confirmation of the algorithms' reliability and validity, is rare in the same line of research. The number of comparisons made by image quality evaluation tools to confirm a standard image quality that meets biometric standards has not been thoroughly investigated. Furthermore, a few images features have been evaluated to confirm the quality of the images[Ahadullah et al. (2020), Ali et al. (2019), Best-Rowden and Jain (2018), Fu et al. (2022), Hernandez-Ortega et al. (2020), Best-Rowden and Jain (2017), Bharadwaj et al. (2014), Wasnik et al. (2017)].

3. To design a secure smart card application based on fractal code, one must first investigate the security of public-key encryption for embedded software applications and how it operates in real-time when encrypting fractal code Gupta and Quamara (2021). Card frauds and financial losses occur as card-based transactions grow in popularity. The standard cryptographic key is maintained on a remote server, and when a Java smart card is being authenticated, the image information is under phishing attack, Dodging attack, Evasion attack, Impersonation attack, Poisoning attack showing a red flag for exploitation that can be assumed cracked or lost [Gupta and Quamara (2021), Ali et al. (2019), Best-Rowden and Jain (2018), Fu et al. (2022), Hernandez-Ortega et al. (2020), Best-Rowden and Jain (2017), Bharadwaj et al. (2014), Wasnik et al. (2017)]

1.3 Objectives

The objective of this thesis are:

The thesis seeks to explore whether fractal coding can be applied to bio-metric images to introduce the advantages of personal authentication. Fractal coding has some severe disadvantages, such as information loss issue or lengthy encoding time, which can be obstructed implying an appropriate fractal coding plan for bio-metric image.

According to the author, the goal of the thesis can be achieved by finding or building a technique of fractal coding that best meets the needs and conditions of biometric image and, at the same moment, provides all the benefits of fractal coding. This investigates the correspondence of current fractal coding techniques with the domains are adapted and improved best suited methods and techniques.

Encoding latency, image compression loss, and image quality loss were all investigated experimentally. This is the only technique to enable biometric facial authentication Jurinić and Domović (2017).

The specific objectives of this study are:

1. to implement the algorithms for CPM and BPBM fractal coding methods and to achieve an improved, optimized technique for balancing the time required to encode the bio-metric image and decoded fractal image quality.

2. to compare the results of some images of various sizes generated by the proposed algorithms with the results of other fractal coding methods to confirm the algorithms' reliability and validity.

3. to propose a novel way of integrating Fractal coding into a digital smart card with embedded cryptographic encryption and decryption guarding against all possible assaults stated in problem statements, according to cryptoanalysis research.

This thesis presents five substantial improvements to Jacquin's partition iterated function system as contributions. The first four contributions use the Central Pixel Method, CPM, and the fifth contribution uses the Binarize Pixel Block Method, BPBM. Finally, the third method is a Unique Biometric Face Authentication Method (UBFAM), which is an application of BPBM.

This study aims to develop or create a fractal coding approach that best meets the needs and conditions of biometric images while still giving all of the fractal coding's benefits. This research compares current fractal coding techniques to the most appropriate methodologies and strategies.

However, nowadays, the bio-metric sector has appeared to reduce the delay in the operation and enhance the degree of accuracy. Therefore, this thesis seeks to evaluate bio-metric parameters of a fractal nature that can be updated to enhance personal authentication. This thesis suggests a few modifications of the existing method to encode an image, and we called it Central Pixel Method (CPM). Specifically, this thesis optimizes the domain block numbers by optimal block size, confirming the diagonally and centrally symmetric central block pixel. The study proposed the best-fit spatial contraction (pooling mechanism). Through this research, we first proposed a novel matching approach between domain to corresponding range block, implying the central pixels of the blocks. The designed algorithms of CPM in Chapter 3 created IFS-Data from best-fit domain block in search space results in optimizing the contrast and brightness of block pixels to produce a good quality fractal image by analyzing sufficiently enough fractal image quality features, optimized compression ratio, and reduced encoding time complexity. Thus we achieve better encoding time and compression ratio with optimized image quality.

Besides, we proposed another scheme of Binarize Pixel Block Method (BPBM in Chapter 4) to design each pixel value of a block in a gray-scale image with eight bits into two bits, resulting in faster encoding. We changed over the block dimension, the spatial contraction method, and block searching process in this approach. Consequently, we obtained the result of 97.48% of encoding time reduction on average compared to existing known methods.

In addition, Chapter 5 will show a Biometric Face Authentication system created with BPBM inside a micro-controller smart card.

1.4 Scope and limitations

This study focuses on developing more effective and efficient designs of methodologies to implement a full-search fractal image coding from Jacquin's baseline method. The results of the designed algorithms are the fractal encoding data (IFS-Data), encoding time, fractal image, and its quality features. Besides, the research investigates the extent to which the aforementioned results are applicable to what is supposed to be expected in biometric face authentication using smart card.

Standard available eighty-five images collected from internet have been divided into two sets. With the proposed method, CPM the obtained results are investigated displayed in chapter three. Another proposed method, BPBM analyze the results in chapter 4. In Chapter 5, fractal code of only BPBM is utilized to demonstrate the actual implementation of facial biometrics. All investigations are done by a core i7 Machine in the platform of Matlab.

The suggested schemes are limited to the image shape (square), gray scale image (not RGB), block size (odd size pixel), pooling size (odd size), pooling method (Max pooling). Decoding time is not measured. Reference and non-reference Fractal image quality are measured. Exhaustive (full) search based on vector norm which is equivalent to Hausdorff metric is applied, domain classification is absent. As a biometric modality, the only face image is considered. The efficiency of the algorithms in both methods are checked by only one computer system. Computer processor time is not celebrated prior investigation.

Last but not least, a microprocessor-based smart card is essentially imperative to run the secure authentication in Chapter 5.

1.5 Basic Concepts

Fractal geometry is an active branch of modern mathematics that deals with an irregular shape, scale, form, geometry and fractal interrelations. All Mathematical definitions and theories are being available to establish a deep connection to fractal image coding and its application.

1.5.1 Set and Topology

Definition 1.1 *Limit Point of a Set : A number* $x \in (X,d)$*such that* $\forall \varepsilon > 0$, $\exists$ *a number of set* $y \in A$ *different from x such that* $|y - x| <$ ”*, (Ahadullah et al. (2020)).*

Definition 1.2 *Neighborhood : in a metric space* $M = (X,d)$*, a set N is a neighborhood of a point p if* $\exists$ *a open ball with center p and radius* $\varepsilon > 0$ *such that* $N_{”}(p) = N(p, ”) = \{x \in X | d(x,p) < ”\}$ *is contained in N, (Ahadullah et al. (2020)).*

Definition 1.3 *Self-similar Fractal Set : A set is self-similar fractal if it is the invariant set of an Iterated Function System. Mathematically a scaling law* $\mathbb{S}$ *is an N-tuple* $(S_1, S_2 \ldots\ldots S_N)$ *,* $N \geq 2$ *of Lipschitz's maps* S_i*:* $\mathbb{R}^n \to \mathbb{R}^n$ *where Lipschitz constant,* $r_i = LipS_i$*. If* $K \subset \mathbb{R}^n$ *then* $\mathbb{S}K \subset \mathbb{R}^n$*is defined by* $\mathbb{S}K = \bigcup_i^N S_i(K)$*. We say* $K \subset \mathbb{R}^n$ *satisfies the scaling law of* $\mathbb{S}$*, or self-similar fractal set , if* $K = \mathbb{S}K$*, (Ahadullah et al. (2020)).*

Definition 1.4 *Strictly self Similar Set : A compact set* $A \subset (X,d)$ *is strictly self-similar if A is strictly invariant when it is composed of with an finite number of regular subsets* A_i *scaling by* $\mathbb{S} = \{S_1, S_2 \ldots\ldots S_N\}$ *. Mathematically* $A = \bigcup_i^N S_i(A)$*, (Ahadullah et al. (2020)).*

1.5.2 Metric and Space

Definition 1.5 *Hausdorff Metric : Let set* $A = \{a_1, a_2 \ldots\ldots a_p\}$ *and* $B = \{b_1, b_2 \ldots\ldots b_q\}$ *both A and* $B \in (X,d)$ *then Hausdorff metric,* $H(A,B) = max\{h(A,B), h(B,A)\}$*, (Ahadullah et al. (2020)).*

Definition 1.6 *Norms of Vectors: To define how close two vectors or two matrices or two gray image windows x and y are, and to define the convergence of sequences of vectors or matrices x and y, we use the concept of the norm. and Hausdorf Metric is also measured by Norms, (Ahadullah et al. (2020)).*

1.5.3 Space

Definition 1.7 *Metric Space: A set $X \in R^n$ and a distance function $d : X \times X \to X$ combined togather makes a metric space if $x, y, z \in X$ (Ahadullah et al. (2020)) such that*

1. *$d(x,y) \geq 0$ property of non-negativity*

2. *$d(x,y) = d(y,x)$ properties of symmetry*

3. *$d(x,y) = 0$ iff $x = y$ properties of zeroness*

4. *$d(x,z) \leq d(x,y) + d(y,z)$ properties of triangle inequality.*

Definition 1.8 *Compact Metric Space : A metric space (X,d) is sequentially compact if every sequence of points in (X,d) has a convergent sub-sequence converging to a point in (X,d),(Ahadullah et al. (2020)) .*

Definition 1.9 *Complete Metric Space : A metric space (X,d) is called complete if every Cauchy sequence in (X,d) converges in (X,d), (Ahadullah et al. (2020)).*

Definition 1.10 *Hausdorff Space : A metric space (X,d) said to be Hausdorff Space if for any $x, y \in (X,d)$, $x \neq y$ $\exists$ open sets such that $x \in U$, $y \in V$ and $U \cap V =$ Œ, (Ahadullah et al. (2020)).*

1.5.4 Affine Transformation

Definition 1.11 *Affine Transformation : is a linear mapping method that preserves points, straight lines, and planes. i.e., An affine transformation is a type of geometric transformation that preserves the collinearity and distance ratios in a line. (Ahadullah et al. (2020))*

Definition 1.12 *Isometry: is a mapping from one metric space to another metric space that preserve distance between each pair points.Example-Translation, Rotation, Reflection, Glide Reflection (is a composition of Rotation about a line and Translation along the same line) (Ahadullah et al. (2020))*

Definition 1.13 *Pixel Intensity : Conceptually, a grayscale image can be interpreted as a function I(x, y) that evaluates the pixel intensity at the position of the pixel (x, y), (Ahadullah et al. (2020)).*

Definition 1.14 *Contrast : The term contrast refers to the amount of difference in color or grayscale that exists in both analog and digital images between different image features. Images with a higher level of contrast usually show a higher degree of difference in color and grayscale than those with lower contrast. The difference between the pixel intensity in an image must be maximum and minimum, Block Symmetric Cetral Pixel, MSE. (Ahadullah et al. (2020))*

Definition 1.15 *Block Central Pixel and Its Neighbors : A micro-image (block) contains some pixels arranged with spatial coordinate system in Figures 3.4, four-neighborhood and eight-neighbor are shown where (x,y) is the Central Pixel (CP) for both the case and another four or eight are neighbors of it.(Liao et al. (2010), Ahadullah et al. (2020))*

Definition 1.16 *Block Symmetric Central Pixel : Due to eight isometric affine transformation on square image block, if spatial coordinates of CP does not change and is termed as Block Symmetric Central Pixel in Figure 3.7. and this is possible when block size is $(2d+1)\times(2d+1)$ and where, $d=[1,2,3,4]$, (Ahadullah et al. (2020))*

Definition 1.17 *Gray Scale Image : It is a Set in a complete metric space such that $f(i,j)=I_p$, where $i,j\in\mathbb{Z}^+$ $(i,j)=$ spatial coordinates system and $i\times j=$ size of image and pixel Intensity, I_p=$[0,1\ldots,255]$.*

In general, Grayscale refers to digital images in which each pixel's value represents the light's intensity information solely. Only the darkest black to the brightest white is often displayed in such an image. The value of each pixel in a grayscale image is proportional to the number of bits of data utilized to represent it. The value of a grey image is commonly represented by 8 bits; that is, a combination of eight binary values represents a pixel value. As a result, pixels have a value range of 0–255, with a total of 256 grayscale levels (Zareai et al., 2021).

Definition 1.18 *Bits per second (bps or bit/sec) is a standard metric of data speed for computer modems and transmission carriers in data communications. The amount of bits transferred or received each second is equal to the speed in bps. (Samad et al. (2018))*

For example, Bits per second(bps) are a unit of measurement of the speed of bit-transformation is bps during the encoding process of an image to IFS.

Definition 1.19 *Bits per pixel(bpp): bits per pixel denotes the number of bits per pixel. At the time of Fractal image encoding, bits allocation is taken palace as follows:*

- *For the domain blocks positioning: For horizontal and vertical locations 6 bits each.(Conci and Aquino (2005), Davoine et al. (1996), Tong and Pi (2001))*
- *For the eight isometrics: 3 bits (Conci and Aquino (2005))*
- *For the brightness adjust: 8 bits (Davoine and Chassery (1994))*
- *For the contrast adjust: 2 bits when quantize to 0.25, 0.50, 0.75 (Bi and Wang (2014))*

so bits required per block are 25 bits and total bits (t_b) required image to encode are, $t_b=\frac{25\times ndb}{total\ pixel\ in\ image}$, (Distasi et al. (2005)).

Definition 1.20 *The compression ratio (CR) is a key metric for assessing the effectiveness of data compression technologies. It is defined as,* $CR = \frac{bpp_{uncomp}}{bpp_{comp}}$.

A ratio of original image data to fractal code (IFS-Data), for instance.

1.5.5 Image Quality Assessment Tools (IQAT)

For reality, the IQA model should be both practical and competitive. The complexity of computation is efficient when predicting accuracy is effective. The increasing number of visual data is being examined as digital imaging and communication software becomes more omnipresent in our daily lives. Therefore, efficiency has become a key issue for IQA algorithms. We have selected several IQAIMs from recent studies and grouped them in a single set. IQAIMs as used in Chapters 3 and 4. We have taken some of them to define according to research papers.

Definition 1.21 *MSE: The MSE is the cumulative square error between the test and the reference images. Mathematically,*

$$MSE = \frac{1}{h \times v} \sum_{i=0}^{h-1} \sum_{j=0}^{v-1} [I(i,j) - A(i,j)]^2$$

So according to Thomos et al. (2005) and Li and Cai (2007) Maximum allowable MSE value should be 650.25 for bit depth is 8 bits, with the help of 1.22.

Theorem 1.1 *SNR(Signal to Noise Ratio): is the ratio between the discrete signal of power and the discrete signal of noise. If* P_{dsp} *is defined as discrete signal power, Mathematically we can write,*

$$P_{dsp} = \sum_{-\infty}^{\infty} f_s^2[i] = |f_s[i]|^2$$

and the Error signal, $e[i]$ *so, the effected signal,* $f_s'[i] = f_s[i] + e[i]$. *Therefore,* $e[i] = f_s'[i] - f_s[i]$. *Thus, the discrete error signal power,* $P_e = |f_s'[i] - f_s[i]|^2$. *So, according to definition,* $SNR = \frac{P_{dsp}}{P_e} = \frac{|f_s[i]|^2}{|f_s'[i] - f_s[i]|^2}$, *Phonon (2013). The SNR ratio in imaging is used as a practical measure of an imaging system's sensitivity. According to, Wikipedia (2013), Industry standards measure and define ISO film speed equivalent sensitivity;*

1. *SNR: 32.04 dB = excellent image quality;*

2. *SNR: 20 dB = acceptable image quality;*

But for lossy compression, the SNR value in dB might be less.

Definition 1.22 *PSNR (Peak Signal to Noise Ratio): The PSNR block calculates the maximum signal-to-noise ratio of two images in decibels. This is the ratio used as a measurement of the quality between the original and the decoded image. The higher the PSNR, the better the quality of the test image (Zheng et al. (2020)). Mathematically,*

$$PSNR = 10log_{10}(\frac{I_{max}^2}{MSE})$$

Definition 1.23 *WPSNR: Weighted Peak Signal to Noise Ratio - Mathematically,* $WPSNR = 10 \times \log_{10}(\frac{(2^{bd}-1)^2}{WMSE})$*, where WMSE is Weighted Mean Square Error and bd is bit depth, (Erfurt et al. (2019))*

Theorem 1.2 *SSIM: The Structural Similarity Index (SSIM) is a measure which quantifies deterioration of image quality due to processing, such as data compression or data transmission loss. It is a full metric of reference that requires a reference picture and the processed image, two pictures from the same image. The SSIM, an image- quality estimation metric, is calculated primarily supported the computation of three major elements called correlation of luminance,* $f_L(x,y)$*, contrast,* $f_C(x,y)$*, and structural* $f_S(x,y)$*. Mathematically,*

$$f_L(x,y) = \frac{2\mu_x\mu_y + \alpha}{\mu_x^2 + \mu_y^2 + \alpha}$$

$$f_C(x,y) = \frac{2\sigma_x\sigma_y + \beta}{\sigma_x^2 + \sigma_y^2 + \beta}$$

$$f_S(x,y) = \frac{\sigma_{xy} + \gamma}{\sigma_x\sigma_y + \gamma}$$

The SSIM index is calculated on different image windows. Measured between two prevalent same size windows of x and y, the combined metric is as follows,

$$f_{SSIM}(x,y) = [f_L(x,y)]^{\zeta}.[f_C(x,y)]^{\eta}.[f_S(x,y)]^{\theta}$$

where ζ*,* η *and* θ *are positive constant and if* $\zeta = \eta = \theta = 1$*, we get*

$$f_{SSIM}(x,y) = \frac{(2\mu_x\mu_y + \alpha)(2\sigma_{xy} + \beta)}{(\mu_x^2 + \mu_y^2 + \alpha)(\sigma_x^2 + \sigma_y^2 + \beta)}$$

SSIM measures the difference between perceptions of two similar images. It is unable to assess which of the two is better: it must be concluded that it is the "original" and that additional processing has occurred, such as data compression. SSIM is based on visible structures in the image as opposed to PSNR. For more details, reader can study Lahoulou et al. (2017); Wang et al. (2004, 2003); Dosselmann and Yang (2011); Brooks et al. (2008)

Definition 1.24 *Feature Similarity Index Metric (FSIM): is actually Phase congruency model, where two image frames,* $f_1(x)$ *and* $f_2(x)$ *and we calculate* $f_{FSIM} =$

$\frac{\sum f_{S_L}(x).f_{PC_m}(x)}{\sum f_{PC_m}(x)}$, *(Zhang et al. (2011)).*

Definition 1.25 *Edge Strength Similarity Index Metric(ESSIM): Two same size image windows, x and y. The edge strength correlation,* $f_E(x,y) = \frac{\mu'_{xy}+\alpha}{\mu'_x\mu'_y+\alpha}$. *So the ESSIM,*

$$f_{ESSIM}(x,y) = [f_L(x,y)]^{\zeta}.[f_C(x,y)]^{\eta}.[f_E(x,y)]^{\theta}$$

(Chen et al. (2006)), and (Zhang et al. (2013))

Definition 1.26 *MS-SSIM: Multi-scale Structural Similarity Index- MS-SSIM approach is a practical means of integrating descriptions of the digital image at various resolutions, and is evaluated by combining the measurement at different scales using:-*

$$f_{MS-SSIM}(x,y) = [f_{L_M}(x,y)]^{\zeta_M}\prod_{j=1}^{M}.[f_{C_j}(x,y)]^{\eta_j}.[f_{S_j}(x,y)]^{\theta_j}$$

where the contrast and the structure are measured at the jth scale and are denoted as $f_{C_j}(x,y)$ *and* $f_{S_j}(x,y)$ *respectively. The luminance is determined only at Scale M and is denoted as* $f_{L_M}(x,y)$, *(Wang et al. (2003))*

Definition 1.27 *SR-SIM: Spectral Residual based Similarity Index- Approximately, the SR-SIM reflects the creativity portion of an image by eliminating statistically redundant components. Mathematically,* $SR-SIM = \frac{\sum_{\lambda} S_{\mu}(x)\times[S_V]^{\rho}\times R_{\varphi}(x)}{\sum_{\lambda} R_{\varphi}}$, *(Zhang and Li (2012))*

Definition 1.28 *Contrast Per Pixel (CPP): is a measure of image quality. It is defined as the average intensity difference between a pixel and its adjacent pixels. Mathematically,* $f_{cpp} = \frac{\sum_{x=0}^{X}\sum_{y=0}^{Y}(\sum_{x,y}|f(x,y)-f(x',y')|)}{XY}$, *where* $f(x',y')$ *are the neighboring pixels of* 3×3 *window, (Chang et al. (2015)).*

Definition 1.29 *GMSD: Gradient Magnitude Similarity Deviation is a perceptual image quality index that is highly efficient. A lower GMSD value means a picture of higher quality and gives the right conclusion, but GMSM (Gradient Magnitude Similarity Mean) fails, (Xue et al. (2013))*

Definition 1.30 *IFM:Image Fidelity Measure- Mathematically,*

$$IFM = 1-(\sum_{x=1}^{M}\sum_{y=1}^{N}[f(x,y)-\hat{f}(x,y)]^2/\sum_{x=1}^{M}\sum_{y=1}^{N}[f(x,y)]^2)$$

(Eskicioglu and Fisher (1993))

Definition 1.31 *SFF: Sparse Feature Fidelity-The SFF index is determined using the combination of* SFF_m *and* SFF_f *in a quality score.* $SFF = \pi\times SFF_m + (1-\pi)\times SFF_f$, *where* $0<\pi<1$ *is a parameter to change the relative quality of the two elements, its proper value is 0.8, (Chang et al. (2013))*

1.5.6 Mathematical Cryptography

It is known that the science of cryptography was born along with the art of writing. Human beings were organized into clans, communities, and kingdoms as cultures developed. This led to concepts like power, wars, hegemony, and politics emerging. Such theories further intensified people's natural need to secretly interact with the selected recipient, which in effect also meant that cryptography continued to evolve. The origins of cryptography are located in the cultures of Rome and Egypt and Cryptography and Cryptanalysis were overly mathematical during the Second world war. A person of ages had two inherent needs:-

1. interacting and sharing information, and
2. communicating selectively

Such two requirements culminated in the art of coding information in such a way that the data could only be obtained by the intended people. This art of coding information we will discuss in this section.

Definition 1.32 *Cryptography: The art and science of concealing messages is recognized as cryptography to introduce secrecy in information security. Two Greek terms,'Krypto' meaning secret and 'graphene' meaning writing, invented the term 'cryptography'(Dhanda et al. (2020)).*

An encrypted communication in which letters are substituted with the other characters is an example of fundamental cryptography.

Definition 1.33 *Cryptanalysis: is the art and science of cracking the ciphertext. Cryptography concerns cryptosystem development, while cryptanalysis studies cryptosystem breakdown (Rawal and Padhye (2020)).*

The British decryption of a telegram from the German foreign minister during World War I is a classic example of cryptanalysis accomplishment.

1.5.7 Biometric Cryptography

Finger printing, Face Recognition, Hand Geometry, Iris, Retinal Scan, DNA, Keystroke Authentication, Voice are the common biometric modalities. To the confirmation of Face Authentication, we need to know the False acceptance rate (FAR) and False Reject Rate (FRR).

Definition 1.34 *False acceptance rate (FAR): is a calculation of the probability that an unauthorized user will be incorrectly identified as a legitimate user by a biometric device (Malik et al. (2014)).*

$$FAR = \frac{I_{fa}}{T_{ss}} \times 100\%$$

where I_{fa} is the incidents of False Acceptance, and T_{ss} is the number of identification attempts.

For example, If the FAR is 0.02 percent, one out of every 5,000 unauthorized users will be get access when they try to log in.

Definition 1.35 *False Reject Rate (FRR): is a measure of the probability that an approved user as an unauthorized user will be wrongly rejected by the biometric device (Malik et al. (2014)).*

$$FRR = \frac{I_{fr}}{T_{ss}} \times 100\%$$

where I_{fr} is the incidents of False Rejection, and T_{ss} is the number of identification attempts.

For example, If the FRR is 0.02 percent, one out of every 5,000 authorized users will be denied access when they try to log in.

1.5.8 Computational Time Complexity

P and NP are two types of issues in computational complexity theory. The readers can look at two groups of difficulties based on the merits of this research:

Definition 1.36 *P-Problem: A Turing machine's running time is defined as a function, $f : \mathbb{N} \mapsto \mathbb{N}$, where f(n) is the maximum number of steps the Turing machine can take on an input of size n. As a result of this function, temporal complexity classes are defined. $Time(h(n))$ denotes the set of all languages decidable by O(g(n)) time Turing machines. The equation P is a set of problems that can be solved by a deterministic Turing machine in Polynomial-time*

$$P = \bigcup_{k} Time(n^k)$$

Where k is a constant (Peng (2018), Baeldung (2021)).

Many algorithms take polynomial time to complete, for example: Addition, subtraction, division, and multiplication are all basic mathematical operations. Lookups in hashtables, string operations, and sorting issues. For a given set of numbers, Linear and Binary Search Algorithms are used.

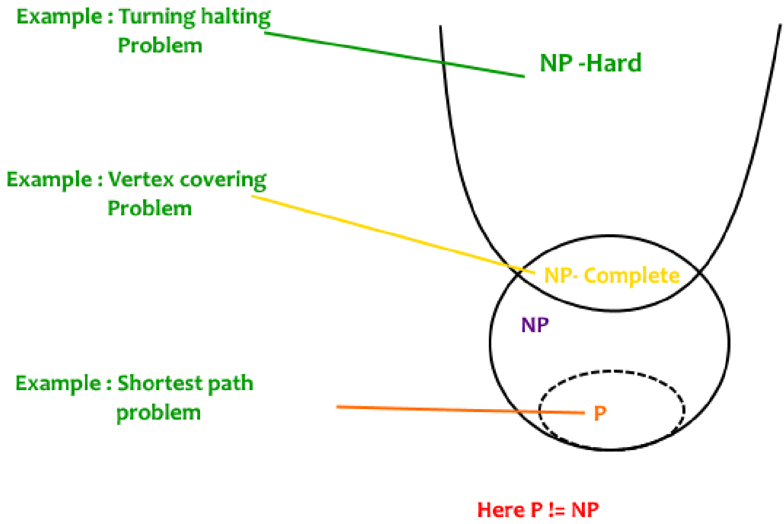

Figure 1.1: Computational Time Complexity: [The Image is adapting from-Freeman (1979)]

Definition 1.37 *NP-Problem: NP is a set of decision problems that can be solved by a Non-deterministic Turing Machine in Polynomial-time. P is a subset of NP (Peng (2018), Baeldung (2021)).*

Integer Factorization and Graph Isomorphism are two of them.

Definition 1.38 *NP-Complete Problem: NP-complete problems are the hardest problems in NP set (Peng (2018), Baeldung (2021)). A decision problem L is NP-complete if:*

1. *L is in NP (Any given solution for NP-complete problems can be verified quickly, but there is no efficient known solution).*
2. *Every problem in NP is reducible to L in polynomial time.*

There are various issues that have been demonstrated to be complete. Traveling Salesman, Knapsack, and Graph Coloring are a few of them.

Definition 1.39 *NP-Hard Problem: A problem is NP-Hard if it follows property 2 mentioned above, it doesn't need to follow property 1. Therefore, the NP-Complete set is also a subset of the NP-Hard set (Peng (2018), Baeldung (2021)).*

They are not just challenging to solve but also to verify. In reality, some of these issues are insurmountable. The following are some of the most challenging NP-Hard problems: Iteration of Jacquin's fractal coding and K-means clustering.

1.6 Thesis organization

In Chapter 2, we focused literature review of the basics on the fractal image encoding method assessed along with linked up to schemes mentioned in Chapters 3 and 4. In Chapter 3, we explored the Research Methodology of the First Scheme, the Central Pixel System. In Chapter 4, the second method, we portrayed the Block Pixel Binarizing Mechanism. In Chapter 5, we implemented the proposed schemes listed in Chapter 4 for the purpose of Biometric Authentication. Finally, in Chapter 6, we concluded and proposed more potential changes for further development.

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