UNIVERSITI PUTRA MALAYSIA

THE SIMULATION OF TRANSIENT HEAT TRANSFER IN A THERMAL BOUNDARY LAYER

TAN KAH YAW

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By

TAN KAH YAW

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LIST OF SYMBOLS

$\tilde{a}_c$ dimensionless wave number

B rate of surface temperature variation, °C/s

d depth of fluid layer, m

g gravity, m^2/s

h heat transfer coefficient , $W/(m^2\ °C)$

k thermal conductivity of the fluid, W/(m°C)

l depth of fluid layer for defining transient Biot number, m

q° constant heat flux, W/m^2

t_c critical time for onset of convection, s

T temperature, °C

T_0 initial water temperature, °C

T_S water surface temperature, °C

T_{air} temperature of the surrounding air, °C

Greek Symbols

α volumetric coefficient of thermal expansion, K^{-1}

δ thickness of effective thermal layer, m

κ thermal diffusivity, m^2/s

λ wavelength, m

ν	kinematic viscosity, m^2/s

Abbreviations

CFD	Computational Fluid Dynamic
CHF	constant heat flux boundary condition
FST	fixed surface temperature boundary condition
LSA	linear stability analysis
LTR	linear temperature rate

Subscripts

c	critical
0	initial state
s	surface
max	maximum

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TAN KAH YAW

June 1999

Chairman: Associate Professor Tan Ka Kheng, Ph.D., P.Eng.

Faculty: Engineering

The onset of convection in a horizontal layer is widely recognised as the simplest example of hydrodynamic instability where *nonlinear* temperature profile prevails. A newly defined *transient Rayleigh number* that incorporates the mode of heat transport and the critical time can be used to predict the onset of convection. The mode of heat transport is characterized by a thermal boundary condition that determines the *Biot number* and its corresponding critical Rayleigh number. The study of onset of convection can be verified by using Computational Fluid Dynamic (CFD) to solve the governing partial differential equations for heat conduction, momentum, energy and chemical species. The simulation of heat transfer in the thermal layer was carried out by the commercial software, FLUENT. The occurrence of the convection was observed for two boundary conditions: constant heat flux and fixed surface temperature. Most of the observed values were in good agreement with the theoretical values. The critical time and critical depth for transient heat transfer

can be determined accurately. However, the accuracy of modeling of transient heat transfer can be further improved.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan ijazah Master Sains

SIMULASI UNTUK PEROLAKAN DI SEMPADAN LAPISAN TERMAL

Oleh

TAN KAH YAW

Jun 1999

Pengerusi: **Profesor Madya Tan Ka Kheng, Ph.D., P.Eng.**

Fakulti: **Kejuruteraan**

Kejadian perolakan di lapisan termal adalah dikenali sebagai contoh ketidakseimbangan hidro-dinamik di mana profail suhu tidak linear berlaku. Satu *nombor Rayleigh transient* yang bekerjasama dengan mode permindahan haba and masa kritikal boleh digunakan untuk meramalkan kejadian perolakan. Mode permindahan haba adalah dikenali sebagai keadaan sempadan termal di mana nombor Biot dan nombor Rayleigh yang berkenaan boleh ditentukan. Penyelidikan kejadian perolakan boleh ditentusahkan dengan menggunakan Pegiraan untuk Bendalir Dinamik (CFD) yang menyelesaikan persamaan pembezaan separa untuk konduktiviti, momentum, tenaga dan bahan kimia. Simulasi permindahan haba di lapisan termal dilakukan oleh perisian komersial, FLUENT. Kejadian perolakan adalah diperhatikan untuk dua keadaan sempadan: fluks haba tetap and suhu permukaan tetap. Kebanyakan data yang diperhatikan adalah sama dengan teori. Masa kritikal and kedalaman kritikal boleh ditentukan dengan tepat. Akan tetapi, ketepatan permodelan permindahan haba sementara boleh dimajukan lagi.

CHAPTER I

INTRODUCTION

When a layer of fluid is heated below and cooled above, its density generally changes. If the layer of fluid is in a gravitational field, the hotter, lighter fluid rises while the colder, heavier fluid sinks. This kind of motion, due solely to non-uniformity of fluid temperature in the presence of a gravitational field, is called natural convection.

Convection currents induced in fluid by normal bottom heating and top evaporative cooling are natural phenomena commonly found in any lake, pond, or a glass of water. Thus, the prediction of the onset of convection is useful in astrophysics, geology, oceanography, climatology, meteorology and other natural sciences. Also, one major area of application where natural convection represents the key phenomenon is in energy systems such as cooling system for nuclear reactors, buoyant flow in the furnaces, performance of solar collectors and energy storage system. Material processing is another area in which buoyant flow plays a dominant role. Solidification of alloys as in casting is affected strongly by natural convection. The purity of crystals in crystal growing will be reduced by natural convection if impurity is present. In the cooling of microelectronics components, natural convection again decides the physical mechanism. In addition, other examples can be found in environmental processes such as cryogenic system including insulation and low-temperature energy transfer; thermosyphons and gravity-assisted heat pumps.

Therefore, the study of natural convection has become very important area of research in heat transfer. Spangenberg and Rowland (1961) have observed the plunging plume produced by top cooling. This was confirmed experimentally by Foster (1965b) using amplification theory, but he calculated extremely large Rayleigh number exceeding 10^8. Rayleigh numbers for the onset of convection based on the total fluid depth are clearly unrealistic. Tan and Thorpe (1995) have defined a new transient Rayleigh number that incorporated the mode of heat transport and the critical time can be used to predict the onset of convection. The mode of heat transport is characterized by a thermal boundary condition that determines the Biot number and its corresponding critical Rayleigh number. When transient Rayleigh number exceeding certain critical Rayleigh number, convection will occur.

The simulations using numerical techniques now far outweigh in detail those obtained from direct laboratory experiments. With the advent of the high-speed computer, the mechanism of the convection caused by buoyancy effect can be observed from the simulation results. The difficulties of measuring small temperature change of ±0.01 °C in the laboratory experiments can be avoided. Very expensive laboratory equipment like infrared radiometer is no longer required in the simulation. Therefore, the experiment to study the onset of convection is carried out by using FLUENT computer simulation program. There will be more than 2000 nodes in the simulation experiments to measure temperature, velocity, and physical properties of the fluids. Hence, the critical time and sizes of plume at the onset of convection can be verified. Both two-dimensional and three-dimensional simulation of heat transfer at the thermal layer will be carried out by the software. The onset of convection will be demonstrated for various boundary conditions, modes and rates of heat transfer.

Objectives

The objectives of this project include:

- To use a newly derived transient Rayleigh number to predict the onset of convection in water and glycerin.
- To use a CFD (Computational Fluid Dynamic) package to simulate the steady state and the unsteady state heat transfer in the thermal boundary layer.
- To study flow pattern during the onset of convection.
- To estimate the rate of heat transfer, velocity of the liquid, critical time and the plume sizes at the onset of convection.

CHAPTER II

LITERATURE REVIEW

Introduction

Convection induced by buoyancy forces has widespread application in many branches of physics and engineering recently. The occurrence of convection in a thin layer of fluid confined between two surfaces can be predicted by linear stability analysis (LSA) for steady-state heat conduction. The literature review will study the stability criterion in order to predict the onset of convection. Its results are summarised in terms of critical Rayleigh number. There is a new feature of stability to be discussed here. The point of instability in transient heat conduction will be examined in which the onset of convection can be predicted from transient Rayleigh number that incorporates the mode of heat transport and critical time, t_c. Analytical study of the boundary conditions and the critical time can thus be understood from the discussion. The related laboratory and numerical experiments will be presented to study the buoyancy effects explicitly.

Natural Convection Driven by Buoyancy

Natural convection is observed when the adverse density gradient in a fluid-phase heat or mass transfer process produces an unstable hydrodynamic situation. The stabilising effect of viscosity will dissipate the energy and thereby stabilise a flow. Viscosity has also the more complicated effect of diffusing momentum. Thermal conductivity or molecular diffusion of heat has some effects similar to those of viscosity, which tends to smooth out the temperature differences of an unstable temperature gradient. When the temperature difference across the layer is great enough, the stabilising effects of viscosity and thermal conductivity are overcome by destabilising buoyancy, and overturning instability ensues as thermal convection. This natural convection may be distinguished from free convection, such as due to a hot *vertical* plate, for which hydrostatic equilibrium is impossible. A basic flow of free convection may itself be unstable. Our concern here is with convection driven by buoyancy effect.

Natural convection seems to have been first described by Thomson (1882), but the first quantitative experiments were made by Benard (1900). Stimulated by Benard's experiments, Rayleigh (1916) formulated the theory of convective instability of a layer of fluid between horizontal planes and solved the idealised case when the top and bottom layers are free surface with a linear temperature gradient. He used a perturbation expansion of linearized Boussinesq approximation. The basis of this approximation is that there is a flow in which the temperature varies little, and therefore the density varies little, yet in which the buoyancy drives the motion. Then the variation of density is neglected everywhere except in the buoyancy. He assumed a convection pattern which varies sinusoidally in the horizontal (x and y) directions and showed that there is a critical value of Rayleigh number ($Ra = 657$) for free boundary condition at the top and bottom surfaces of the fluid. At the value above the critical number, cells will fill the gap between the boundaries and hence become unstable. Convection will begin when $Ra > Ra_c$. Its results are put in terms of the critical Rayleigh number, Ra_d,

$$Ra_d = \frac{g\alpha d^3 \Delta T}{\nu\kappa} \qquad [2.1]$$

where g is gravity, d is depth of fluid layer, α is thermal expansion, ν is kinematics viscosity, κ is thermal diffusivity and ΔT is drop of temperature. His linear stability analysis for steady-state heat conduction has been extended to other boundary conditions. Jeffreys (1928), Low (1929) and Nield (1964) obtained the critical Rayleigh number (Ra_d) of 1709, 1108 and 669 respectively in various boundary conditions. Later, Sparrow (1964) solved for a wide range of critical Rayleigh numbers, which are dependent on the Biot number ($=hl/k$), as shown in Table 1. Biot number is a dimensionless parameter, which provides a measure of the temperature difference between the convecting phase and the fluid. In a cooling process at a gas-liquid interface, high conducting gas at the interface will render the Biot number high, whereas insulating gas leads to low Biot number.

Block (1956) showed physically and Pearson (1958) analytically that most of the motions observed by Benard, being in very thin layers with a free surface, were driven by the variation of surface tension with temperature, not by buoyancy effect. However, Rayleigh's linear stability analysis is in accord with experiments in the deep fluid. The importance of the variation of surface tension relative to that of buoyancy diminishes as the thickness of the layer increases. Tan and Thorpe (1999) found that for thin liquid layers of thickness less than transient depth d_m, about 4 mm for many liquids, the convection is induced by surface tension. When fluids deeper than 10 mm, buoyancy-driven convection dominates.

Table 1: The Critical Rayleigh Numbers and Wave Numbers for Various Boundary Conditions (From Sparrow *et al.*, 1964)

	Top surface free		Top surface solid	
Bi	$\tilde{a}_c$	Ra_C	$\tilde{a}_c$	Ra_C
	Bottom surface at fixed temperature			
0*	2.09	669.0 (Nield's)	2.55	1295.8
0.1	2.115	682.4	2.58	1309.5
0.3	2.17	706.4	2.64	1334.1
1	2.30	770.6	22.75	1398.5
3	2.46	872.5	2.9	1497.6
10	2.59	989.5	3.03	1607.1
30	2.65	1055.3	3.08	1667.1
100	2.67	1085.9	3.11	1694.6
∞**	2.68	1100.7 (Low's)	3.12	1707.8 (Jeffrey's)
	Bottom surface at constant heat flux			
0*	0	320.0	0	720.0
0.1	1.015	381.7	1.23	807.7
0.3	1.3	428.3	1.57	869.2
1	1.64	513.8	1.94	974.2
3	1.92	619.7	2.24	1093.7
10	2.11	725.2	2.44	1204.6
30	2.18	780.2	2.51	1259.9
100	2.20	805.0	2.53	1284.3
∞**	2.21	816.7	2.55	1295.8

* Corresponds to constant heat flux
** Corresponds to fixed surface temperature

Steady State Convection

The occurrence of cellular convection in a fluid with an adverse linear density gradient has been studied theoretically and experimentally. Schmidt and Milverton (1935), Schmidt and Saunders (1937) performed the heating experiments in the layer of water between two horizontal plates. They found that the occurrence of the convection agreed and conformed to the theoretical value of Jeffreys (1928) when $Ra_d = 1709$. Silverstone (1958) also showed experimentally that the convection occur at the critical Rayleigh number of 1700-1800 in many liquids.

Koschmieder (1967) repeated Benard's experiments in silicone oil, using various shapes of convection chamber with free and solid top boundaries. He showed that the initial cellular pattern to form when the bottom is gradually and uniformly heated is the two-dimensional roll. A series of concentric annular rings was appeared in a circular chamber, as shown in Figure 1.

Steady-state convection that spans the whole depth of the fluid is difficult to be observed in experiments. It is shown in the previous literature that the experiments have to be manipulated artificially with slow heating at the initial heating stage in order to obtain linear temperature gradient in the liquid. It is unrealistic because in nature, heat transport commences with a non-linear temperature gradient, which is responsible for the onset of convection. It is also impossible to obtain a situation when the heat flux and the surface temperature are constant at the same time. The surface temperature or heat flux will change with time as any of the parameters is fixed. Therefore, the study of unsteady state convection is much more important in this literature review.

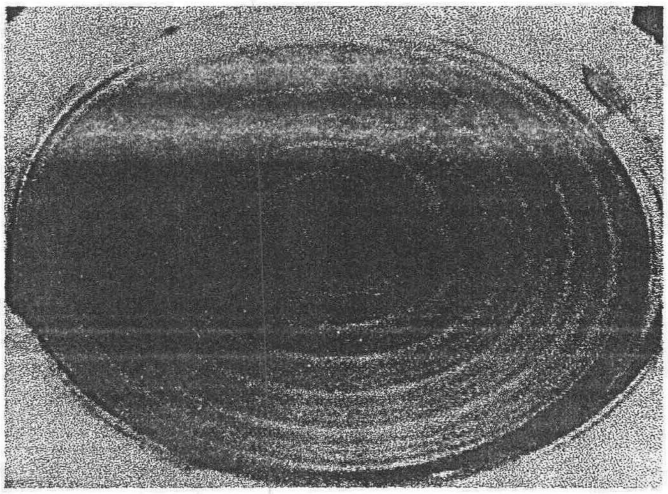

Figure 1: Convection Rolls Forming in a Layer of Silicone Oil with a Free Surface. The Concentric Ring Pattern Shows the Strong Influence of the Circular Boundary. (From Koschmieder 1967)

Unsteady State Convection

Many geophysical phenomena, such as convection in the atmosphere induced by heating from below or in the ocean induced by cooling from above, are time-dependent processes. Townsend's (1959) detailed observations, as well as common experience in the lower atmosphere, make it clear that the flux from a heated boundary is intermittent rather than steady. Buoyant fluid slowly accumulates and then breaks away, either as a thermal, or as an unsteady plume. Thermals are masses of relatively hot fluid, which ascend through the environment above a heated horizontal surface. Likewise, falling thermal plumes are masses of relatively cold fluid, which descend below a cooled horizontal surface. The generation of thermals is thought to be the result of instability in the conduction layer adjacent to the heated or cooled surface. Thermals are believed to play an important role in certain thermal convection phenomena, such as evaporative cooled layer at a liquid surface and the heated unstable layer of the atmosphere near the earth's surface that rises as thermal move through the troposphere.

When Spangenberg and Rowland (1961) conducted experiments to study convection induced by evaporative cooling in a deep tank of water, they observed plunging sheets and columns that later would develop into inverted expanding mushrooms, as shown in Figure 2. Streamers of liquid then plunged precipitously from the cooled layer. These plunging sheets grow and fade and are of continuously changing form until the circulation reaches a steady state.

They found that *nonlinear* temperature profile was responsible for the onset of convection. Therefore, conventional steady-state stability analysis cannot be applied. The total fluid depth of the water is no longer relevant in the analysis. They found that the convection occurred at 70 sec when the surface temperature has dropped about 0.36 °C.

All the early theories presented fail to bring out this feature, but Howard (1964) has proposed a model which predict a mean temperature profile and heat flux by averaging the error function profile over the period. He defined a Rayleigh number based on thickness of boundary layer δ when it become unstable, but he simply determined critical time by $Ra_{\delta} = 1000$ without considering its corresponding Biot number.

a)

b)

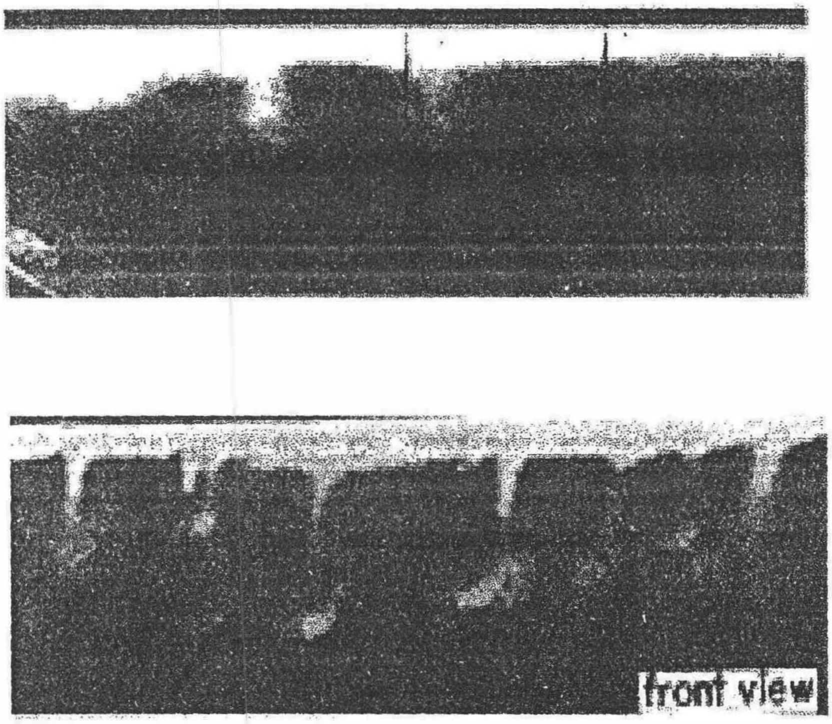

Figure 2: Front View of Photograph of Water Showing Surface Layer Cooled by Natural Evaporative Cooling, and Simultaneous Sheet and Columnar Plunging during the Onset of Convection. [Photo from Spangenberg and Rowland (1961)]

The time-dependent stability problem has been studied by Foster (1965a). He developed a simplified mathematical model in which the velocity and temperature perturbations vanish at the top and bottom boundaries but which takes into account nonlinear, time-dependent temperature profiles. Foster applied his velocity