



UNIVERSITI PUTRA MALAYSIA

**EFFECT OF NON-UNIFORM TEMPERATURE GRADIENT AND
MAGNETIC FIELD ON MARANGONI AND BENARD-MARANGONI
CONVECTION**

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**EFFECT OF NON-UNIFORM TEMPERATURE GRADIENT AND MAGNETIC
FIELD ON MARANGONI AND BENARD-MARANGONI CONVECTION**

By

SITI SUZILLIANA PUTRI BT MOHAMED ISA

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in
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January 2010



To My Family and Friends.



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirements for the degree of Master of Science

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January 2010

Chairperson : Norihan Md Arifin, PhD

Institute : Institute for Mathematical Research

The problem of thermal convection in a fluid layer driven by either buoyancy (Bénard) or thermocapillary (Marangoni) effects has recently been assumed importance in material processing. In this study, the problems of Marangoni and Bénard-Marangoni convection in a horizontal fluid layer are theoretically considered. The fluid layer is bounded from below by a rigid boundary and above by a non-deformable free surface. A linear stability analysis is applied to the problem, and the effect of non-uniform temperature profiles and magnetic field are examined. The critical Marangoni numbers are obtained for free-slip and isothermal, and no-slip and adiabatic lower boundary with adiabatic temperature on upper free surface. Six non-uniform basic temperature profiles which are linear, inverted parabola, parabola, step function (superposed two-fluid layer), piecewise linear (heated from below) and piecewise linear (cooled from above) are considered. The eigenvalues are obtained and solved using single-term Galerkin



expansion procedure. The influence of various parameters such as Chandrasekhar number and thermal depth on the convection has been analysed. Finally, we showed that the inverted parabola is most stabilizing basic temperature distribution, and the step function is the most destabilizing basic temperature distribution. We have also proved that magnetic field suppressed Marangoni and Bénard-Marangoni convection.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

**KESAN KETIDAKSERAGAMAN KECERUNAN SUHU DAN MEDAN
MAGNET KE ATAS OLAKAN MARANGONI DAN BÉNARD-MARANGONI**

Oleh

SITI SUZILLIANA PUTRI BT MOHAMED ISA

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Masalah olakan haba dalam lapisan bendalir yang disebabkan oleh samaada kesan apungan (Bénard) atau termokapilari (Marangoni) kebelakangan ini telah dianggap penting dalam pemprosesan bahan. Dalam kajian ini, masalah tentang olakan Marangoni dan Bénard-Marangoni ke atas lapisan mengufuk bendalir dikaji secara teori. Lapisan bendalir dibatasi oleh sempadan bawah yang tegar dan sempadan atas tak terancang. Analisis kestabilan linear diaplikasikan dalam masalah, dan kesan ketidakseragaman kecerunan suhu dan medan magnet telah diuji. Nilai kritikal bagi nombor Marangoni telah diperoleh untuk bebas gelincir dan berkonduksi, serta tak gelincir dan berpenibat di sempadan bawah dengan suhu yang tertebat pada permukaan atas bebas. Enam jenis profil ketidakseragaman kecerunan suhu yang terdiri daripada linear, parabola songsang, parabola, fungsi langkah (pertindihan dua bendalir), linear cebis demi cebis (dipanaskan dari bawah) dan linear cebis demi cebis (disejukkan dari atas) telah dipertimbangkan.



Nilai eigen telah diperoleh dan diselesaikan dengan menggunakan prosedur sebutan tunggal pengembangan Galerkin. Kesan pelbagai parameter yang berbeza ke atas olakan seperti nombor Chandrasekhar dan kedalaman suhu telah dianalisis. Akhirnya, kami menunjukkan bahawa parabola songsang adalah taburan suhu asas paling menstabilkan dan fungsi langkah adalah taburan suhu asas paling tidak menstabilkan. Kami juga membuktikan bahawa medan magnet berupaya menghentikan olakan Marangoni dan Bénard-Marangoni.



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I certify that a Thesis Examination Committee has met on 26 January 2010 to conduct the final examination of Siti Suzilliana Putri Bt Mohamed Isa on her thesis entitled “Effect of Non-Uniform Temperature Gradient and Magnetic Field on Marangoni and Benard-Marangoni Convection” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations, which have been duly acknowledged. I also declared that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

SITI SUZILLIANA PUTRI BT MOHAMED ISA

Date: 17 March 2010



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LIST OF ABBREVIATIONS

a	-	wave number
B_i	-	Biot number
B_o	-	Bond number
C_r	-	Crispation number
d	-	depth of fluid layer
g	-	gravity field
h	-	heat transfer coefficient between the fluid and the gas phases
H	-	Hartmann number
k	-	thermal conductivity of fluid
M	-	Marangoni number
p	-	pressure inside the fluid
p_a	-	gas pressure
P_r	-	Prandtl number
P_m	-	Magnetic Prandtl number
Q	-	Chandrasekhar number
R	-	Rayleigh number
s	-	growth rate
t	-	time
T	-	temperature of fluid
Ta	-	Taylor number



T_∞	-	temperature of the ambient gas
u	-	velocity in x-axis
v	-	velocity in y-axis
w	-	velocity in z-axis
x, y, z	-	Cartesian coordinates

Greek Symbols

α	-	positive coefficient of the thermal fluid expansion
β	-	adverse temperature gradient maintained between the upper and lower boundaries of the fluid
δ	-	surface deflection of the fluid
ε	-	thermal depth
ϕ	-	amplitude of the perturbation
γ	-	rate of change of surface tension with respect to temperature
κ	-	thermal diffusivity
μ	-	magnetic permeability
ν	-	kinematic viscosity
ρ	-	density of the fluid
τ	-	surface tension of the fluid



Subscript

o - reference quantity

c - critical state



CHAPTER 1

INTRODUCTION

1.1 Heat Transfer and Convection

Heat transfer is thermal energy transit due to temperature difference. Based on the second law of thermodynamics, heat transfer always occurs from a heated region to a cooler region. When an object or fluid is at a different temperature than its surrounding or another object, heat transfer will occur in such a way that the object and the surroundings reach thermal equilibrium. When the temperature difference between object and surrounding in proximity, heat transfer cannot be stopped, it can only be slow down.

There are three types of heat transfer: conduction, convection and radiation. Conduction is the heat transfer across the stationary medium (solid, fluid in motionless) when a temperature difference exist. Convection refers to heat transfer that will occur between a surface and a moving fluid when they are at different temperatures. In radiation, all surfaces of finite temperature emit energy in the form of electromagnetic waves. There is net heat transfer by radiation between two surfaces at different temperatures in the absence of an intervening medium.



In this thesis, we will study one of the types of heat transfer, which is convection. Convection is of two types: natural and forced. In natural convection, the heated fluid (at the bottom boundary) becomes less dense and rises. The surrounding, cooler fluid then moves down, becomes heated and the process continues. This process forms the convection. For natural convection, the flow is induced by buoyancy force which arises from density differences caused by temperature variations in the fluid. Forced convection occurs when the flow is caused by external means, such as by a fan, a pump or atmospheric winds.

1.2 Applications of Convection

In recent years, considerable attention has been given to convection because it plays an important role in numerous applications such as in welding process, coating operation and bubble motion in the melts.

1.2.1 Welding Process

Convection occurs in weld pool, which is formed by the welding process. Welding is a process in which materials of the same fundamental type or class are brought together and caused to join (and become one) through the formation of primary (and, occasionally, secondary) chemical bonds under the combined action of heat and pressure (Messler, 1993). Weld pool contains molten weld metal, the liquid which



produced by the welding process when the materials are heated and become melt. In weld pool, the driving forces for convection are buoyancy force and surface tension gradient force.

The effect of buoyancy force in weld pools can cause the convection pattern and the change of shape of the weld pool. The molten metal at the center of the pool becomes hotter and less dense. This is because thermal energy breaks down the attractive force that keeps the atoms or molecules comprising the liquid together, so that the density decreases. The molten metal at the edge of the pool is cooler and denser. It will sink under the force of gravity, causing hotter and less dense molten metal to be displaced and rises. The resulting convection pattern causes the weld pool becomes wider and shallower than its shape before convection.

The convection caused by surface tension gradient in a weld pool strongly effects the shape of the weld pool. As temperature increases, the surface tension decreases. The hotter, lower surface tension liquid at the center of the weld pool will move to the cooler, higher surface tension at the pool edges. The moving liquid (superheated molten metal) tends to erode surrounding unmelted base material at the edges of the pool, because the molten metal redistributes superheat and can cause melting. As a conclusion, the weld pool becomes wider and shallower than the shape of weld pool before convection. This change of the shape of weld pool is same as the change of the shape of weld pool caused by buoyancy force. The convection pattern and the change



of shape of the weld pool, which induced by buoyancy or surface tension gradient have been shown in Figure 1.1 and 1.2.

But, by the addition of surface-activating agents such as O, S, Se and Te, the surface gradient at the weld pool could be change (Heiple and Roper, 1981; Heiple et al., 1983). With the presence of surface-activating agents, the surface tension will increase as the temperature increases. So, the liquid at the lower surface tension at the pool edges (cooled region) moves to the higher surface tension at the center of the weld pool (heated region). The moving molten metal will erode surrounding unmelted based material at the bottom of the weld pool. The schematic of the circulation and weld pool shape due to convection dominated by surface tension gradient force (with the effect of surface-active agent) has been shown in Figure 1.3 and 1.4. As a conclusion, the weld pool becomes narrower and deeper than the shape of the weld pool before convection.



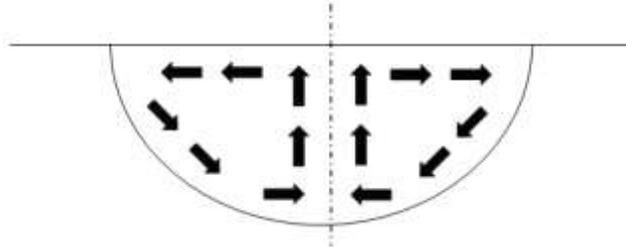


Figure 1.1: Schematic of the circulation or convection pattern in weld pool induced by either a buoyancy or surface tension gradient force

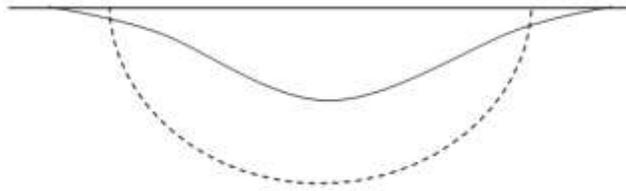


Figure 1.2: Schematic showing the shallower but wider weld pool shape due to convection dominated by either a buoyancy or surface tension gradient force

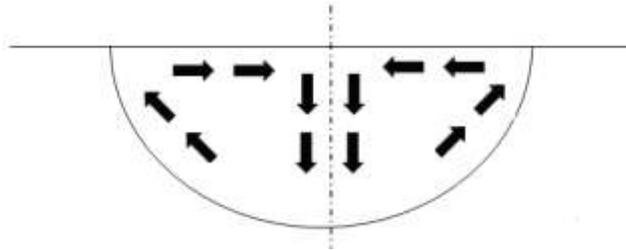


Figure 1.3: Schematic of the circulation or convection pattern induced by surface tension gradient force (with the effect of surface-active agent)

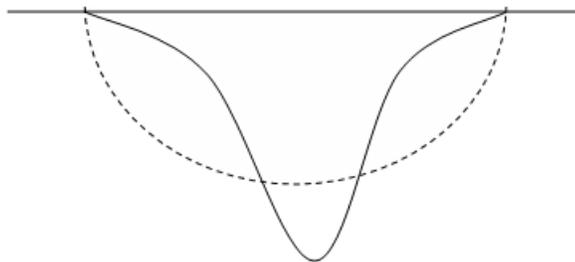


Figure 1.4: Schematic showing the deeper but narrower weld pool shape due to convection dominated by surface tension gradient force (with the effect of surface-active agent)

1.2.2 Coating Operation

In coating operations the coating liquid is spread out as a thin layer, creating large surface areas. As does every surface, these surfaces have associated surface tensions, which tend to reduce the areas of the surfaces. These surface tensions can cause defects that occur in coating process, such as convection. There are two types of convection occur in coating process, which are Bénard convection and Marangoni convection. Bénard convection is caused by density gradients. Marangoni convection is caused by surface tension gradients, which occurs from temperature gradients. For almost all coating, convection is almost due to surface tension gradients when the wet layers are less than 1 mm thick (Koschmieder and Biggerstaff, 1986).

The convection in coating process can be reduced by having thinner layers (for reducing Bénard convection) and by having higher viscosities (for reducing Marangoni convection). Evaporative cooling may cause temperature fluctuations in the surface. The temperature fluctuations will lead to surface tension fluctuations, since surface tension is a function of temperature. Surface tension fluctuations can induce Marangoni convection. Drying at a slower rate can reduce the convection. To increase the quality of coating, adding a lower-volatility solvent is always helpful.

