



**UNIVERSITI PUTRA MALAYSIA**

**LIMIT CRITERIA OF POST-TENSIONED  
THIN WEB BEAMS**

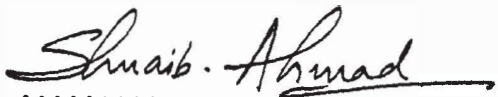
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**LIMIT CRITERIA OF POST-TENSIONED  
THIN WEB BEAMS**

By

LEE TEANG SHUI

A Thesis Submitted in Partial Fulfilment of the  
Requirements for the Degree of Doctor of Philosophy  
in the Faculty of Engineering  
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## LIST OF SYMBOLS

$f_{ci}$	Concrete strength at transfer
$f_{cu}$	Concrete cube compressive strength, $N/mm^2$
$G_k$	Dead load
$Q_k$	Applied load
$\gamma_m$	Material partial safety factor
$V_{TSR}$	Total shear resistance of a section
$V_s$	Shear resistance of a $45^\circ$ strut inclination truss analogy
$V_c$	Ultimate shear cracking resistance of a section of the concrete
$V_{co}$	Web shear cracking capacity of a section-uncracked in flexure
$V_{cr}$	Flexural shear cracking capacity of a section cracked in flexure
$b_w$	Web width of T beam
$h$	Total depth of T beam
$f_t$	Maximum principal tensile stress
$f_{cp}$	Compressive stress at the centroidal axis due to prestress, taken as positive
$f_{pe}$	Effective prestress in the tendons after all losses have occurred.
$f_{pu}$	Strength of a prestressing tendon
$v_c$	Concrete shear stress
$d_e$	Effective depth of steel reinforcements



M o	Moment necessary to produce zero stress in the concrete at the extreme tension fibre.
M u	Bending moment value at the section due to the particular ultimate load condition
V	Shear force value at the section due to the particular ultimate load condition
A ps	Total area of prestressing tendon section
A nps	Total area of nonprestressed steel section
A sv	Cross-sectional area of two legs of a link/stirrup
S v	Link/stirrup spacing
f y	Strength of a nonprestressed steel bar
f yv	Strength of the shear reinforcement
d t	Depth to the bottom most longitudinal reinforcement
v u	Maximum shear stress of concrete
b we	Effective web width of T beam
f ps	Stress in prestressed tendon
ε ps	Strain in prestressed tendon
x	Depth to neutral axis
m	Modular ratio
E s	Elastic modulus of steel
E c	Elastic modulus of concrete
h f	Flange depth



$f_{c'}$	6 inch diameter x 12 inch cylinder compressive strength at 28 days
$f_{pb}$	Design tensile stress in the tendons
$d_n$	Depth to the centroid of the compression zone or $0.45x$ if flange thickness equal or more than $0.9x$
$\epsilon_{cu}$	Concrete strain at extreme compression fibre
$\sigma$	Stress



## **ABSTRACT**

Abstract of thesis submitted to the Senate of Universiti Pertanian Malaysia, in partial fulfilment of the requirements for the degree of **Doctor of Philosophy**.

### **LIMIT CRITERIA OF POST-TENSIONED THIN WEB BEAMS**

By

**LEE TEANG SHUI**

October, 1989

Supervisor : Dr. S.A. Salam  
Faculty : Engineering

A total of 45 partially prestressed post-tensioned thin web T-beams were tested to investigate their ultimate shear and flexural strengths and the serviceability limits of deflection and crack width. The varying parameters included in the study are the amount of prestress, amount of supplementary nonprestressed reinforcements, prestressing tendons being bonded or unbonded, absence or presence of shear reinforcements and width of the web. A strain compatibility method with the associated bond factors is presented for the ultimate flexural strength analysis of the beams. For prediction of theoretical loads, material characteristics are taken at their ultimate with no materials safety factor associated. Equilibrium of forces and a trilinear stress strain relationship for

