UNIVERSITI PUTRA MALAYSIA

STATISTICAL PROCESS CONTROL ANALYSIS

IBARHIM SHEIKH ABDUL KADIR HAGI

FSAS 1999 35

STATISTICAL PROCESS CONTROL ANALYSIS

IBARHIM SHEIKH ABDUL KADIR HAGI

MASTER OF SCIENCE
UNIVERSITI PUTRA MALAYSIA

1999

STATISTICAL PROCESS CONTROL ANALYSIS

By

IBRAHIM SHEIKH ABDUL KADIR HAGI

Thesis Submitted in Fulfillment of the Requirements for the
Degree of Master of Science in the Faculty of
Science and Environmental Studies
Universiti Putra Malaysia

March 1999

TO MY PARENTS

ACKNOWLEDGEMENTS

Firstly, praise be to God, for giving me the strength and patience to complete this work. I would like to thank my supervisor Dr. Habshah Binti Midi for her excellent supervision, invaluable guidance, helpful discussions and continuous encouragement, without which this work could not be finished. My thanks also to the member of my supervisor committee, Associate Professor Dr. Muhamad Idreess Ahmad and Dr Kassim bin Haron for their invaluable discussions, comments, and help. Similar thanks must go to Dr Mahendran Shitan for his help in S-plus programming Language.

TABLE OF CONTENTS

LIST OF TABLES

Table

LIST OF FIGURES

FIGURES **Page**

Abstract of thesis submitted to the Senate of Universiti Putra Malaysia in fulfillment of the requirements for the degree of Master of Science

STATISTICAL PROCESS CONTROL ANALYSIS

By

IBRAHIM SHEIKH ABDULKADIR HAGI

March 1999

Chairman: Habshah Bt Midi, Ph.D.

Faculty: Science and Environmental Studies

This thesis is concerned with the investigation of the two key aspects of statistical process control. The first aspect is maintaining a stable process so that the pattern of variation of process out-put is not changing. In order to maintain a stable process, the study includes an examination of the state of control of the process. A traditional variable control charts, $\bar{x}$ and R charts and also the $\bar{x}$ control chart based on sample median and median control chart in conjunction with a chart for sample range were used for both normal and non-normal process. The second aspect depicts the process capability. Assuming that the processes have reached the state of statistical control, capability measurements were proceeded in this study for both normal and non-normal processes.

A simulation studies are carried out to compare the performance of the traditional and the robust control chart. Likewise, the classical capability index is compared to two robust capability index. The results of the study indicate that the traditional and the robust control chart are equally good when no contamination in the data. However, the later performs better than the former in the presence of outliers in the data. Similarly, the traditional process capability index are almost as good as the robust capability index as proposed by Clement (1989) and John Kot (1993) in a well behaved data. Nevertheless, the robust capability index were found to be better compared to the traditional index when contamination occurs in the data.

The study also carried out an investigation of properties of the three types of bootstrap confidence interval for estimating the process capability index (C_{pk}), namely the standard, percentile and bias corrected and accelerated for two processes (normal and skewed). The average lengths displayed a consistent pattern where the longest intervals were the standard intervals, and with the shortest intervals being the percentile and bias corrected and accelerated intervals for both normal and highly skewed processes. The results of the study seem to be consistent for sample size $n = 25$ to $n = 50$.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia bagi memenuhi keperluan untuk ijazah Master Sains.

ANALISA PROSES KAWALAN BERSTATISTIK

Oleh

IBRAHIM SHEIKH ABDUL KADIR HAGI

March I999

Pengerusi: Habshah Bt Midi, Ph.D.

Fakulti: Sains dan Pengajian Alam Sekitar

Tesis ini berkenaan dengan penyelidikan dua aspek utama dalam proses kawalan berstatistik . Aspek yang pertama ialah penyelenggaraan proses yang stabil supaya corak variasi proses output tidak berubah. Bagi menentukan proses yang stabil, kajian ini meliputi pemeriksaan keadaan bagi proses yang terkawal. Carta pembolehubah kawalan tradisi , $\bar{x}$ dan carta R dan juga carta kawalan $\bar{x}$ berasaskan median sampel dan carta kawalan median yang berkaitan dengan carta bagi sampel julat telah digunakan bagi proses normal dan tak normal. Aspek yang kedua menerangkan proses keupayaan. Dengan menganggapkan bahawa proses telah sampai ke tahap kawalan berstatistik, ukuran keupayaan diteruskan dalam kajian ini bagi kedua dua proses normal dan tak normal.

Kajian simulasi telah dijalankan untuk membandingkan carta kawalan tradisi dan carta kawalan teguh. Index keupayaan klasik juga dibandingkan dengan dua index keupayaan teguh. Keputusan kajian menunjukkan bahawa prestasi carta kawalan teguh adalah sama baiknya dengan carta kawalan tradisi apabila data dalam keadaan 'bersih'. Walabagaimana pun, prestasi carta kawalan teguh adalah lebih baik daripada carta kawalan tradisi apabila wujud data terpencil di dalam data. Begitu juga dengan keputusan perbandingan diantara index keupayaan proses tradisi dan index keupayaan proses teguh yang dicadangkan oleh Clement (1989) dan John Kot (1993). Kedua-dua kaedah menunjukkan prestasi yang agak sama baiknya apabila data dalam keadaan bersih. Namun begitu, index keupayaan teguh didapati lebih baik jika dibandingkan dengan index keupayaan tradisi apabila terdapat data `kotor` di dalam data.

Kajian ini juga dijalankan bagi memeriksa sifat-sifat bagi tiga jenis selang keyakinan bootstrap bagi menganggarkan index keupayaan proses (Cpk), iaitu selang keyakinan piawai , persentil dan pincang dibetulkan dan dipecut (Bca) bagi dua proses (normal dan pencung). Purata panjang selang menunjukkan corak yang konsisten dengan selang yang paling panjang ialah selang keyakinan piawai dan kedua-dua selang keyakinan persentil dan BCa adalah lebih pendek bagi kedua dua proses normal dan proses pencung. Keputusan kajian ini kelihatan konsisten bagi setiap saiz sampel n=25 dan n=50.

CHAPTER I

INTRODUCTION

Statistical methods provide a very effective means for the development of new technology and quality control in manufacturing process. Many leading manufacturers have been striving for an active use of statistical methods, and some of them spend more than 100 hours annually in in-company education on this subject. While knowledge of statistical methods is becoming part of the normal fixture of an engineer, the fact that one knows statistical methods does not immediately lead to the ability to use it. The ability to treat maters from the statistical viewpoint is more important that the individual methods. In addition, it is not easy to chose among the different statistical techniques that can be used in solving a given problem.

One of the most important statistical tools for the quality control in manufacturing process is Statistical Process Control (SPC); the goal of statistical process control is to maintain a stable process where the pattern of variation of the process output is not changing. Statistical process control may be addressed in terms of three key aspects:

a) Process control: maintaining the process on target with respect to centering and spread.

b) Process capability: Determining the inherent spread of a controlled process for establishing realistic specifications.

c) Process change: Implementing process modification as a part of process improvements and troubleshooting.

Process capability indices (an aspect of SPC) are one of the focuses of this study.

The use and abuse of process capability indices have been the subject of considerable controversy in the last few years, their widespread and often uncritical use may almost inadvertently have led to some improvement in quality, but also, almost certainly, have been the cause of many unjust decision, which might have been avoided by better knowledge of their properties.

The fundamental task of process capability indices are to determine whether a manufacturing production process is capable of producing items within specification limits and that is customer requirements (Kane, 1986; Rado, 1989; Montgomery, 1985).

Process capability indices are used widely throughout industries, to give a relatively quick indication of process capability in a format that is easy to compute and understand.

Among the widely used process capability indices are C_p and C_{pk}. Each of these indices are indicative of process ability to satisfy customer requirements (i.e. specification limits). Process capability indices such as C_p and C_{pk} are assumed to have the properties of normality. The C_p index is used to measure the potential of a process to satisfy a customer requirements (i.e. specification limits). The manufacturer in the form of the specification limits usually sets the allowable process spread, and the actual process spread is based on the distribution (usually estimated) of the product obtained from manufacturing process. The allowable upper and lower specification limits as denoted as (USL and LSL) respectively, the actual process spread usually contained in the natural tolerance limits which are considered to be six times the true standard deviation of the process from manufacturing product. The C_{pk} index is developed to indicate how process confirms to two-sided specification limits. This C_{pk} index is used to measure process performance. The manufacturing engineer recommended value of C_p and C_{pk} equals to 1.33 is the minimum value that should be observed for the acceptable process capability.

One of the advantages of the process capability indices are that it provides an easily understandable aggregate measure of goodness of the process performance. The ability to meet specifications is the criterion used for measuring the attractiveness of the process. The capability described above are nondimensional, which makes them more versatile and appealing because they do not depend on the specific process parameter units (Kane, 1986).

The determination of the capability of a process should begin only after the process has been brought to a state of statistical control. A process is said to be in statistical control when the only source of variation in the system is a result of chance causes. The use of control chart is an important step, which must be taken in early stages in an SPC program to eliminate assignable causes, reduce variability, and stabilize process performance.

In this study, a traditional variable control charts commonly known as mean $(\bar{x})$ and range (R) control charts were used to examine the state of control of the process. This will be the first stage of SPC procedure. The variable control charts such as $\bar{x}$ and R charts were chosen because it allows studying a process regardless of its specifications. The $\bar{x}$ and R charts are also allow to employ both instantaneous variable (short-term process capability), and variability across the (long-term process capability). It is particularly helpful if the data for a process capability study are collected in two to three different time periods, such as different shift and different day's. The variable control charts are important in the quality program of many industries, their ability to identify process improvement opportunities. Inherent in the construction of control charts for variables is the assumption that the process under examination is normally distributed with independence and identical observations. Continuous follow process often has outocorrelated observations, which violate the independence assumption and render standard control charts for variables unreliable. In order to rectify this problem a more robust control charts are needed such as mean ($\bar{x}$) control chart based on sample median and median control chart in conjunction with a chart for sample range R for individual observations. The final solution is not easily effected by outocorrelation.

Assuming that the processes have reached a state of statistical control, the capability measurements can then proceeded. The C_p and C_{pk} indices have been calculated to determine whether a manufacturing production process is capable of producing items within specification limits. The manufacturing engineers recommend that the values of C_p and C_{pk} which equals to 1.33 is the minimum value that should be observed for acceptance process capability. But the question is often arises is whether it can meet the tolerance. As mentioned earlier, it is sensible to estimate the ability of a process to meet specification limits only when it is in a state of statistical control. In that state, the process has no assignable causes, so exhibited process variability is a reflection of what the process can achieve. A process should first be analyzed to verify that it is in control before its capability is estimated. An assumption that always is made is that, the process output (which is, the distribution of the quality characteristic under consideration) is normal. The assumption of normality can be validated by means of empirical plots of histogram, normal probability plots, or statistical test for goodness of fit, such as Chi-square tests or the Kolmogorov- Smirnov Test (Cochran,1952; Duncun, 1986; Massey, 1952; Nelson, 1986; Shapiro, 1980). A study by Gunter (1989), have criticized the validity of traditional method in using the capability measurements specially when the underlying distribution is not normal, such approach ignores the fact that the C_p and C_{pk} are random variables. However, when capability measures are used, it is worth noting that some processes do not follow normally distributions, perhaps, due to the presence of autocorrelations or outliers in the data. The discussion of non-normality falls into two main parts. The first and easier of the two, is investigation of the properties of process capability indices and their estimators when the distribution of process has specific non-normal forms. The second and more difficult, is development of the methods to cuter for the non-normality and consideration for the use of new process capability indices specially designed to be robust (i.e., not too sensitive) to non-normality. Only recently statisticians have provided a new methods which are considered to be an alternative to traditional method when underlying distribution is not normal.

Many studies to improve process capability indices when underlying distribution is not normal have been done. The papers are too numerous to be reviewed on non-normal process capability indices (Kane 1986; Gunter 1991; Pean et al., 1992; English and Bates 1991, Kocherlakota 1992; Subramaniam 1966,1968; Karl pearson 1983; Clement 1989;John-Kot 1992; Munecchilka 1986,1992; Johnson and Kot 1970; Chan 1988; Dovich 1991,a, b; McCoy 191; Chan, L.K., Cheng, S.W., and Spring, F.A. 1988; Chan and Zhang 1990; Johnson, Kortz and Pearn 1992; Hall, P. and Martin 1988,1989; Owen and Borrego 1990; Marucucci and Beazaly 1988; Mirbella 1991; Spring 1991 a; Subbaih and Toom 1991; Kushler and Hurly 1992; Johnson 1993; Balistskaye and Zoatuhina 1988; Hall 1992; Gunter 1989; Franklin and Wassermann 1991; Price 1992; Goh 1994; Mooney and Duvall 1993).

However, the effects of non-normality on properties of process capability indices have not been major research items until quite recently. The number of different capability indices have increased and so lead to confusion among practitioners, as such were unable to provide an adequate and clear explanation of the meaning of various indices, and more importantly when the underlying distribution is not normal. Literature review indicated that some aspects of these problems have not been solved completely, even the problems solved so far have different approaches. It is important to recognize non-normality to avoid gross errors, not only in capability measurements but also for correct prescription of control chart techniques and other aspects of statistical analysis. Therefore, we should incorporate robust method in our study so as to correct the problems of outliers and outocorrelations if it exist in the data.

Statement of the Problem

Many manufacturing production processes have problem in determination of process capability whether, it can produce items within specification limits, and this is because capability indices such as C_p and C_{pk} are assumed to have properties of normality. Gunter (1989) pointed out, many processes are normal with skewed distribution, however, and approximate technique based on traditional method may perform badly and can be misleading, especially when underlying distribution is not normal. Literature showed that some researchers have dealt with non-normal process capability indices but their approaches in dealing with non-normal process are different, and non-recommendable.

This research is concerned with the three key aspects of SPC as mentioned earlier. To examine the state of control of the process, variable control charts such as $\bar{x}$ and R charts were used as a traditional method, where robust standard mean control chart and median in conjunction with a chart for sample range were used as robust method. The study also focused on the estimation of capability measurements for normal and non-normal processes. Traditional method used normal process, whereas non-normal process used robust method, which was discussed by (Clement, 1989; John-Kot, 1992).

The study also focuses on the estimation of confidence interval limits for C_{pk} by using non-parametric bootstrap method. The percentile, bias corrected and accelerated confidence intervals for C_{pk} index were calculated from bootstrap method. Such intervals represent a major step toward the correct understanding and interpretation of C_{pk} index. Bootstrap confidence interval for estimating C_{pk} does not depend on the usual assumption of normality and in fact can be calculated regardless of whatever the underlying process distribution was.

Some Key Words and Definitions

For the sake of completeness, a brief review of some important concepts that were used frequently in this study are given below. Some important key words include Specification limits and tolerance limits, assignable causes, and common causes.

Specification and Tolerance Limits

Specification and tolerance limits are often used interchangeable and are defined by the ANSI/ASQC standard A1 (1987) as the limits that define the conformance boundaries for an individual unit of a manufacturing or service operation. The standard suggests that the tolerance limits are generally preferred in evaluating the manufacturing or service requirements, whereas specification limits are more appropriate for categorizing materials, products, or services in terms of their stated requirements. In general, tolerances are subsets of specifications. Usually, tolerances pertain to all requirements.

Tolerance limits may be two-sided (with upper and lower limits) or one-sided with either upper or lower limits. A lower tolerance limit defines the lower conformance boundary for an individual unit of a manufacturing or service operation; an upper specification limit is determined by the needs of the customer. What the customer wants in the product service is analyzed by means of market research and incorporated through product or service design. These limits are placed on a product characteristic by designers and engineers for an individual unit in order to ensure an adequate functioning of the product. Some part of this study uses the term's specification limits and tolerance limits interchangeably because of the ANSI/ASQC standard A1 (1987) makes no distinction between them.

Assignable Causes or Special Causes

Variability caused by special causes or assignable cause that is not inherent in the process. That is, it is not part of the process as designed and does not effect all items. Special causes can be use of a wrong tool, an improper raw material, or an operator error. If an observation falls outside the control limits or non-normal pattern is exhibited, special causes are assumed to exist, and the process is said to be out of control. One objective of control chart is to detect the presence of special causes as soon as possible to allow appropriate corrective action. Once special causes are eliminated through remedial actions, the process is again brought to state of statistical control. Deming (1982) believed that 15% of all problems are due to special causes. Actions on the part of both management and workers will reduce the occurrence of such causes.

Chance Causes or Common Causes

Variability due to common causes is inherent to a process. It exists as long as the process is not changed and is referred to, as the natural variation in a process. It is inherent part of the process design and effects all items. This variation in the effect of many small causes and cannot be totally eliminated. When this variation is consisiting, we have what is known as a stable system of common causes. A process operating under a stable system of common causes is said to be in statistical control. Examples include inherent variation of incoming raw materials from a qualified vendor, a lack of adequate supervision skills, the vibration of machines, and fluctuations in working condition. Management alone is responsible for common causes. Deming (1982) believed that about 85% of all problems are due to common causes and hence can be solved only by action on the part of management. In a control chart, if quality characteristic values are within the control limits and no non-random pattern is visible, it is assumed that a system of common causes exists and the process is in state of statistical control.

The Objectives of the Study

The main objectives of the study are

1. To ensure the stability of the manufacturing process by using robust median control chart and median control chart in conjunction with chart for a sample range R.

2. To employ a robust method which may improve capability of the process when underlying distribution is not normal. Our focus of the study is limited to the statistical process control for variables.

Other aspects that will also be considered in this study are

i) To compare the performance of traditional and robust control charts

ii) To compare the performance of traditional and robust process capability index

iii) To adopt the non-parametric bootstrap method on the Cpk index.

Organisation of the Thesis

This thesis is comprised of five chapters.

Chapter I, introduces the objectives of the research, and depicts the need to use robust estimation procedure instead of traditional methods. Chapter II describes the construction of control charts by using both traditional and robust methods. The traditional control chart was based on sample mean whereas the robust control charts were based on sample median. A complete discussion on process capability measurement was exhibited, in Chapter III after assuming that the processes have reached a state of statistical control. The discussions address the process capability index in both situations where the process having a normal and non-normal distribution. In this chapter the classical C_{pk} is compared to the robust methods such as Clement and John Kot techniques. In Chapter IV the parametric bootstrap method for evaluating percentile, bias corrected and accelerated confidence