



UNIVERSITI PUTRA MALAYSIA

**DESIGN OF A FUZZY LOGIC CONTROLLER FOR SKID STEER
MOBILE ROBOT**

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**DESIGN OF A FUZZY LOGIC CONTROLLER FOR SKID STEER MOBILE
ROBOT**

By

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**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,
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The control problem of four-wheeled skid steering mobile robots is quite challenging mainly because the skid steering system is an underactuated system and its mathematical model is highly uncertain. Skid steering configurations employ a differential-drive technique in which the wheels rotation is limited to around one axis and the lack of a steering wheel causes the navigation to be determined by the change of speed in either side of the robot for turning. Equal speed in both sides causes a straight-line motion. However, the implementation of the dead reckoning technique on skid-steer mobile robots will limit the precision of current robot's position because skid-steer configuration intentionally relies on wheel slippage for normal operation and this possesses some difficulties when implementing motion control using the odometric system.

The thesis describes the design of a fuzzy logic controller to compensate the dead reckoning limitation and implementation on a skid-steer mobile robot. The fuzzy controller has two inputs (angle error and distance), two outputs (translational and rotational speed) and 14 rules. These inputs are computed from the dead-reckoning



method that is totally reliant on the odometry readings and data are fuzzified to be the inputs of the fuzzy controller. The outputs are the analogue voltages to the left and right motors, which drive the mobile robot. For simplicity, membership functions consisting of triangular and trapezoid shapes have been adopted. The membership functions of the fuzzy sets are chosen by trial-and-error based on experimentation. The heuristic rules control the orientation of the robot according to the information about the distances from the desired positions. The crisp output values from the fuzzy logic controller are decoded and fed into a decision module where the ratios of both sides motor voltage are determined for every smooth change in speed of the motors.

To facilitate the implementation of control system, real-time execution is done in an indoor environment. Data acquisition is done in a LABVIEW and a MATLAB control algorithm is called in LABVIEW. A real mobile robot, PUTRABOT2 was used to conduct the experiment. Performance evaluation is observed from the accumulated error in orientation and its trajectory obtained after mapping the information gathered from the real world via odometry sensors. Few features such as the rise time, settling time and peak time of the output responses are analyzed. Comparisons are made between fuzzy logic and PD controllers. Comparative results among these two controllers indicate the superiority of the fuzzy approach with the ability to minimize the position and orientation errors. Moreover, the trajectory accuracy is very high and more reliable in the presence of unreliable odometry readings.



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**REKA BENTUK PENGAWAL LOGIK SAMAR UNTUK ROBOT
BERGERAK KEMUDI KELINCIR**

Oleh

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Masalah kawalan robot bergerak empat roda secara kemudi kelincir adalah agak mencabar kerana sistem kemudi kelincir merupakan sistem yang mempunyai jumlah penggerak yang kurang dari darjah kebebasannya dan model matematikanya adalah tidak pasti. Konfigurasi-konfigurasi kemudi kelincir menggunakan teknik pacuan pembezaan dimana putaran roda-roda adalah terhad keliling satu paksi dan ketiadaan roda stereng menyebabkan navigasi ditentukan oleh perubahan laju di salah satu bahagian tepi sesebuah robot itu untuk membelok. Laju yang sama pada kedua-dua belah bahagian menyebabkan ia bergerak lurus. Namun begitu, pelaksanaan teknik jangkaan kedudukan keatas robot kemudi kelincir akan menghadkan kejitian kedudukan semasa robot kerana konfigurasi kemudi kelincir bergantung secara keseluruhannya keatas gelinciran roda sebagai operasi normal dan mempunyai kesukaran apabila melaksanakan kawalan gerakan dengan menggunakan sistem odometri.

Tesis ini menerangkan rekabentuk pengawal logik samar untuk memampas pembatasan teknik jangkaan kedudukan dan melaksanakannya keatas robot mudah bergerak empat roda secara kemudi kelincir. Pengawal logik samar tersebut



mempunyai dua input (ralat orientasi dan jarak), dua keluaran (kelajuan terjemahan dan putaran) dan juga 14 aturan. Dua input tersebut dikira melalui teknik jangkauan kedudukan yang sepenuhnya bergantung kepada bacaan-bacaan dari odometri dan data-data tersebut disamakan sebagai input-input untuk pengawal samar. Keluarannya adalah voltan analog yang dihantar ke sebelah kiri dan kanan motor yang memacu robot tersebut. Untuk kemudahan, fungsi-fungsi keahlian set samar dipilih melalui kaedah cuba-cuba berdasarkan eksperimen. Aturan-aturan heuristik mengawal orientasi robot dan jarak dari kedudukan yang dikehendaki. Keluaran-keluaran jitu dari pengawal logik samar kemudiannya di nyahkod dan dihantar ke modul di mana nisbah voltan kedua-dua belah motor ditentukan untuk setiap perubahan laju yang setara untuk motor-motor tersebut.

Untuk memudahkan pelaksanaan sistem kawalan itu, pelaksanaan masa sebenar dilakukan di dalam persekitaran tertutup. Perolehan data dan algoritma kawalan MATLAB dibangunkan dalam LABVIEW. Robot bergerak sebenar, PUTRABOT2 telah digunakan untuk menjalankan eksperimen tersebut. Penilaian prestasi dilihat pada ralat terkumpul orientasi robot dan trajektori navigasinya setelah memetakan maklumat yang dikumpul melalui penderia odometri dalam masa sebenar. Beberapa ciri seperti masa naik, masa ketetapan dan masa puncak dari reaksi keluaran dianalisa. Perbandingan dibuat antara pengawal logik samar dan pengawal PD. Keputusan-keputusan secara perbandingan diantara kedua-dua pengawal tersebut menunjukkan kelebihan pendekatan logik samar dengan kebolehan meminimumkan ralat-ralat kedudukan dan orientasi. Tambahan lagi, ketepatan trajektori adalah tinggi dan boleh dipercayai dengan kehadiran bacaan odometri yang tidak tepat.

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I certify that a Thesis Examination Committee has met on **3 April 2009** to conduct the final examination of Mohd Azizi Bin Abdul Rahman on his thesis entitled “**Design of a Fuzzy Logic Controller for Skid Steer Mobile Robot**” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science degree.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

MOHD AZIZI ABDUL RAHMAN

Date: 17 July 2009



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CHAPTER 1

Introduction

1.1 Overview

The development of the control strategy of autonomous mobile robots in real-world environments constitutes one of the major trends in the current research on robotics [1, 2]. A significant problem in autonomous control strategies is the need to cope with the large amount of uncertainty that permanently exists in a natural environment. The goal of autonomous mobile robotics is to build physical systems that can be controlled without human intervention in unstructured or unpredictable environments, i.e. in real-world environments that have not been specifically engineered for the robots.

In future's flexible manufacturing system (FMS) environment, the autonomous mobile robot (AMR) will play a very important role. It will transport parts from one workstation to others, load and unload parts, remove any undesired objects from floors, and so on [2]. Without AMR, the workstations will be isolated and will not become a manufacturing system. In addition to indoor FMS, there are some other outdoor occasions that an AMR may take care on heavy responsibilities. For example, in automated construction site, exploration of hardly accessible areas, military missions, harmful-materials handling, maintenance robots operate in hazardous environments, interplanetary exploration, and so on [1]. It is clear that the development of mobile robots is still continuing considering present ongoing research.



1.2 Problem Statement

Despite the impressive advances in the field of autonomous robotics in recent years, a number of issues remain. The main issues in mobile robot research include navigation strategy, mission planning, maneuvering, and manipulation [2]. Some of the problems are solved as stated in [12].

The choice of skid-steer configuration other than typical mobility configurations for a mobile robot platform represented a challenging part of research element to this work. The control problem for a skid-steering mobile robot is quite challenging mainly because of two facts [3]. Firstly, a skid-steering system is an under-actuated system and secondly, its mathematical model is uncertain. Any approach to controlling a dynamic system needs to use some knowledge, or model, of the system to be controlled. In the case of a robot, this system consists of the robot itself and the environment in which it operates. However, while a model of the robot on its own can normally be obtained, the situation is different when a robot is embedded in the real world and in an unstructured environment. These environments are characterized by the ubiquitous presence of uncertainty or even worse, the nature of the involved phenomena which is not able to be precisely modeled or quantified.

The accuracy of odometric measurements for performing dead-reckoning is to a great extent a direct function of the kinematics design of a vehicle [4] and the realities of mechanical systems will limit the accuracy of dead reckoning application [5]. Such a skid-steer configuration intentionally relies on wheel slippage for normal operation and however provides rather poor dead-reckoning information. This configuration poses special difficulties when implementing motion control by using odometry [6-

8]. Furthermore, skid-steering kinematics is not straightforward, since it is not possible to predict the exact motion of the vehicle based solely upon its control inputs. Thus, pure rolling and no-slip assumptions usually considered in kinematics models for non-holonomic wheeled vehicles do not apply [9]. Therefore, a fuzzy logic approach becomes more attractive in this study as a low level controller as proposed in [33].

Fuzzy logic unlike classical logic is tolerant to imprecision, uncertainty, and nonlinearity. This makes it easier to implement fuzzy logic controller (FLC) to nonlinear models compared to other conventional control techniques. In the context of mobile robot navigation, a fuzzy logic-based system has the advantage in that it allows an intuitive nature of rule-based navigation to be easily modeled using linguistic terminology.

The computational loads of typical fuzzy inference systems are relatively light. As a result, fuzzy control systems permit more or less intelligent decisions to be made in the real-time implementation, thus allowing smooth and uninterrupted motion.



1.3 Aim and Objectives

The aim of this research is to design a control system by using a fuzzy logic approach in order to be implemented on a real skid steering mobile robot and the research objectives are:

- To apply a fuzzy control system by experiment applying to control of a mobile robot using MATLAB and LABVIEW.
- To implement a fuzzy system that is able to perform heading control of the mobile robot to the desired position(s).
- To prove the reliability and compare the performance of a fuzzy-controlled mobile robot in terms of heading control and position estimation.

1.4 Scope of Work

The fuzzy system approach is the control framework of choice in the studies covered in this thesis. Here, it is applied to a mobile robot, whose primary task is to control the robot from any starting position to the desired position(s) in an indoor environment. A coordinate system is primarily used to locate the desired position(s) in the environment.

It is worth noticing that the angle and distance errors computed by the odometric system are accurate since four-wheeled skid-steer configuration may cause instability in term of the mechanical part. Moreover only forward velocity is considered in the work of this thesis.

1.5 Thesis Outline

The main topic of this thesis, fuzzy control system, is covered in the next two chapters. Literature review is discussed in chapter 2; where an overview of previous works done by researchers is given with emphasis on mobile robot control problems and intelligent techniques used in designing the control system. The fuzzy logic approach is reviewed and applied in designing the controller. The methodology for designing a fuzzy system is described in chapter 3; where issues on designing a fuzzy system for a skid-steer mobile robot are presented. Then, a simple individual behavior design for a fuzzy logic controller is introduced and implemented to the low level control system. Chapter 4 provides results and evaluations on the experimental works; results that emphasis on the localization and errors in orientation are discussed thoroughly and some evaluation results are presented followed by a discussion. Comparisons are made between the fuzzy logic controller and a Proportional-Derivative (PD) controller. Finally, conclusions are drawn in chapter 5; the thesis is summarized with particular focus on the control system that is specially designed for skid steering mobile robot as a whole. Moreover, the major open issues of future research are discussed.



CHAPTER 2

Literature Review

This chapter reviews previous works done by several researchers related to the control strategy of mobile robots in structured and unstructured environments. The literature review starts with the concept of behavior-based robotics, fuzzy control scheme, positioning technique of mobile robots using dead-reckoning, and the concept design of a mobile robot platform.

2.1 Behavior-based Robotics

Early intelligent systems were dominated by approaches from classical artificial intelligence. Perfect knowledge of the environment and a deterministic outcome of actions were presumed [1]. Then, a symbol based planner calculates an action sequence to be executed in order to accomplish the given goal. The applicability of this methodology to mobile robots was rather limited due to various deficiencies. Complete knowledge of the environment is usually not available and the outcome of the control actions is subject to noise influenced by imperfect actuators and other outside influences from the surrounding. Hence the robot must make use of sensors to update its perception of the environment.

The first systems for autonomous robots were designed based on these sensory data and a reasoning system inspired from classical artificial intelligence (AI) [10]. Extensive planning and re-planning due to the noise in sensory data and outcome of motor actions are carried out. Thus, the plans used a large amount of computation time which made the robots move very slowly. Furthermore, environments are often

changing in an unpredictable way, which means that it is impossible to obtain all of their properties even in theory.

A new paradigm, behavior based robotics, has established itself over the last decade to solve the flaws of classical AI approaches. Behaviors as a method of controlling robots were inspired by Brooks' Subsumption Architecture [11]. In a behavior-based approach, the control of the platform is distributed to several behaviors. Each of these behaviors is tightly coupled to sensory data and controls the robot in a reactive manner to accomplish some sub-problem of the navigational task.

Typical examples of such subtasks are "target approach", "obstacle avoidance", or "wall following" as indicated in [11]. Intelligence comes through the combination of these perception-action loops rather than through symbolic reasoning. However, these initial purely reactive approaches were rather limited in overcoming complex tasks, for example the reaching of a distant goal in a large-scale environment. Hence, elements of the classical approach were incorporated again to enable planning and reasoning on a symbolic level using some predefined model of the world.

2.1.1 Behavior-based in Navigation

Target approach is a typical example of a subtask of the navigation problem. Research work recorded by [12] stressed out an approach to employ a supervision layer based that makes context-based decisions as to which behavior(s) to be activated rather than processing all behaviors and then blending the appropriate ones. In the same work, an individual behavior design and action coordination technique was proposed. Basic behaviors, such as move to the target, goal-seeking, obstacle

avoidance and go to predefined position can be subdivided into simple tasks which are easier to manage. This divide-and-conquer approach has been deployed in their work and proved to be a successful approach for it makes the system modular. Their work was inspired by several researchers who were previously working on behavior-based navigation approaches such as the use of reactive behaviors or motor schema [13], the subsumption architecture [11], a distributed architecture for mobile navigation (DAMN) [14] and the coordination behavior technique used in their work inspired by Seraji et al. [15]. For the evaluation of their proposed scheme, some typical cases were simulated in which a robot is to move from a given current position to a desired goal in various unknown environment. It was successfully tested, in which the robot managed to navigate its way towards the goal while avoiding obstacles.

Payton et al. [16] have presented a command fusion method for combining outputs of multiple behaviors in a mobile robot navigation system to reduce information loss due to command fusion. This approach was motivated by their observation that a fixed-priority-based command arbitration usually results in loss of information causing increased difficulty in decision making.

2.2 Dead Reckoning

Dead-reckoning is the most widely used technique for estimating the position of a mobile robot, taking into account prior position and amount of distance traveled. The parameters of dead-reckoning are direction of motion and distance traveled. In most practical applications dead-reckoning provides easily accessible real-time positioning information in between periodic absolute position measurement [17]. Location is a

basic part of navigation. A basic position estimation method often employed in mobile robot applications is relative positioning [18, 19]. Relative positioning is based on dead-reckoning (i.e. monitoring the wheel revolution to compute the offset from a known starting position). Dead-reckoning is simple, inexpensive, and easy to accomplish in real-time. However, the disadvantage of dead-reckoning is its unbounded accumulation of errors. An improved dead-reckoning can reduce the installation costs of mobile robot systems because it simplifies the fundamental problem of position determination [17].

Using geometric equations, it is straight-forward to compute the momentary position of the vehicle relative to a known starting position and that is called dead-reckoning. Suppose that at sampling interval I , the left and right encoders show a pulse increment of N_L and N_R , respectively. Suppose that [19] the conversion from encoder pulses to distance is

$$c_m = \frac{\pi D_n}{nC_e} \quad (2.1)$$

where c_m is conversion factor that translates encoder pulses into linear wheel displacement; D_n is nominal wheel diameter (in meter); C_e is encoder resolution (in pulses per revolution) and n is gear ratio of the reduction gear between the motor (i.e. ≈ 1). The incremental travel distance for the left and right wheel $\Delta U_{L,I}$ and $\Delta U_{R,I}$ can be computed. The equation for calculating the distance traveled by each wheel is

$$\Delta U_{L,R,I} = c_m \cdot N_{L,R,I} \quad (2.2)$$

The distance traveled by each wheel and the length of the wheelbase are crucial pieces needed to calculate the midpoint of the robot, the angle of the turned and the

heading angle of the robot. The wheelbase used to calculate the angle is the distance between the two wheels is in contact with the floor. The incremental linear displacement of the robot's midpoint, C denoted by ΔU_i is given as

$$\Delta U_i = (\Delta U_{R,i} + \Delta U_{L,i})/2 \quad (2.3)$$

Next, the robot's incremental change of orientation is calculated by

$$\Delta \theta_i = (\Delta U_{R,i} - \Delta U_{L,i})/b \quad (2.4)$$

where b is the wheelbase of the platform, ideally measured as the distance between the two contact points between the wheel and the floor. The calculation of the heading angle, θ_i is given as

$$\theta_i = \theta_{i-1} + \Delta \theta_i \quad (2.5)$$

With the information from 2.3 and 2.5, it is now possible to calculate the position of the midpoint of the robot using simple trigonometric functions. The relative position of the midpoint, C in a Cartesian coordinate system is given as

$$x_i = x_{i-1} + \Delta U_i \cos \theta_i \quad (2.6)$$

$$y_i = y_{i-1} + \Delta U_i \sin \theta_i \quad (2.7)$$

$$d_i = \sqrt{x_i^2 + y_i^2} - \sqrt{x_{i-1}^2 + y_{i-1}^2} \quad (2.8)$$

where x_i, y_i is the relative position of the robot's midpoint, C as instant I and d_i is the distance between the desired position(s) and current position.

As can be seen in Equations (2.1) to (2.8), dead-reckoning is based on simple equations that are easy to implement, and utilizes data from inexpensive incremental encoders. However, dead-reckoning is also based on the assumption that the wheel revolutions can be translated into linear displacement relative to the floor or in other words, slippage-free. This assumption is only of limited validity since the mobile