

**THE EFFECT OF THIN-LAYER ELEMENTS IN STRUCTURAL
MODELING OF RAIL-TRACK SUPPORTING SYSTEM**

By

YEAT CHOOI FONG

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,
in Fulfilment of the Requirement for the Degree of Master of Science**

December 2006

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

THE EFFECT OF THIN-LAYER ELEMENTS IN STRUCTURAL MODELING OF RAIL-TRACK SUPPORTING SYSTEM

By

YEAT CHOOI FONG

December 2006

Chairman: Associate Professor Jamaluddin Noorzaei, PhD

Faculty: Engineering

A conventional track system consists of rails, sleepers, ballast, sub-ballast and sub-grade. A track foundation on the basis of live load response and permanent settlement, it is necessary to use an analytical model that will realistically represent the actual behaviour of this track system subjected to actual load. This study deals with the development of numerical modeling of Malaysian railway track along with the supporting system. The model is capable to simulate the sleeper, ballast, sub-ballast, soil layers and their interaction as a single compatible unit. Under plane strain condition, the coupled finite-infinite elements were implemented to represent the near field and far field behaviour of media. Thin-layer elements have been used to represent the interfacial behaviour between sleepers and ballast. The following constitutive relationships were adopted in this study:

(i)-Linear Elastic

(ii)-Elasto-plastic

Based on the above physical and material modeling, an existing two dimensional finite element program has been extensively modified in view of including the new elements as well as the new constitutive law. After verification of the modified

version of the program, the applicability of the program was shown in analysis of railway track supporting system. The response of the railway track supporting system has been presented under static and dynamic loading. The behavior of the railway track supporting media has been discussed with respect to displacements, accelerations and rate of plastic flow.

This analysis shows that the thin layer element is reliable to be used as an interface element to represent the contact surface between two different materials.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

**KESAN BAGI ELEMENT LAPISAN NIPIS DALAM PEMODELAN SISTEM
SOKONGAN RANGKAIAN KERETAPI**

Oleh

YEAT CHOOI FONG

Disember 2006

Pengerusi: Profesor Madya Jamaluddin Noorzaei, PhD

Fakulti: Kejuruteraan

Secara umumnya, sistem landasan keretapi terdiri daripada rel, penidur, ballast, sub-ballast dan bahagian tanah. Model yang bersifat analitik dimama dapat mempersembahkan kelakuan sebenar bagi asas landasan keretapi yang menanggung beban hidup dan mengalami pemindahan yang kekal adalah amat diperlukan. Kajian ini merangkumi pemodelan sistem sokongan yang direkabentuk untuk landasan keretapi di Malaysia. Model ini berupaya mempersembahkan kelakuan rel, penidur, ballast, sub-ballast, lapisan tanah dan juga keadaan lapisan antara dua jenis bahan pembinaan yang berbeza. Hubungan yang dikaji dalam pemodelan ini adalah::

- (i) Elastic linear
- (ii) Elasto-plastic

Dalam proses memodel sistem sokongan landasan keretapi secara fizikal dan bahan pembinaan, satu program analisis unsure terhingga yang sedia ada telah diubahsuai dari segi penggunaan unsure yang baru dimana merupakan suatu constitutive bahan yang baru. Pengesahan telah dilakukan ke atas program yang telah diubahsuai bagi memastikan bahawa program tersebut dapat menganalisis sistem sokongan landasan keretapi di Malaysia. Kelakuan ke atas sistem rangkaian keretapi Malaysia telah

dibentang dan dibincangkan di bawah beban static dan dinamik. Di samping itu, perubahan bentuk ke atas sistem sokongan landasan keretapi juga dibincangkan.

Kajian ini telah membuktikan bahawa pemodelan suatu lapisan dapat mempersembahkan kelakuan sebenar di antara dua jenis bahan pembinaan yang berlainan.

ACKNOWLEDGEMENTS

As with any other text, the number of individuals who have made it possible far exceeds those whose names grace the cover. At the hazard of leaving someone out, I would like to explicitly thank the following individuals for their contribution.

The following professor and friends helped to solve problems, proofread text, prepare illustrations, raise embarrassing questions and generally make sure the students could understand it: Prof. Madya Ir. Dr. Jamaloddin Noorzai, Prof. Madya. Ir. Dr. Mohd. Saleh Jaafar, Prof. Madya Dr. Waleed A. Thanoon and Huda A. Thanoon, University Putra Malaysia. To them a hearty thank you!

To my family, who, I believe, is inquisitive and questioning in the space beyond, which is congruent to that of mine.

To those giants of mechanics, physics and philosophy, on whose contributions I stand and extend.

This thesis submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Master of Science. The members of the Supervisory Committee are as follows:

Jamaloddin Noorzaei, PhD

Associate Professor
Faculty of Engineering
Universiti Putra Malaysia
(Chairman)

Mohd Saleh Jaafar, PhD

Associate Professor
Faculty of Engineering
Universiti Putra Malaysia
(Member)

Waleed Abdul Malik Thanoon, PhD

Associate Professor
Faculty of Engineering
Universiti Putra Malaysia
(Member)

AINI IDERIS, PhD
Professor/Dean
School of Graduate Studies
Universiti Putra Malaysia

Date: 08 MARCH 2007

DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

YEAT CHOOI FONG

Date: 13 FEBRUARY 2007

TABLE OF CONTENTS

| | Page |
|---|-------------|
| ABSTRACT | ii |
| ABSTRAK | iv |
| ACKNOWLEDGEMENTS | iv |
| APPROVAL | vii |
| DECLARATION | ix |
| LIST OF TABLES | x |
| LIST OF FIGURES | xi |
| LIST OF NOTATIONS/GLOSSARY OF TERMS | xvi |
| | |
| CHAPTER | |
| | |
| I INTRODUCTION | |
| Historical Background of Malaysian Railway Network | 1 |
| General | 2 |
| Statement of the Problem | 5 |
| Objectives | 7 |
| Scope | 7 |
| Organization of the Thesis | 10 |
| Limitation of the Present Work | 10 |
| Assumption | 11 |
| | |
| II LITERATURE REVIEW | |
| Introduction | 12 |
| Earlier Work on Modeling of the Railway Track-Supporting System | 12 |
| Concluding Remarks | 33 |
| Summary | 34 |
| | |
| III FINITE ELEMENT FORMULATION | |
| Finite Element Analysis and Solution of Dynamic-Equation | 36 |
| Introduction | 36 |
| Formulation Finite Element of Linear Continuum Mechanics | 37 |
| Isoparametric Elements | 40 |
| Eight- and Six-Node Isoparametric Finite Element | 41 |
| Five-Node Isoparametric Infinite Element. | 43 |
| Interface Element | 44 |
| Thin-Layer Element | 47 |
| Formulation in Local Coordinate System | 49 |
| Formulation in Global Coordinate System | 54 |
| Proposed Physical Modeling Railway Track-Supporting System | 57 |

| | | |
|------------|---|-----|
| III | Numerical Integration in Isoparametric Element | 58 |
| | Formulation of Non-linear Continuum Mechanics | 59 |
| | Yield Criterion | 60 |
| | Work Hardening | 62 |
| | Plastic Stress-Strain Relationship Suitable for Finite | 63 |
| | Element Method | 66 |
| | Elasto-Plastic Formulation Thin-Layer Element | 67 |
| | Elasticity Condition | 70 |
| | Perfectly Plastic Condition | 72 |
| | Constitutive Modeling | 77 |
| | Finite Element Formulation of Dynamic Problem | 80 |
| | Solution of Dynamic Equation | 80 |
| | Host Finite Element Program | 86 |
| | Calibration for Static Analysis and Learning Process | 91 |
| | Example 1 – Cantilever Beam | 99 |
| | Example 2 – Column and Beam | 100 |
| | Example 3 - Foundation | 101 |
| | Calibration for Dynamic Analysis | 102 |
| | Thin-Layer Element | 109 |
| | Spherical Shell Example | |
| | Foundation | |
| | Concluding Remarks | |
| | | |
| IV | ANALYSIS OF RAILWAY TRACK SUPPORTING MEDIA | 112 |
| | Introduction | |
| | Finite Element Analysis of Railway Track Bedding System | 113 |
| | Problem Definition | 114 |
| | Loading | 116 |
| | Finite Element Mesh | 118 |
| | Result and Discussion on Static Analysis | 119 |
| | Linear Static Response | 124 |
| | Elasto-Plastic Static Analysis | 128 |
| | Result and Discussion on Dynamic Analysis | 131 |
| | Linear Dynamic Response | 138 |
| | Elasto-Plastic Dynamic Behaviour | 146 |
| | Concluding Remarks | |
| | | |
| V | CONCLUSION | 147 |
| | REFERENCES | 150 |
| | APPENDICES | 156 |
| | <i>BIODATA OF THE AUTHOR</i> | 176 |

LIST OF TABLES

| Table | Page |
|---|-------------|
| 3.1 Physical Modeling of Malaysian Railway | 57 |
| 3.2 Gauss Points and Weights for Gaussian Quadrature | 58 |
| 3.3 Proposed Material Modeling in Analysis | 70 |
| 3.4 Newmark's Algorithm | 74 |
| 3.5 Explicit Predictor-Corrector Algorithm | 75 |
| 3.6 Comparison of Deflection for Cantilever Beam | 82 |
| 3.7 Comparison of Maximum Deflection for Cantilever Beam | 85 |
| 3.8 Comparison of Maximum Deflection for Column and Beam | 91 |
| 3.9 Comparison of Deflection for Foundation | 98 |
| 3.10 Shear Stress (psi) in Thin-Layer Element at Gauss Point | 100 |
| 3.11 Comparison of Displacement for Spherical Shell | 102 |
| 3.12 Comparison of Displacement for Load Type 1 | 104 |
| 3.13 Comparison of Displacement for Load Type 2 | 106 |
| 3.14 Comparison of Displacement for Load Type 3 | 107 |
| 4.1 Material Properties | 114 |
| 4.2 <i>Train Traffic Data</i> | 116 |
| 4.3 Types of Static Analysis Performed | 118 |
| 4.4 Selection of Time Step | 130 |
| 4.5 Comparison No. of Gaussian Point Yielded against Static and Dynamic Analysis | 146 |

LIST OF FIGURES

| Figure | | Page |
|--------|---|------|
| 1.1 | Malaysian Railway Network System | 8 |
| 1.2 | Support System of Railway Track in Malaysia | 8 |
| 2.1 | Track Model - European Rail Research Institute (ERRI) | 14 |
| 2.2 | Dynamic Wheel Load during Passage over a Weld | 18 |
| 2.3 | Track Model to describe Nikex Construction | 18 |
| 2.4 | Impulse Force and Displacement of Nikex Construction Determined with TILLY | 19 |
| 2.5 | Ballasted Track Model for Typical Dynamic Track Behavior | 23 |
| 2.6 | Test Arrangement for Clamping Force Measurement | 24 |
| 2.7 | Loading Procedure for Clamping Force Measurement | 25 |
| 2.8 | Test Arrangement to Measure the Longitudinal Force | 26 |
| 2.9 | Characteristic of Longitudinal Load / Displacement | 26 |
| 2.10 | Measurement of Vertical Stiffness | 26 |
| 2.11 | Test Arrangement for Cyclic Repeated Loading Test | 27 |
| 2.12 | Test Procedure for Cyclic Loading Testing | 28 |
| 2.13 | Mechanic Model of Railway Track | 29 |
| 2.14 | Finite Element Model | 30 |
| 2.15 | Impulse-Shape Load Type 1 | 31 |
| 2.16 | Impulse-Shape Load Type 2 | 32 |
| 2.17 | Impulse-Shape Load Type 3 | 33 |
| 3.1 | Linear Continuum Mechanics | 37 |
| 3.2 | Six-Nodded Isoparametric Finite Element | 41 |

| | | |
|-------|---|----|
| 3.3 | Eight-Nodded Isoparametric Finite Element | 42 |
| 3.4 | Five-Nodded Infinite Element | 43 |
| 3.5 | Parabolic Interface Element | 45 |
| 3.6 | Thin-Layer Interface Element | 49 |
| 3.7 | Mohr Circle Representation of the Mohr-Coulomb Yield Criterion | 60 |
| 3.8 | Flow Chart of Finite Element Program Procedures for Elasto-Plastic Static Problems | 78 |
| 3.9 | Flow Chart of Finite Element Program Procedures for Dynamic Problems | 79 |
| 3.10 | Cantilever Beam Mesh | 81 |
| 3.11 | Deflection along Beam Length with Various Analyses | 83 |
| 3.12 | Deflection along Beam Length with Various Poisson Ratio | 83 |
| 3.13 | Deflection along Beam Length with Various Thickness of Thin-Layer Element | 84 |
| 3.14a | Displacement without Thin-Layer Element | 84 |
| 3.14b | Displacement with Six-Nodded Thin-Layer Element | 84 |
| 3.14c | Displacement with Eight-Nodded Thin-Layer Element | 85 |
| 3.15 | Column and Beam | 87 |
| 3.16 | Deflection along Section A-A with Various Analyses | 88 |
| 3.17 | Deflection along Section A-A with Various Thickness of Thin-Layer Element | 89 |
| 3.18a | Overall Deflection without Thin-Layer Element | 90 |
| 3.18b | Overall Deflection with Six-Nodded Thin-Layer Element | 90 |
| 3.18c | Overall Deflection with Eight-Nodded Thin-Layer Element | 90 |
| 3.19 | Finite Element Modeling of Foundation Soil-Interaction | 94 |

| | | |
|------------|--|------------|
| 3.20 | Distribution of Displacement with Various Analyses | 95 |
| 3.21 | Distribution of Displacement with Various Stiffness (Normal or Shear) | 96 |
| 3.22 | Distribution of Displacement with Various Thickness of Thin-Layer Element | 96 |
| 3.23a | Overall Distribution of Displacement (without Thin-Layer) | 97 |
| 3.23b | Overall Distribution of Displacement (with Six-Nodded Thin-Layer) | 97 |
| 3.23c | Overall Deformation of Structures (with Eight-Nodded Thin-Layer) | 98 |
| 3.24 | Thin-Layer Element | 100 |
| 3.25 | Spherical Shell and Finite Element Mesh | 101 |
| 3.26a | Impulse-Shape Load Type 1 | 103 |
| 3.26b | Impulse-Shape Load Type 2 | 103 |
| 3.26c | Impulse-Shape Load Type 3 | 104 |
| 3.27 | Horizontal Displacement Behaviour with Impulse Load Type 1 | 105 |
| 3.28 | Vertical Displacement Behaviour with Impulse Load Type 1 | 105 |
| 3.29 | Horizontal Displacement Behaviour with Impulse Load Type 2 | 106 |
| 3.30 | Vertical Displacement Behaviour with Impulse Load Type 2 | 107 |
| 3.31 | Horizontal Displacement Behaviour with Impulse Load Type 3 | 108 |
| 3.32 | Vertical Displacement Behaviour with Impulse Load Type 3 | 108 |
| 4.1 | Typical Geometry for Railway Track | 113 |
| 4.2 | <i>KTMB Loading Diagram</i> | 115 |
| 4.3 | Typical Transverse Section | 117 |
| 4.4 | Distribution of Vertical Displacement along Depth of Structure | 119 |

| | | |
|------|--|-----|
| 4.5 | Distribution of Horizontal Displacement along Depth of Structure | 120 |
| 4.6 | Distribution of Horizontal Stress (σ_x) along Width of Structure | 121 |
| 4.7 | Distribution of Contact Pressure (σ_y) along Width of Structure | 122 |
| 4.8 | Variation of Maximum Principle Stress (σ_1) at different section | 123 |
| 4.9 | Variation of Minimum Principle Stress (σ_3) at Different Section | 124 |
| 4.10 | Spread of Plastic-Yielding with Various Load Factor | 125 |
| 4.11 | Relationship between No. Gaussian Point Yielded with Load Factor | 126 |
| 4.12 | Normal Stresses at Section F – F | 126 |
| 4.13 | Maximum Principle Stress at Section F – F | 127 |
| 4.14 | Vertical Displacement at Section F – F | 127 |
| 4.15 | Impulse-Shape Load Type 3 | 129 |
| 4.16 | Time History of Acceleration when Time Step = 100, 200 and 300 | 130 |
| 4.17 | Contour of the Peak Horizontal Displacement | 132 |
| 4.18 | Contour of the Peak Vertical Displacement | 132 |
| 4.19 | Contour of the Peak Horizontal Acceleration | 133 |
| 4.20 | Contour of the Peak Vertical Acceleration | 133 |
| 4.21 | Distribution of Horizontal Stress at ISTEP = 200 | 135 |
| 4.22 | Distribution of Vertical Stress at ISTEP = 200 | 135 |
| 4.23 | Distribution of Shear Stress at ISTEP = 200 | 136 |
| 4.24 | Distribution of Maximum Principle Stress at ISTEP = 200 | 137 |
| 4.25 | Distribution of Minimum Principle Stress at ISTEP = 200 | 137 |
| 4.26 | Variation of Horizontal Displacement | 138 |
| 4.27 | Variation of Vertical Displacement | 139 |
| 4.28 | Distribution of Maximum Principle Stresses (σ_1) at Various Plane | 140 |

| | | |
|------|--|-----|
| 4.29 | Distribution of Minimum Principle Stresses (σ_3) at Various Plane | 140 |
| 4.30 | Load Factor = 5 | 141 |
| 4.31 | Load Factor = 10 | 141 |
| 4.32 | Load Factor = 25 | 142 |
| 4.33 | Spread of Plastic-Yielding with Various of Load Factor | 142 |
| 4.34 | Impact Load Duration Time = 0.15 sec | 143 |
| 4.35 | Impact Load Duration Time = 0.20 sec | 143 |
| 4.36 | Impact Load Duration Time = 0.50 sec | 144 |
| 4.37 | Spread of Plastic-Yielding with Various Duration of Time | 145 |
| 4.38 | Relationship between No. of Gaussian Point Yielded with Load Factor | 145 |

LIST OF NOTATIONS/GLOSSARY OF TERMS

| | | |
|---|---|---|
| $[a]$ | = | Flow vector |
| $[a_1, a_2, a_3]$ | = | Co-efficient used in material modal parameters |
| $[\alpha]$ | = | Factor of end of elasticity |
| $[B]$ | = | Strain-displacement transformation matrix |
| $[\beta]$ | = | A parameter |
| $[C]$ | = | Constitutive matrix in global coordinate |
| $[\bar{C}^e]$ | = | Elastic constitutive matrix in local coordinates |
| $[\bar{C}^{ep}]$ | = | Elasto-plastic constitutive matrix in local coordinates |
| $[C_i]$ | = | Constant parameter |
| $[c]$ | = | Co-efficient value material |
| $[D]$ | = | Elasticity matrix in global coordinates |
| $[\Delta]$ | = | Displacement in global coordinates |
| $[\Delta t]$ | = | Incremental of time |
| $[\delta]$ | = | Displacement in local coordinates |
| $[\partial\delta]$ | = | Virtual displacement vector |
| $[\phi]$ | = | Friction angle |
| $[E]$ | = | Young's modulus / Modulus of Elasticity |
| $[\varepsilon]$ | = | Strains |
| $[\varepsilon_0]$ | = | Initial Strains |
| $[\varepsilon_x, \varepsilon_y, \varepsilon_z]$ | = | Nodal strains in global coordinates |

| | |
|--|--|
| $[\bar{\epsilon}_x, \bar{\epsilon}_y, \bar{\epsilon}_z] =$ | Nodal strains in local coordinates |
| $[\xi, \eta] =$ | Local coordinates of gauss point |
| $[\zeta] =$ | Mass / unit volume |
| $[F_x, F_y, F_z] =$ | Concentrate load |
| $[F_x, F_y, F_z]_b =$ | Body force |
| $[F_x, F_y, F_z]_s =$ | Traction force / pressure |
| $[F(\sigma_{ij}) / f / g] =$ | Failure function |
| $[f_{n+1}] =$ | Internal force vector |
| $[G] =$ | Modulus rigidity |
| $[\gamma] =$ | A parameter |
| $[J_2, J_3] =$ | Second and third invariant of deviatoric stress tensor |
| $[J_2'] =$ | Second invariant of deviatoric strain tensor |
| $[\psi] =$ | Interface dilatancy angle |
| $[K] =$ | Stiffness matrix |
| $[K_{nn}] =$ | Normal stiffness matrix |
| $[K_{ss}] =$ | Shear stiffness matrix |
| $[k] =$ | Hardening parameter |
| $[M] =$ | Mass matrix |
| $[N] =$ | Shape functions matrix |
| $[N_i] =$ | Nodal shape function |
| $[\sigma] =$ | Stress |
| $[\sigma_0] =$ | Initial Stress |

| | | |
|---|---|---|
| $[\sigma_1]$ | = | Maximum Principle Stress |
| $[\sigma_3]$ | = | Minimum Principle Stress |
| $[\sigma_x, \sigma_y, \sigma_z]$ | = | Nodal stress in global coordinates |
| $[\bar{\sigma}_x, \bar{\sigma}_y, \bar{\sigma}_z]$ | = | Nodal stress in local coordinates |
| $[P_{nel}]$ | = | Acceleration vector |
| $[Q(k)]$ | = | Yield stress / plastic potential |
| $[T]$ | = | Transformation matrix from local to global coordinate |
| $[t]$ | = | Thickness of thin layer element |
| $[\tau_{xy}, \tau_{yz}, \tau_{zx}]$ | = | Nodal shear in global coordinates |
| $[\bar{\tau}_{xy}, \bar{\tau}_{yz}, \bar{\tau}_{zx}]$ | = | Nodal shear in local coordinates |
| $[u, v, w]$ | = | Nodal displacements in global coordinates |
| $[\bar{u}, \bar{v}, \bar{w}]$ | = | Nodal displacements in local coordinates |
| $[u, \dot{u}, \ddot{u}]$ | = | Displacement, velocity and acceleration |
| $[\mu]$ | = | Damping co-efficient |
| $[\nu]$ | = | Poisson's ration |
| $[x, y]$ | = | Global coordinates of a point |