



**UNIVERSITI PUTRA MALAYSIA**

**SIMULATION OF LIQUID-LIQUID DISPERSED FLOW IN HORIZONTAL  
PIPE USING COMPUTATIONAL FLUID DYNAMICS**

**RASHMI G WALVEKAR**

**FK 2007 17**



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**By**

**RASHMI G WALVEKAR**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in  
Fulfillment of the Requirements for the Degree of Master of Science**

**February 2007**





**MOHD KHALID**

**MASTERS OF SCIENCE**

**2006**

**A COMPARATIVE STUDIES ON  
POLYPROPYLENE-CELLULOSE AND  
POLYPROPYLENE- OIL PALM EMPTY  
FRUITBUNCH BIOCOMPOSITES**

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*DEDICATED TO MY FAMILY*



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment  
of the requirement for the degree of Master of Science

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By

**RASHMI G WALVEKAR**

**February 2007**

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Liquid-liquid dispersed flows are commonly encountered in many of the industrial applications such as polymerization, emulsification, batch and continuous stirred reactors and pipeline flows such as in petroleum industries. Liquid-liquid two phase flows are very complex due to the existence of several flow patterns and mechanisms. Numerical approaches offer the flexibility to construct computational models which can adapt large variety of physical conditions without constructing large scale prototypes.

The present work focuses on predicting the phase hold-up across a pipe cross-section and *ambivalence range* for phase inversion phenomena at different mixture velocity and range of input water fraction. The Computational Fluid Dynamics (CFD) computations were carried using FLUENT 6.2.16 while the geometry was created in pre-processor, GAMBIT 2.2.3. Dispersed phase dynamics and the turbulent continuous phase are modeled using an Eulerian-Eulerian approach and standard  $k - \varepsilon$  turbulence model. To check the reliability of the CFD code, the predicted





results were validated with experimental results of previous work at different mixture velocities and phase fractions.

CFD predicted the flow phenomenon such as phase transition from water-in-oil dispersion to oil-in-water dispersion and flow development along the length of the pipe. CFD code also predicted the phase hold-up distributions at pipe cross section. The pressure gradient trends similar to those observed in previous experimental results were obtained using CFD code. The phase inversion point obtained was within the ambivalence range suggested in literature. The numerical CFD simulations also confirmed that interphase drag, lift and turbulent dispersion forces has significant influence on spatial phase distribution. CFD simulations so obtained were subsequently compared with experimental results from previous researchers and correlation featuring range of mixture velocities and phase inputs. The predicted hold-up profiles were in good agreement with the previous experimental results for high mixture velocities and were in reasonable agreement with those of lower mixture velocity. Overall good qualitative agreement was achieved between physical model and simulated results.

Abstrak tesis ini dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Master Sains

**SIMULASI ALIRAN SERAKAN BENDALIR DIDALAM PAIP MENDATAR  
MENGUNAKAN PENGIRAN DINAMIK BENDALIR**

Oleh

**RASHMI G WALVEKAR**

**Februari 2007**

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Kebanyakan aliran tersebar pelbagai cecair boleh ditemui dalam pelbagai aplikasi industri seperti pemolimeran, pengemulsifikasian, reaktor teraduk aliran kelompok dan aliran terus serta aliran dalam paip yang biasanya ditemui dalam industri petroleum. Sistem aliran cecair dua fasa adalah sangat rumit disebabkan kewujudan pelbagai corak aliran dan mekanisme. Pendekatan secara numerikal menawarkan fleksibiliti untuk membina model pengiraan yang berpadanan dengan pelbagai keadaan fizikal tanpa keperluan untuk membina prototaip yang sebenar.

Kajian ini tertumpu kepada pengramalan isi tertahan fasa yang berlaku pada keratan rentas paip dan julat ambivalens untuk fenomena penyonsangan fasa pada halaju bancuhan serta julat masukan pecahan air yang berlainan. Komputasi CFD dilakukan menggunakan FLUENT 6.2.16 manakala geometri dihasilkan menggunakan GAMBIT 2.2.3. Dinamik fasa sebaran dan fasa pengeloraan berterusan diselesaikan menggunakan model Eulirian-Eulerian dan model pengeloraan piawaian,  $k - \varepsilon$ .

Untuk menentusahkan kebolehpercayaan kod CFD, keputusan pengramalan telahpun dibandingkan dengan keputusan eksperimen daripada kerja-kerja terdahulu.

CFD meramalkan fenomena pengaliran seperti peralihan fasa dari penyebaran air-dalam-minyak kepada minyak-dalam air dan perkembangan aliran sepanjang paip. Kod CFD juga meramalkan pengagihan isi tertahan fasa pada keratan rentas paip dan memberikan trend kecerunan tekanan yang serupa dengan yang diperhatikan dalam kerja-kerja eksperimen terdahulu. Titik penyongsangan fasa yang diramalkan juga didapati berada dalam julat ambivalens yang dicadangkan oleh bahan rujukan. Simulasi numerikal CFD juga menentusahkan seretan antara fasa, apungan dan daya penyebaran pergolakan mempengaruhi ruangan taburan fasa. Simulasi CFD juga dibandingkan dengan keputusan eksperimen daripada penyelidik-penyelidik terdahulu dan korelasi pada halaju adukan dan input fasa yang berlainan. Pengramalan ke atas profil isi tertahan adalah berserasi dengan keputusan eksperimen yang dijalankan pada halaju adukan tinggi dan agak berserasi dengan yang dijalankan pada halaju adukan rendah. Secara keseluruhan, keserasian kualitatif yang tinggi wujud antara model fizikal dan keputusan simulasi.

## ACKNOWLEDGEMENTS

I greatly acknowledge my research supervisor Dr. Siti Aslina Hussain for her guidance and support throughout the course of this research. I would like to express my deep appreciation of gratitude to my co-supervisor Dr. Thomas Choong Shean Yaw for his guidance, encouragement, providing with the necessary facilities and scholarship funding throughout my study. I also thank the Head of the Department Dr. Robiah Yunus for being there with us with ready ear and immense erudition despite of her busy schedule.

My sincere thanks to Srinivas Viyyuri (FLUENT India Pvt. Ltd), whose guidance and tutorials were immensely useful in saving a lot of time and effort. I would also like to thank all faculty members and staff at the Chemical and Environmental Engineering Department for their help and cooperation.

Special thanks are expressed to Dr. Ram Prakash, Madhavan, Arjun and Podila for sharing their knowledge, constant help and support throughout. Last but not the least I am highly obliged to my parents, sisters and my husband for their constant support and affection, and their boundless dedication to my education.



I certify that the Examination Committee met on 16.02.2007 to conduct the final examination of Rashmi G Walvaker on her Master of Science in Environmental Engineering thesis entitled “Simulation of Liquid-Liquid Dispersed Flows in Horizontal Pipe Using CFD” in accordance with Universiti Pertanian Malaysia (Higher degree) Act 1980 and Universiti Pertanian Malaysia (Higher degree) Regulation 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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## **DECLARATION**

I hereby declare that the thesis is based on my original work except for quotations and citations, which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

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**RASHMI G WALVEKAR**

Date:

|   | <b>Page</b> |
|---|-------------|
| <b>DEDICATION</b>                                 | ii          |
| <b>ABSTRACT</b>                                   | iii         |
| <b>ABSTRAK</b>                                    | v           |
| <b>ACKNOWLEDGEMENTS</b>                           | vii         |
| <b>APPROVAL</b>                                   | viii        |
| <b>DECLARATION</b>                                | x           |
| <b>LIST OF TABLES</b>                             | xiii        |
| <b>LIST OF FIGURES</b>                            | xiv         |
| <b>NOMENCLATURE</b>                               | xviii       |
| <br>  |             |
| <b>CHAPTER</b>                                    |             |
| <b>1 INTRODUCTION</b>                             | 1.0         |
| 1.1 Classification of Flow Patterns               | 1.2         |
| 1.2 Overview of Computational Fluid Dynamics      | 1.4         |
| 1.3 Problem Statement                             | 1.5         |
| 1.4 Objectives of the Study                       | 1.6         |
| 1.5 Scope of Work                                 | 1.6         |
| 1.6 Structure of the Thesis                       | 1.7         |
| <br>  |             |
| <b>2 LITERATURE REVIEW</b>                        | 2.1         |
| 2.1 Background on Flow Pattern                    | 2.1         |
| 2.2 Factors Affecting Liquid-Liquid Flow Pattern. | 2.3         |
| 2.2.1 Mixture Velocity                            | 2.3         |
| 2.2.2 Density                                     | 2.4         |
| 2.2.3 Viscosity                                   | 2.5         |
| 2.2.4 Wetting Property of the Pipe                | 2.7         |
| 2.3 Pressure Drop                                 | 2.8         |
| 2.3.1 Phenomenological Model for Dispersed Flow   | 2.8         |
| 2.3.2 Phase Inversion Phenomena                   | 2.10        |
| 2.3.3 Factors Affecting Phase Inversion Phenomena | 2.13        |
| 2.4 Phase Hold-Up                                 | 2.16        |
| 2.4.1 Factors Affecting Phase Hold-Up             | 2.19        |
| 2.5 Sauter Mean Drop Diameter ( $d_{32}$ )        | 2.26        |
| 2.6 Interphase Forces in Dispersed Flow           | 2.34        |
| 2.6.1 Drag Force                                  | 2.35        |
| 2.6.2 Lift Force                                  | 2.39        |
| 2.6.3 Turbulent Dispersion Force                  | 2.39        |
| 2.7 Multiphase Models in FLUENT                   | 2.42        |
| 2.7.1 Volume of Fluid (VOF) Model                 | 2.42        |
| 2.7.2 Mixture Model                               | 2.44        |
| 2.7.3 Eulerian-Lagrangian Model (E-L)             | 2.45        |
| 2.7.4 Eulerian-Eulerian Model (E-E)               | 2.46        |
| 2.8 CFD Simulation of Liquid-Liquid Flow          | 2.46        |
| 2.9 Summary                                       | 2.52        |
| <br>  |             |
| <b>3 NUMERICAL METHODOLOGY</b>                    | 3.1         |
| 3.1 Computational Fluid Dynamics (CFD)            | 3.1         |
| 3.1.1 Structure of Computational Fluid Dynamics   | 3.2         |
| 3.2 Computational Resource                        | 3.5         |
| 3.3 Geometry and Grid Generation                  | 3.5         |
| 3.4 Boundary Conditions                           | 3.7         |
| 3.5 FLUENT Methodology                            | 3.7         |
| 3.5.1 Selection of Model                          | 3.8         |
| 3.5.2 Eulerian-Eulerian Approach                  | 3.8         |
| 3.5.3 Simulation Conditions                       | 3.10        |
| 3.5.4 Turbulence Model                            | 3.12        |
|   | 3.14        |





## LIST OF TABLES

| Table |  | Page |
|-------|--|------|
| 2.1   | Drag expressions on drops which take into account the presence of adjacent drops.            | 2.37 |
| 2.2   | Previous studies using CFD in liquid-liquid system.  | 2.48 |
| 3.1   | Summary of discretization schemes used in present study.                                     | 3.12 |
| 3.2   | Physical properties of the liquids used in the present study.                                | 3.13 |
| 3.3   | Standard turbulent $k - \varepsilon$ model constants.  | 3.17 |
| 3.4   | Convergence criteria for flow variables used in present study.                               | 3.23 |
| 3.5   | Under-relaxation factors used in present study.  | 3.24 |
| 4.1   | Experimental conditions and physical properties of liquid used in present study.             | 4.2  |
| 4.2   | Values of interphase forces used in the present study.                                       | 4.3  |
| 4.3   | List of Sauter mean drop diameter ( $d_{32}$ ) of the dispersed phase used in present study. | 4.4  |
| 4.4   | Summary of statistical analysis of the results based on average values of phase hold-up.     | 4.28 |



## LIST OF FIGURES

| Figure |   | Page |
|--------|---|------|
| 1.1    | Schematic diagrams of flow patterns appearing in liquid-liquid flows in horizontal pipe.  | 1.3  |
| 2.1    | Flow regime map for oil-water mixture for oil viscosity of 65 mPa.s and density 988 kg/m <sup>3</sup> .   | 2.5  |
| 2.2    | Flow regime map for oil-water mixture for oil viscosities of 6.29 mPa.s and 16.8 mPa.s and density 988 kg/m <sup>3</sup> .                            | 2.6  |
| 2.3    | Mixture viscosities as a function of input water fraction for low viscosity oils.   | 2.7  |
| 2.4    | Effect of input water fraction on viscosity.  | 2.11 |
| 2.5    | Pressure gradient measured between points 4.75 m to 5.75 m from inlet of test section without mixers.   | 2.12 |
| 2.6    | Phase inversion mechanism for oil-water dispersion system.  | 2.13 |
| 2.7    | Pressure gradient in steel pipe at different mixture velocities.  | 2.14 |
| 2.8    | Pressure gradient in steel pipe for two initial conditions used at 4.0 m/s and 4.5 m/s mixture velocities.  | 2.15 |
| 2.9    | Comparison of pressure gradient in steel and acrylic pipe at mixture velocity of 4.5 m/s and water dominated mixtures.                                | 2.16 |
| 2.10   | Slip ratios as a function of oil/water volume ratios for different water superficial velocities, with oil viscosity of 18 mPa.s.                      | 2.17 |
| 2.11   | Slip ratios as a function of oil/water volume ratios for different water superficial velocities and oil viscosity of 6.29 mPa.s.                      | 2.18 |
| 2.12   | Variation of water volume fraction along the horizontal diameter of the pipe at 1.7 m/s mixture velocity and 62.5% input water in “Transpalite” pipe. | 2.19 |
| 2.13   | Contour plots of phase distribution at 2.12 m/s mixture velocity and 46% input water (a) GDS (b) HFP measured between 4.75-5.75 m from pipe inlet.    | 2.20 |
| 2.14   | Vertical distribution of water fraction using GDS and HFP techniques at 2.12 m/s and 46% input water.   | 2.21 |
| 2.15   | Contour plot of phase distribution at 3 m/s mixture velocity and 46% input water cut. (a) GDS (b) HFP measured between 4.75-5.75 m from pipe inlet.   | 2.21 |

|      |  |      |
|------|--|------|
| 2.16 | Vertical distribution of water phase fraction using GDS and HFP at 3.0 m/s and 46% input water.                                    | 2.22 |
| 2.17 | Cross sectional phase distribution at distance of 7.72 m from inlet as a function of mixture velocity and 40% input water.         | 2.23 |
| 2.18 | Cross sectional phase distribution at distance of 7.72 m from inlet as a function of mixture velocity and 60% input water.         | 2.23 |
| 2.19 | Cross-sectional phase distributions (using Gamma tomographic technique) at 2.76 m/s mixture velocity and 1.0 m from the inlet.     | 2.24 |
| 2.20 | Insitu water fractions vs. input water fraction at different mixture velocity.   | 2.25 |
| 2.21 | Phase distribution at 2.76 m/s mixture velocity and 40% input water at a distance of (a) 1.0 m (b) 5.85 m (iii) 7.72 m from inlet. | 2.26 |
| 2.22 | Experimental mean drop diameter ( $d_{10}$ ) at different continuous phase velocity of oil and water.                              | 2.28 |
| 2.23 | Friction factor at different oil and water continuous phase velocity.  | 2.29 |
| 2.24 | Maximum drop diameters at different oil and water continuous phase velocity.   | 2.31 |
| 2.25 | Sauter mean drop diameter at different oil and water continuous phase velocity.  | 2.31 |
| 2.26 | Variations of continuous phase velocity and maximum drop diameter along friction factor.   | 2.33 |
| 2.27 | Variations of Sauter mean drop diameter and continuous phase velocity along friction factor.                                       | 2.34 |
| 2.28 | Comparison of various drag models which takes into account presence of adjacent drops at $d_e = 2.0$ mm.                           | 2.38 |
| 2.29 | Effect of various dispersion coefficients on dispersed phase hold-up for Lopez de Bertodano model (1992).                          | 2.41 |
| 2.30 | Effect of various dispersion coefficients on dispersed phase hold-up for Simonin and Viollet model (1990).                         | 2.41 |
| 3.1  | Structure of CFD describing the solution procedure.  | 3.4  |
| 3.2  | Schematic representation of flow in horizontal circular cylinder and boundary conditions.  | 3.5  |
| 3.3  | Structured grid composed of hexahedral cells.  | 3.6  |

|      |  |      |
|------|--|------|
| 3.4  | Flow structure near wall.  | 3.15 |
| 3.5  | Near-wall turbulence modeling approaches.  | 3.16 |
| 3.6  | Schematic diagram showing <i>rake</i> of 24 points.  | 3.25 |
| 4.1  | Phase distribution profile at 2.76 m/s and 60% input water fraction using steady state solver.   | 4.5  |
| 4.2  | Phase distribution profile at 2.76 m/s and 60% input water fraction using unsteady state solver.   | 4.6  |
| 4.3  | Mean velocity profiles of water for 60% input water at different mixture velocities at 4.75 m from pipe inlet.                               | 4.7  |
| 4.4  | Phase distribution profile at the pipe cross section as a function of solution time at 46% input water and 2.12 m/s mixture velocity.        | 4.8  |
| 4.5  | Phase distribution profiles along the distance from pipe inlet at 1.8 m/s, 2.17m/s and 2.76 m/s mixture velocities and 60% input water.      | 4.10 |
| 4.6  | Phase distribution profiles at the pipe cross section at 4.75 m from inlet and at 1.8 m/s mixture velocity.                                  | 4.12 |
| 4.7  | Phase distribution profiles at the pipe cross section at 4.75 m from inlet and at 2.76 m/s mixture velocity.                                 | 4.14 |
| 4.8  | Phase distribution profile at different mixture velocity for 60% input water.  | 4.16 |
| 4.9  | Pressure gradient profiles at different mixture velocity and input water fraction.   | 4.18 |
| 4.10 | Comparison of radial phase distribution profile at 62.5% input water and 1.7 m/s mixture velocity for data of Angeli (1996).                 | 4.21 |
| 4.11 | Comparison of vertical phase distribution profile at 46% and 60% input water and 3.0 m/s mixture velocity for data set of Soleimani (1999).  | 4.22 |
| 4.12 | Phase distribution contour profiles at the pipe cross section at 4.75 m from pipe inlet and at 3.0 m/s mixture velocity.                     | 4.22 |
| 4.13 | Comparison of vertical phase distribution profile at 46% and 60% input water and 2.12 m/s mixture velocity for data set of Soleimani (1999). | 4.23 |
| 4.14 | Phase distribution contour profiles at the pipe cross section at 4.75 m from pipe inlet and at 2.76 m/s mixture velocity.                    | 4.24 |
| 4.15 | Comparison of vertical phase distribution profiles at 46% input water with data of Soleimani (1999) and Hussain (2004).                      | 4.25 |

- 4.16 Phase distribution contour profiles at the pipe cross section at 4.75 m from pipe inlet and at 2.12 m/s mixture velocity. 4.25
- 4.17 Comparison of average insitu water fraction at different mixture velocity. 4.26

## LIST OF NOMENCLATURE

|  |  |           |
|--|--|-----------|
| $A$  | Flow area  | $[m^2]$   |
| $C_{DM}$   | Drag coefficient in the presence of adjacent drops | $[-]$     |
| $C_{D0}$   | Drag coefficient                                   | $[-]$     |
| $C_L$  | Lift coefficient                                   | $[-]$     |
| $C_{1\varepsilon}, C_{2\varepsilon}, C_\mu, C_v$ | Model constants                                    | $[-]$     |
| $D$  | Pipe diameter                                      | $[m]$     |
| $D'$   | Diffusivity  | $[m^2/s]$ |
| $D_t$  | Dispersed phase turbulent viscosity                | $[kg/ms]$ |
| $D$  | Drop diameter                                      | $[mm]$    |
| $d_e$  | Equivalent drop diameter                           | $[mm]$    |
| $d_{32}$   | Sauter mean drop diameter                          | $[mm]$    |
| $d_{10}$   | Linear mean drop diameter                          | $[mm]$    |
| $d_j$  | Drop diameter of size $j$                          | $[mm]$    |
| $d_i$  | Drop diameter of sampling size interval $i$        | $[mm]$    |
| $d_{\max}$                                       | Maximum drop diameter                              | $[mm]$    |
| $F$  | Force  | $[N/m]$   |
| $F_D$  | Drag force   | $[N/m]$   |
| $F_L$  | Lift force   | $[N/m]$   |
| $F_{vm}$   | Virtual mass force                                 | $[N/m]$   |
| $F$  | Friction factor                                    | $[-]$     |
| $G$  | Acceleration due to gravity (= 9.81)               | $[m/s^2]$ |



|             |  |             |
|-------------|--|-------------|
| $G_k$       | Generation of turbulent kinetic energy       | $[N/m^2s]$  |
| $H$         | Dimensionless height ratio                   | $[-]$       |
| $I$         | Turbulence intensity                         | $[-]$       |
| $K$         | Turbulent kinetic energy                     | $[m^2/s^2]$ |
| $k_c$       | Turbulent kinetic energy of continuous phase | $[m^2/s^2]$ |
| $k'$        | Consistency factor                           | $[-]$       |
| $k_{pq}$    | Interphase exchange coefficient              | $[-]$       |
| $K_L$       | Mass transfer coefficient                    | $[kg/m^2s]$ |
| $L$         | Pipe length                                  | $[m]$       |
| $L_t$       | Length scale of turbulent eddies             | $[m]$       |
| $L$         | Turbulent length scale                       | $[m]$       |
| $\dot{m}$   | Mass flow rate                               | $[kg/s]$    |
| $N$         | Number of drops in distribution              | $[-]$       |
| $n'$        | Power law index                              | $[-]$       |
| $n_i$       | Number of drops in interval $i$              | $[-]$       |
| $P$         | Pressure                                     | $[N/m^2]$   |
| $R$         | Pipe roughness                               | $[-]$       |
| $Re$        | Reynolds number                              | $[-]$       |
| $Re_c$      | Reynolds number of continuous phase          | $[-]$       |
| $Re_y$      | Near wall Reynolds number                    | $[-]$       |
| $Re_{flow}$ | Flow Reynolds number                         | $[-]$       |
| $Re_{DH}$   | Reynolds number using hydraulic diameter     | $[-]$       |
| $S$         | Slip ratio                                   | $[-]$       |

|                              |                                       |                |
|------------------------------|---------------------------------------|----------------|
| $S_\phi, S_k, S_\varepsilon$ | Source terms                          | $[kg / m^2 s]$ |
| $T$                          | Time                                  | $[sec]$        |
| $U_{sc}$                     | Continuous phase superficial velocity | $[m/s]$        |
| $U$                          | Velocity                              | $[m/s]$        |
| $U_c$                        | Continuous phase velocity             | $[m/s]$        |
| $U_*$                        | Velocity                              | $[m/s]$        |
| $U'_f$                       | Turbulent velocity fluctuation        | $[m/s]$        |
| $U_x$                        | Velocity in x direction               | $[m/s]$        |
| $U_y$                        | Velocity in y direction               | $[m/s]$        |
| $U_z$                        | Velocity in z direction               | $[m/s]$        |
| $U_{mix}$                    | Mixture velocity                      | $[m/s]$        |
| $U_s$                        | Superficial velocity                  | $[m/s]$        |
| $V_s$                        | Slip velocity                         | $[m/s]$        |
| $V$                          | Velocity of phases                    | $[m/s]$        |
| $v_{dr}$                     | Drift velocity                        | $[m/s]$        |
| $y_w$                        | Distance from pipe wall               | $[m]$          |
| $X$                          | Martinelli and Lockhart parameter     | $[-]$          |
| $Y^+$                        | Wall function                         | $[-]$          |

### **Greek Symbols**

|            |                                  |       |
|------------|----------------------------------|-------|
| $\alpha$   | Volume fraction/ Hold-up         | $[-]$ |
| $\alpha_c$ | Continuous phase volume fraction | $[-]$ |



|                                |  |             |
|--------------------------------|--|-------------|
| $\alpha_d$                     | Dispersed phase volume fraction  | [ - ]       |
| $\varepsilon_M$                | Mean rate of energy dissipation per unit mass in pipe flow             | $[m^2/s^3]$ |
| $\varepsilon_l$                | Local rate of energy dissipation per unit mass in pipe flow            | $[m^2/s^3]$ |
| $\tau_t$                       | Characteristic time of energetic eddies in turbulent flow              | [s]         |
| $\theta$                       | Angle between mean dispersed phase velocity and mean relative velocity | [deg]       |
| $\tau_F$                       | Characteristic particle relaxation time                                | [s]         |
| $\mu$                          | Viscosity  | $[kg/ms]$   |
| $\mu_t$                        | Turbulent viscosity  | $[kg/ms]$   |
| $\mu_g$                        | Viscosity of gas phase   | $[kg/ms]$   |
| $\mu_l$                        | Viscosity of liquid phase  | $[kg/ms]$   |
| $\mu_c$                        | Viscosity of continuous phase  | $[kg/ms]$   |
| $\rho$                         | Density  | $[kg/m^3]$  |
| $\rho_c$                       | Density of continuous phase  | $[kg/m^3]$  |
| $\rho_g$                       | Density of gas phase   | $[kg/m^3]$  |
| $\rho_l$                       | Density of liquid phase  | $[kg/m^3]$  |
| $\rho_d$                       | Density of dispersed phase   | $[kg/m^3]$  |
| $\sigma_{pr}$                  | Dispersion Prandtl number  | [ - ]       |
| $\sigma_k, \sigma_\varepsilon$ | Model constants  | [ - ]       |
| $\lambda$                      | Coefficient of resistance  | [ - ]       |