UNIVERSITI PUTRA MALAYSIA

STABILITY ANALYSIS OF SOME POPULATION MODELS WITH TIME DELAY AND HARVESTING

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By

SYAMSUDDIN TOAHA

Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirement for the Degree of Doctor of Philosophy

December 2006

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Dedication

Specially Dedicated to

My Wife, Nur Rahmatullah

and

My Daughters, Salsa and Dhani

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

STABILITY ANALYSIS OF SOME POPULATION MODELS WITH TIME DELAY AND HARVESTING

By

SYAMSUDDIN TOAHA

December 2006

Chairman: Professor Malik Bin Hj. Abu Hassan, PhD

Faculty: Science

This research presents the development and extension of some models for the growth rates of population. The existing models, i.e., logistic population model for single population, predator – prey model, Wangersky – Cunningham model, competing model and symbiosis model for two interaction populations, are extended by considering time delay, harvesting function and time delay in harvesting term in the models to get some new population models. The time delay is considered in the model to make the model more accurate because the growth rate of population does not only depend on the present size of population but also depends on past information. The current size of the population does not immediately change the growth rate of the population, but there is a time delay. The population as a valuable stock, for example fish population, is then harvested. The considered harvesting functions in the new population models are constant effort and constant quota of harvesting.

The new models are then analyzed to determine the stability of their equilibrium points. Before determining the stability of the equilibrium points, we provide the necessary and sufficient conditions for the existence of the equilibrium points. Since we consider population model, we just investigate the nonnegative equilibrium points. For some models, we determine only the sufficient conditions for the existence of the positive equilibrium points. The value of time delay, level of harvesting, initial size of populations, and parameters of the models need to be controlled so that the populations will not be extinct for a long time and also the populations give maximum profit.

The methods used to study the stability of the equilibrium point are linearization model around the equilibrium point, eigenvalues method, phase plane analysis, and plotting trajectories around the equilibrium point. In order to determine the stability of the equilibrium point, we inspect the sign of real parts of the eigenvalues. The graphs of the trajectories are plotted to visualize the behavior of the trajectories. For the models with constant effort of harvesting, we determine the critical value of the effort that maximizes the profit and does not affect the stability of the equilibrium point. Some new theorems are constructed and proved to determine the time delay margin, stability switches and stability intervals.

We find that there exists a certain condition so that the positive equilibrium point of the models becomes stable. From the analysis we find that the time delay can induce instability, stability switches and bifurcations in all the models except for the symbiosis model with time delay in harvesting term. The analysis also shows that for the models with constant effort of harvesting, there exists a critical value for the effort of harvesting that maximizes the profit function and maintains the stability of the equilibrium point.

When we control the values of the parameters, level of harvesting, and time delay, the positive equilibrium point can be found and possibly stable. The existence of the populations also depends on the initial value of the population since we just consider local stability. For the models without time delay and harvesting, we find the global stability of the positive equilibrium point. For the models with a time delay, there exists either a time delay margin or some stability switches so that the positive equilibrium point remains stable on the stability interval. The maximum profit can be found without affecting the stability of the equilibrium point when the values of parameters and the level of constant efforts of harvesting are strictly controlled.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

ANALISIS KESTABILAN BAGI BEBERAPA MODEL POPULASI DENGAN MASA LENGAH DAN TUAIAN

Oleh

SYAMSUDDIN TOAHA

Disember 2006

Pengerusi: **Profesor Malik Bin Hj. Abu Hassan, PhD**

Fakulti: **Sains**

Penyelidikan ini mengemukakan pembangunan dan pengembangan beberapa model untuk kadar pertumbuhan populasi. Model yang sedia ada, iaitu, model populasi logistik bagi satu populasi, model pemangsa – mangsa, model Wangerksy – Cunningham, model persaingan dan model simbiosis bagi dua populasi yang berinteraksi, dikembangkan dengan mempertimbangkan masa lengah, fungsi tuaian dan masa lengah dalam tuaian ke dalam model untuk mendapatkan beberapa model populasi yang baru. Masa lengah dipertimbangkan dalam model untuk membuat model menjadi lebih jitu sebab kadar pertumbuhan populasi bukan hanya bergantung pada saiz kini populasi tetapi ianya juga bergantung pada maklumat pada masa lepas. Saiz semasa populasi tidak secara langsung mengubah kadar pertumbuhan populasi, tetapi wujud suatu masa lengah. Populasi sebagai stok yang bernilai, sebagai contoh populasi ikan,

kemudiannya dituai. Fungsi tuaian yang dipertimbangkan ke dalam model populasi baru adalah tuaian dengan usaha malar dan tuaian dengan kuota malar.

Model-model baru kemudian dianalisis untuk menentukan kestabilan bagi titik keseimbangan. Sebelum menentukan kestabilan bagi titik keseimbangan, kita sediakan syarat perlu dan cukup bagi kewujudan titik keseimbangan. Oleh kerana kita pertimbangkan model populasi, kita hanya menyiasat titik keseimbangan yang tak negatif. Bagi beberapa model, kita hanya menentukan syarat cukup bagi kewujudan titik keseimbangan yang positif. Nilai masa lengah, aras tuaian, saiz awal populasi dan parameter bagi model perlu di kawal supaya populasi tidak akan pupus untuk masa yang panjang dan juga populasi memberi keuntungan maksimum.

Kaedah yang digunakan untuk mengkaji kestabilan bagi titik keseimbangan adalah pelinearan model di sekitar titik keseimbangan, kaedah nilai eigen, analisis satah fasa, dan melakar trajektori di sekitar titik keseimbangan. Untuk menentukan kestabilan titik keseimbangan, kita memeriksa tanda bahagian nyata nilai eigen. Graf bagi trajektori dilakar untuk menggambarkan telatah trajektori. Bagi model dengan tuaian usaha malar, kita tentukan suatu nilai genting bagi usaha tuaian yang memaksimumkan fungsi keuntungan dan tidak menjejaskan kestabilan titik keseimbangan. Beberapa teorem yang baru dibina dan dibuktikan untuk menentukan sut masa lengah, pertukaran kestabilan dan selang kestabilan.

Kita mendapati bahawa wujud suatu syarat tertentu supaya titik keseimbangan positif bagi model menjadi stabil. Daripada analisis kita dapati bahawa masa lengah boleh menyebabkan ketidakstabilan, boleh berlaku petukaran kestabilan dan bifurkasi bagi semua model kecuali model simbiosis dengan masa lengah dalam tuaian. Analisis juga menunjukkan bahawa bagi model dengan tuaian usaha malar, wujud suatu nilai genting bagi usaha tuaian yang memaksimumkan fungsi keuntungan dan mengekalkan kestabilan bagi titik keseimbangan.

Apabila kita mengawal nilai parameter, aras tuaian dan masa lengah, titik keseimbangan dapat diperolehi dan berkemungkinan stabil. Kewujudan populasi juga bergantung kepada saiz awal populasi kerana kita hanya mempertimbangkan kestabilan setempat. Bagi model tanpa masa lengah dan tuaian, kita perolehi kestabilan sejagat bagi titik keseimbangan positif. Bagi model dengan masa lengah, wujud suatu sut masa lengah atau beberapa pertukaran kestabilan supaya titik keseimbangan positif kekal menjadi stabil pada selang kestabilan. Keuntungan maksimum boleh diperolehi tanpa menjejaskan kestabilan titik keseimbangan apabila nilai parameter dan aras usaha tuaian yang malar dikawal dengan ketat.

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I certify that an Examination Committee has met on 12 December 2006 to conduct the final examination of Syamsuddin Toaha on his Doctor of Philosophy thesis entitled "Stability Analysis of Some Population Models with Time Delay and Harvesting" in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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This thesis submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for the degree of Doctor of Philosophy. The members of the Supervisory Committee are as follows:

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Date: 8 FEBRUARY 2007

DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

SYAMSUDDIN TOAHA

Date: 12 DECEMBER 2006

CHAPTER 1

INTRODUCTION

1.1 Introduction

A number of problems in the world usually involve continuously changing quantities such as distance, velocity, acceleration, or force. On the other hand, many problems in the life sciences deal with aggregates of individuals (human being, animals, fishes, trees, etc.) that are clearly discrete rather than continuous. Since derivatives and hence differential equations are meaningful only for quantities that change continuously, we might think that differential equations would arise only in the formulation of physical problem. If we consider the population in discrete time, the rate of change of the population can be denoted as a system of difference equations. However, if the population in an ecological problem is sufficiently large in quantity, it can usually be approximated or modeled in terms of a continuous system in which the growth rates of the populations can be expressed as first derivatives and the behavior of the system can be described by a system of differential equations.

In modeling the growth rate of the population in terms of a system of differential equations, the growth rate usually only depends on the population size in the present time. In fact, the growth rate of the population does not only depend on the present population size but also on the past. The present population does not immediately affect the growth rate of the population, but there is a time delay. In