DIAGONALLY IMPLICIT RUNGE-KUTTA METHOD FOR SOLVING ORDINARY DIFFERENTIAL EQUATIONS

By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of Master of Science

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Dedicated to

Nasaruddin Abd. Rahman
Muhammad Luqman
Nurjannah
Surayya Aqilah
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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Master of Science

BLOCK DIAGONALLY IMPLICIT RUNGE-KUTTA METHOD FOR SOLVING ORDINARY DIFFERENTIAL EQUATIONS

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March 2004

Chairman: **Associate Professor Fudziah bt Ismail, Ph.D.**

Faculty: **Science and Environmental Studies**

Block diagonally implicit Runge-Kutta (BDIRK) method of second order is derived using Butcher analysis and equi-distribution of errors (EDE) approach to outperform standard Runge-Kutta formulae. BDIRK method produces simultaneously approximation to the solution of the initial value problem (IVP) at a block of two points x_{n+1} and x_{n+2}. Numerical results are given to illustrate the performance of the method and comparison is made to the standard Runge-Kutta method.

In this thesis, we also derived block method using embedded diagonally implicit Runge-Kutta method. The conventional one step embedded diagonally implicit Runge-Kutta (DIRK) 3(2) method which is known to be reliable and has been tested extensively is used to approximate the solution at the first point of the block, and also to generate the solution at the second point of the block. The efficiency of the method is supported by some numerical results.

Finally, block embedded diagonally implicit Runge-kutta method is extended from approximating solutions at two points to three points simultaneously by increasing the order of the method. The results indicate that the block methods are competitive, both in terms of efficiency and reliability with conventional Runge-Kutta method. The stability polynomial for all the methods are obtained and their regions of stability are presented. Based on the numerical results and the stability regions it can be said that the method is suitable for solving stiff differential equations.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

KAEDAH BLOK RUNGE-KUTTA PEPENJURU TERSIRAT UNTUK MENYELESAIKAN PERSAMAAN PEMBEZAAN BIASA

Oleh

YONG FAEZAH BT RAHIM

March 2004

Pengerusi: **Profesor Madya Fudziah bt Ismail, Ph.D.**

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Kaedah blok Runge-Kutta pepenjuru tersirat (BRKPT) peringkat kedua telah diterbitkan menggunakan analisis Butcher dan juga pendekatan taburan sama rata ralat (TSRR) bagi menandingi kaedah Runge-Kutta biasa. Kaedah ini memberi penganggaran penyelesaian secara serentak kepada masalah nilai awal (MNA) di dua titik dalam satu blok iaitu di x_{n+1} dan x_{n+2}. Keputusan berangka telah diberi bagi menunjukkan pelaksanaan kaedah ini dan perbandingan dengan kaedah Runge-Kutta biasa telah dibuat.

Dalam tesis ini, kami juga telah menerbitkan kaedah blok menggunakan kaedah terbenam Runge-Kutta pepenjuru tersirat. Kaedah satu langkah biasa iaitu kaedah terbenam Runge-Kutta pepenjuru tersirat (RKPT) 3(2) yang telah diuji akan keberkesanannya telah dipilih untuk menganggarkan penyelesaian pada titik yang pertama dalam blok ini dan juga digunakan sebagai penjana untuk menghasilkan penyelesaian pada titik yang kedua

dalam blok. Keberkesanan kaedah ini telah disokong dengan beberapa keputusan berangka.

Akhir sekali kaedah terbenam blok Runge-Kutta pepenjuru tersirat telah diperkembangkan daripada menganggarkan penyelesaian pada dua titik kepada tiga titik serentak dengan menambahkan peringkat bagi kaedah tersebut. Keputusan yang didapati menunjukkan kaedah blok ini menjadi saingan dari segi keberkesanan dan kebolehannya, kepada kaedah Runge-Kutta biasa . Polinomial kestabilan bagi semua kaedah tersebut telah didapati dan rantau kestabilannya telah dibentangkan. Daripada keputusan berangka dan rantau kestabilan yang didapati, boleh dikatakan kaedah ini sesuai untuk menyelesaikan persamaan pembezaan kaku.

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I certify that an Examination Committee met on **11 of March 2004** to conduct the final examination of **Yong Faezah Rahim** on her **Master of Science** thesis entitled " **Block Diagonally Implicit Runge-Kutta Method for Solving Ordinary Differential Equations**" in accordance with University Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulation 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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This thesis submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirements for the degree of Master of Science. The members of the Supervisory Committee are as follows:

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DECLARATION

I hereby declare that the thesis is base on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

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YONG FAEZAH RAHIM

Date: 7 June 2004

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