

CROSSING NUMBERS OF CERTAIN GRAPHS

By

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**Thesis Submitted to the School of Graduate Studies,
Universiti Putra Malaysia, in Fulfilment of the Requirement
for the Degree of Master of Science**

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Dedication

To

My Dear wife

Liew Lin Kiow

For Her Great Patience and Support

My Lovely Daughters

Yee Yin and Yee Hann

For Their Understanding

And

My Parents

For Their Encouragement

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in
Fulfilment of the requirement for the degree of Master of Science

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May 2004

Chairman: Professor Peng Yee Hock, Ph.D.

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Since Harary, Kainen and Schwenk conjectured in 1973 that the crossing number of the Cartesian product $C_m \times C_n$ of an m -cycle with an n -cycle is $(m - 2)n$ for $m \leq n$, many results on the crossing number of the Cartesian product of graphs have been obtained. In this thesis, we study and investigate the crossing numbers of $P(k,1) + K_1$ and $P(k,1) \times P_n$.

The thesis begins with an explanation of the necessary terminology. The thesis continues by finding the crossing number of $P(3,1) + K_1$ and $P(4,1) + K_1$. We show that the crossing number of $P(3,1) \times P_n$ is $4(n - 1)$ for $n \geq 1$. We then proved that the crossing number of $P(4,1) \times P_n$ is $8(n - 1)$ for $n \geq 1$.

We end the thesis by investigating the crossing number of $P(5,1) \# P_n$. We also show that, the crossing number of $P(k,1) \# K_1$ is k . The determination of the crossing number of $P(k,1) \# P_n$ for $k \geq 5$ is left as an open problem for future research.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

CROSSING NUMBERS OF CERTAIN GRAPHS

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Sejak Harary, Kainen and Schwenk membuat konjektur dalam tahun 1973 bahawa nombor silang bagi hasil darab Cartesian $C_m \times C_n$ daripada kitar- m dan kitar- n ialah $(m - 2)n$ untuk $m \leq n$, banyak hasil kajian nombor silang bagi graf telah diperolehi. Dalam tesis ini, kita mengkaji nombor silang bagi $P(k, 1) + K_1$ dan $P(k, 1) \times P_n$.

Tesis ini bermula dengan menjelaskan terminologi yang penting. Tesis ini seterusnya mengenalpasti nombor silang bagi $P(3, 1) + K_1$ dan $P(4, 1) + K_1$. Kita buktikan nombor silang bagi $P(3, 1) \times P_n$ ialah $4(n - 1)$ untuk $n \geq 1$. Kita buktikan juga bahawa nombor silang $P(4, 1) \times P_n$ ialah $8(n - 1)$ untuk $n \geq 1$.

Kita akhiri tesis ini dengan menyiasat nombor silang bagi $P(5, 1) \times P_n$. Kita juga tunjukkan bahawa nombor silang $P(k, 1) + K_1$ ialah k . Penentuan nombor silang bagi $P(k, 1) \times P_n$ untuk $k \geq 5$ masih merupakan masalah terbuka untuk kajian lanjutan

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I certify that an Examination Committee met on 1st May 2004 to conduct the final examination of Yiew Yip Chong on his Master of Science thesis entitled “The Crossing Number Of Certain Graphs” in accordance with Universiti Pertanian Malaysia (Higher Degree) Act 1980 and Universiti Pertanian Malaysia (Higher Degree) Regulations 1981. The Committee recommends that the candidate be awarded the relevant degree. Members of the Examination Committee are as follows:

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DECLARATION

I hereby declare that the thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at UPM or other institutions.

YIEW YIP CHONG

Date:

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