Malaysian Journal of Mathematical Sciences 8(1): 1- 14 (2014)

MALAYSIAN JOURNAL OF MATHEMATICAL SCIENCES

Journal homepage: http://einspem.upm.edu.my/journal

Generating Mutually Unbiased Bases and Discrete Wigner Functions for Three-Qubit System

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ABSTRACT

It is known that there exists $2^N + 1$ mutually unbiased bases for N qubits system. Between the different MUB construction algorithms of the three-qubit case, we focus on Wootters method with discrete phase space that leads naturally to a complete set of $2^N + 1$ mutually unbiased bases for the state space. We construct discrete Wigner function using mutually unbiased bases from the discrete phase space for three-qubit system by explicitly calculating the Wigner functions for exemplary three-qubit pure states such as the GHZ state, the W state and the embedded Bell state. We also highlight some quasi-probability characteristics of these entangled states.

Keywords: Qubit, MUB, Discrete Wigner function, GHZ state, W state, Bell state.

1. INTRODUCTION

The Wigner function based on continuous phase space is an alternative way of representing quantum states and it serves the role of a quasi-probability distribution whose values can be negative. In the same way as the original Wigner function, Wootters (1987), Wootters (2004) and Gibbons *et al.* (2004) constructed Wigner functions on discrete phase spaces to describe finite-dimensional quantum systems. For determining this discrete phase space, they labeled the axes of phase space with finite field (Galois field) of *N* elements where *N* is power of prime. For example, the system of two qubits described by a state space whose dimensionality is $N=2^2$, meets this condition, and this leads to five different striations (in phase space, any set of parallel lines is called a "striation" of the phase space). Equivalently they labeled the horizontal and vertical axes of their phase space by quantum states of two-qubit system. From this labeling and using suitable translation vectors, one can produce five mutually unbiased bases for the state space.

Romero *et al.* (2005) and Björk *et al.* (2007) have shown that there are other approaches to MUB construction for the three-qubit case and there exist four different MUB structures, with respect to their entanglement properties. They have forms of (3,0,6), (2,3,4), (1,6,2), and (0,9,0), where the digits represents the number of triseparable , biseparable, and non separable bases, respectively. Their construction of MUBs is mainly based on the use of the finite Fourier transform, employing the Pauli operators and tensor products. In this paper our intent is to extend Wootters MUB construction method that is naturally based on properties of phase space for three-qubit case. Applying this method, we will explicitly discuss one of those four different structures with (3,0,6) form, where three of the bases are fully separable, and the remaining six bases are nonseparable. This is the only structure which has the "visually straight" lines in their striations, where each line obeys equations $aq + bp = c$ as we describe in following section. After finding the appropriate Galois field and translation vectors for the three-qubit system, we get nine sets of striations while each of them has eight parallel lines. Based on these striations we will obtain the nine mutually unbiased bases for the Hilbert space. In the paper, we will use those definitions found in Wootters (1987), Wootters (2004), Gibbons *et al.* (2004), Paz *et al.* (2005), Nielsen and Chuang (2010) and Kaye *et al.* (2007).

For calculating discrete Wigner function, we have to determine appropriate quantum net $Q(\lambda)$ where λ is a line from one of our bases. There are 8^9 different choices for defining quantum nets of three qubits, but by the same method of Gibbons *et al* (2004) for two qubits using some unitary operators, we can reduce our choices to 8^4 different choices of quantum nets (8^4 similarity classes). Based on eight arbitrary chosen similarity classes, we calculate the Wigner functions for exemplary states like *GHZ* and *W* states, an embedded Bell state which is not completely entangled, and others which are demonstrative enough for the discussion of the properties of their Wigner functions. Finally we compare phase-space point operators of different quantum nets of three-qubit case with tensor-product of three qubit phase-space point operators.

2. LINES AND STRIATIONS IN DISCRETE PHASE SPACE

In the continuous phase space, lines are defined by $aq + bp = c$, where all a, b and c are real constants, while the variables p and q that form our axes take values in the real numbers. Two lines $aq + bp = c$ and $aq + bp = c'$ are parallel when $c \neq c'$. The same properties and definitions can be extended for discrete phase space of n -qubit system by using Galois field element of $N = 2^n$ dimension which is associated with the axis of discrete phase space.

In our work, we apply Galois field for three-qubit $GF(2^3)$. In this case we take the primitive polynomial to be $f(x) = x^3 + x^2 + 1$. Thus, our Galois field is $\mathbb{Z}_2[x]/\langle x^3 + x^2 + 1\rangle$ [Lidl (1994) and Malik *et al.* (1997)], which leads to the field elements that can be listed as: $\{0, 1, w, w^2, w^3, w^4, w^5, w^6\}$, with $w^7 = 1$. The arithmetic of $GF(2^3)$ is shown in the Table 1.

TABLE 1: Addition and multiplication tables in *GF(8)*

$+$	0	1	w	w^2	w^3	w^4	w^5	w^6
0	0	1	w	w^2	w^3	w^4	w^5	w^6
1	1	0	w^5	w^3	w^2	w^6	w	w^4
w	w	w^5	0	w^6	w^4	w^3	1	w^2
w^2	w^2	w^3	w^6	0	1	w^5	w^4	w
w^3	w^3	w^2	w^4	1	0	w	w^6	w^5
w^4	w^4	w^6	w^3	w^5	w	0	w^2	1
w^5	w^5	w	1	w^4	w^6	w^2	0	w^3
w^6	w^6	w^4	w^2	w	w^5	1	w^3	0

$\times$	0	1	w	w^2	w^3	w^4	w^5	w^6
0	0	0	0	0	0	0	0	0
1	0	1	w	w^2	w^3	w^4	w^5	w^6
w	0	w	w^2	w^3	w^4	w^5	w^6	1
w^2	0	w^2	w^3	w^4	w^5	w^6	1	w
w^3	0	w^3	w^4	w^5	w^6	1	w	w^2
w^4	0	w^4	w^5	w^6	1	w	w^2	w^3
w^5	0	w^5	w^6	1	w	w^2	w^3	w^4
w^6	0	w^6	1	w	w^2	w^3	w^4	w^5

The state space for three qubits has $N = 2^3$ dimensions and hence consists of an 8×8 arrays of points for its phase space as shown in Figure 1(a). It can also be seen that we associate axes of our phase space by the discrete variables p and q, which take values in $GF(N) = GF(8)$. By these variables, one can form equations of lines $(aq + bp = c)$ in discrete phase space where *a, b* and *c* are numerical elements of *GF(N).*

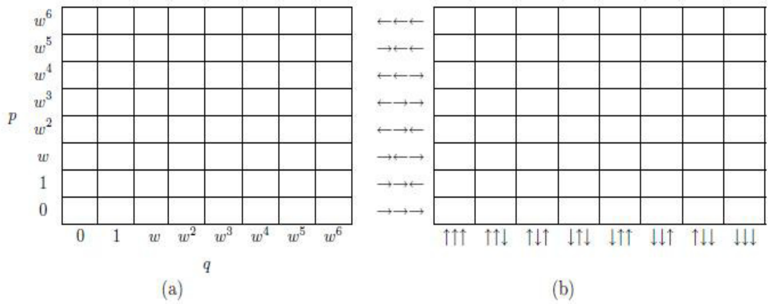

Figure 1: *8* × *8* arrays of points of phase space. The axis of phase space associated (a) by the elements of *GF(N)* and (b) spin states

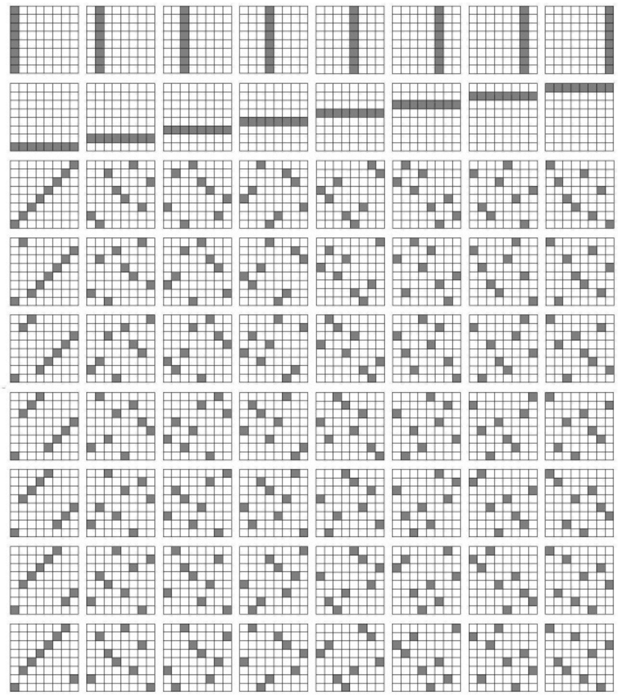

Figure 2: Nine striations of three-qubit system. The equations of them are $q = c$, $p = c$, $q+p = c$, $q+wp = c$, $q+w^2p = c$, $q+w^3p = c$, $q+w^4p = c$, $q + w^5p = c$ and $q + w^6p = c$ respectively

For three qubits, substituting *a*, *b* and *c* from the eight elements of *GF(8)*, we obtain *72* lines which can be divided into nine sets of parallel lines (each set of parallel lines is called a striation of phase space). By employing programming in Matlab we have found the striations for three-qubit system as in Figure 2.

3. MUTUALLY UNBIASED BASES FOR THREE QUBITS

Consider the association of the bases $E=\{e_1,e_2,...,e_n\}$ and $F=\{f_1,f_2,...,f_n\}$ to the horizontal and vertical axes of the phase space respectively. Gibbons *et al.* (2004) showed that if we want to have the same bases for horizontal and vertical direction, these two field bases should be related to each other by $f_i=\alpha\, e'_i$, where α is an element of $GF(N)$ and e'_i is an element of E' (the dual of E). In our work, we get the field bases as $(e_1, e_2, e_3) = (f_1, f_2, f_3) = (1,w,w^3)$, while the multiplication factor α is w^5. Based on it we shall define six basic translations operators for phase space as

$$\begin{aligned} H_1 &= I\otimes I\otimes\sigma_x & V_1 &= I\otimes I\otimes\sigma_z \\ H_w &= I\otimes\sigma_x\otimes I & V_w &= I\otimes\sigma_z\otimes I \\ H_{w^3} &= \sigma_x\otimes I\otimes I & V_{w^3} &= \sigma_z\otimes I\otimes I\,, \end{aligned}$$

where H and V are operators for the horizontal and vertical translations respectively, and their subscripts show the field elements by which one translates the phase space points. All other translations can be obtained by combining these six basic translations. For instance, translation by vector $(1,w^6)$ is equal to the horizontal translation by 1 and the vertical translation by $1+w+w^3$, so it can be associated with the unitary operator: $H_1V_1V_wV_{w^3}=-i\sigma_z\otimes\sigma_z\otimes\sigma_y$.

Beside association of the axes of our discrete phase space by the Galois field elements, one can associate the horizontal axis of phase space by the states $|\uparrow\uparrow\uparrow\rangle, |\uparrow\uparrow\downarrow\rangle,..., |\downarrow\downarrow\downarrow\rangle$, and the vertical axis by the states $|\rightarrow\rightarrow\rightarrow\rangle, |\rightarrow\rightarrow\leftarrow\rangle,..., |\leftarrow\leftarrow\leftarrow\rangle$ of three qubits, as shown in Figure 1(b). Now, for example, consider a horizontal translation, which interchanges the first two columns $(0\Leftrightarrow 1)$ as well as $(w\Leftrightarrow w^5)$, $(w^2\Leftrightarrow w^3)$ and $(w^4\Leftrightarrow w^6)$. These translations correspond to interchanging the third particle state from $\uparrow$ to $\downarrow$ and vice versa.

Now we list the invariance vectors of each set of striation in Table 2. These translation vectors send points in each line of the striation into other points in that line. In other words, under these translations, each line of striation is invariant. These translation vectors can be rewritten using the six basic translations operators. The simultaneous eigenvectors of the invariance operators of each striation constitute the basis which is associated with that striation. We obtain nine bases as listed in Table 3. The vectors in these nine bases have the property of mutually unbiased bases. Note that two orthonormal bases A and A' Hilbert space $\mathbb{C}^d$ are called mutually unbiased (MUB) if and only if $\langle a \mid a' \rangle = \sqrt{1/d}$ holds for all $a \in A$ and $a' \in A'$ (Klappenecker and Rötteler (2004), Bengtsson (2006) and Tselniker *et al.* (2009)).

TABLE 2: Three-qubit invariance vectors; the sequence of bases corresponds to the order of striations in Figure 2.

basis	Invariance Vectors						
0	$(0,1)$	$(0,w)$	$(0,w^2)$	$(0,w^3)$	$(0,w^4)$	$(0,w^5)$	$(0,w^6)$
1	$(1,0)$	$(w,0)$	$(w^2,0)$	$(w^3,0)$	$(w^4,0)$	$(w^5,0)$	$(w^6,0)$
2	$(1,1)$	(w,w)	(w^2,w^2)	(w^3,w^3)	(w^4,w^4)	(w^5,w^5)	(w^6,w^6)
3	$(1,w^6)$	$(w,1)$	(w^2,w)	(w^3,w^2)	(w^4,w^3)	(w^5,w^4)	(w^6,w^5)
4	$(1,w^5)$	(w,w^6)	$(w^2,1)$	(w^3,w)	(w^4,w^2)	(w^5,w^3)	(w^6,w^4)
5	$(1,w^4)$	(w,w^5)	(w^2,w^6)	$(w^3,1)$	(w^4,w)	(w^5,w^2)	(w^6,w^3)
6	$(1,w^3)$	(w,w^4)	(w^2,w^5)	(w^3,w^6)	$(w^4,1)$	(w^5,w)	(w^6,w^2)
7	$(1,w^2)$	(w,w^3)	(w^2,w^4)	(w^3,w^5)	(w^4,w^6)	$(w^5,1)$	(w^6,w)
8	$(1,w)$	(w,w^2)	(w^2,w^3)	(w^3,w^4)	(w^4,w^5)	(w^5,w^6)	$(w^6,1)$

4. DISCRETE WIGNER FUNCTION FOR THREE QUBITS

Each vector of bases in Table 3 can be associated with specific line of corresponding striations in Figure 2. This association is completely arbitrary and each separate choice is named a quantum net $Q(\lambda)$. Gibbons *et al* (2004) classify these quantum nets by N^{N-1} equivalent classes. For example, there are four similarity classes for two-qubit system. For three qubit case there are 8^7 equivalent classes and must be at least 8^4 similarity classes (because we have 8^3-8 unit-determinant linear transformations on the phase space which shows each similarity class can include at most 8^3-8 equivalent classes). Also there is similarity relation between two quantum nets Q and Q' if and only if

$$\Gamma'_{\alpha\beta\gamma} = \Gamma_{L\alpha L\beta L\gamma}, \tag{1}$$

where functions Γ and Γ' correspond to Q and Q' respectively and L is a unit-determinant linear transformation. So we can use function Γ to characterize different similarity classes. Here, by Maple programming, we find eight representatives of $\Gamma_{00\gamma}$ for the eight arbitrary similarity classes of three-qubit system as in Figure 3.

TABLE 3: Nine bases generated by the nine striations. In each vector $|V\rangle = abcdefgh$, the numbers a, b, c, d, e, f, g, h are the coefficients of state $a\,|000\rangle + b\,|001\rangle + c\,|010\rangle + d\,|101\rangle + e\,|100\rangle + f\,|110\rangle + g\,|011\rangle + h\,|111\rangle$.

A bar over the number shows that it is negative number and $i = \sqrt{-1}$

basis	Vectors			
0	$\|0_1\rangle = 10000000$	$\|0_2\rangle = 01000000$	$\|0_3\rangle = 00100000$	$\|0_4\rangle = 00000100$
	$\|0_5\rangle = 00001000$	$\|0_6\rangle = 00000010$	$\|0_7\rangle = 00010000$	$\|0_8\rangle = 00000001$
1	$\|1_1\rangle = 11111111$	$\|1_2\rangle = 1\bar{1}1\bar{1}1\bar{1}1\bar{1}$	$\|1_3\rangle = 11\bar{1}\bar{1}11\bar{1}\bar{1}$	$\|1_4\rangle = 1\bar{1}1\bar{1}\bar{1}1\bar{1}1$
	$\|1_5\rangle = 1111\bar{1}\bar{1}\bar{1}\bar{1}$	$\|1_6\rangle = 11\bar{1}\bar{1}\bar{1}\bar{1}11$	$\|1_7\rangle = 1\bar{1}\bar{1}11\bar{1}\bar{1}1$	$\|1_8\rangle = 1\bar{1}\bar{1}1\bar{1}11\bar{1}$
2	$\|2_1\rangle = 1ii\bar{1}i\bar{1}\bar{1}\bar{i}$	$\|2_2\rangle = 1\bar{i}i1i1\bar{1}i$	$\|2_3\rangle = 1i\bar{i}1i\bar{1}1i$	$\|2_4\rangle = 1\bar{i}i1\bar{i}\bar{1}1\bar{i}$
	$\|2_5\rangle = 1ii\bar{1}\bar{i}11i$	$\|2_6\rangle = 1i\bar{i}1\bar{i}1\bar{1}\bar{i}$	$\|2_7\rangle = 1\bar{i}\bar{i}\bar{1}i11\bar{i}$	$\|2_8\rangle = 1\bar{i}\bar{i}\bar{1}\bar{i}\bar{1}\bar{1}i$
3	$\|3_1\rangle = 1i1\bar{i}i1i\bar{1}$	$\|3_2\rangle = 1\bar{i}1ii\bar{1}i1$	$\|3_3\rangle = 1i\bar{1}ii1\bar{i}1$	$\|3_4\rangle = 1\bar{i}1\bar{i}i1\bar{i}\bar{1}$
	$\|3_5\rangle = 1i1\bar{i}\bar{i}\bar{1}\bar{i}1$	$\|3_6\rangle = 1i\bar{1}\bar{i}\bar{i}\bar{1}i\bar{1}$	$\|3_7\rangle = 1\bar{i}\bar{1}\bar{i}i\bar{1}\bar{i}\bar{1}$	$\|3_8\rangle = 1\bar{i}\bar{1}\bar{i}\bar{i}1i1$
4	$\|4_1\rangle = 1ii11\bar{i}\bar{i}\bar{1}$	$\|4_2\rangle = 1\bar{i}i\bar{1}1\bar{i}\bar{i}1$	$\|4_3\rangle = 1\bar{i}\bar{i}\bar{1}1ii1$	$\|4_4\rangle = 1\bar{i}i\bar{1}\bar{1}ii\bar{1}$
	$\|4_5\rangle = 1ii1\bar{1}\bar{i}i1$	$\|4_6\rangle = 1i\bar{i}\bar{1}\bar{1}\bar{i}\bar{i}\bar{1}$	$\|4_7\rangle = 1\bar{i}\bar{i}11\bar{i}i\bar{1}$	$\|4_8\rangle = 1\bar{i}\bar{i}1\bar{1}\bar{i}i1$
5	$\|5_1\rangle = 11\bar{i}i1\bar{1}ii$	$\|5_2\rangle = 1\bar{1}ii11i\bar{i}$	$\|5_3\rangle = 11\bar{i}i1\bar{1}\bar{i}\bar{i}$	$\|5_4\rangle = 1\bar{1}ii\bar{1}\bar{1}ii$
	$\|5_5\rangle = 11\bar{i}\bar{i}\bar{1}1\bar{i}\bar{i}$	$\|5_6\rangle = 11\bar{i}i\bar{1}1ii$	$\|5_7\rangle = 1\bar{1}\bar{i}\bar{i}11\bar{i}i$	$\|5_8\rangle = 1\bar{1}\bar{i}\bar{i}\bar{1}\bar{1}i\bar{i}$
6	$\|6_1\rangle = 11ii\bar{i}i1\bar{1}$	$\|6_2\rangle = 1\bar{1}i\bar{i}ii11$	$\|6_3\rangle = 11\bar{i}\bar{i}i\bar{i}\bar{1}1$	$\|6_4\rangle = 1\bar{1}i\bar{i}\bar{i}\bar{i}1\bar{1}$
	$\|6_5\rangle = 11ii\bar{i}i\bar{1}1$	$\|6_6\rangle = 11\bar{i}\bar{i}\bar{i}i1\bar{1}$	$\|6_7\rangle = 1\bar{1}i\bar{i}ii\bar{1}\bar{1}$	$\|6_8\rangle = 1\bar{1}i\bar{i}\bar{i}\bar{i}11$
7	$\|7_1\rangle = 1i1i1\bar{i}\bar{1}i$	$\|7_2\rangle = 1\bar{i}1\bar{i}1i\bar{1}\bar{i}$	$\|7_3\rangle = 1i\bar{1}\bar{i}1\bar{i}1\bar{i}$	$\|7_4\rangle = 1\bar{i}1\bar{i}\bar{1}\bar{i}1i$
	$\|7_5\rangle = 1i1i\bar{1}i1\bar{i}$	$\|7_6\rangle = 1i\bar{1}\bar{i}\bar{1}i\bar{1}i$	$\|7_7\rangle = 1\bar{i}\bar{1}i1i1i$	$\|7_8\rangle = 1\bar{i}\bar{1}i\bar{1}\bar{i}\bar{1}\bar{i}$
8	$\|8_1\rangle = 111\bar{1}i\bar{i}\bar{i}i$	$\|8_2\rangle = 1\bar{1}11\bar{i}\bar{i}\bar{i}\bar{i}$	$\|8_3\rangle = 11\bar{1}1iii\bar{i}$	$\|8_4\rangle = 1\bar{1}11i\bar{i}ii$
	$\|8_5\rangle = 111\bar{1}\bar{i}\bar{i}\bar{i}\bar{i}$	$\|8_6\rangle = 11\bar{1}1\bar{i}\bar{i}\bar{i}i$	$\|8_7\rangle = 1\bar{1}\bar{1}\bar{1}i\bar{i}ii$	$\|8_8\rangle = 1\bar{1}\bar{1}\bar{1}\bar{i}\bar{i}\bar{i}\bar{i}$

$$\frac{1}{64}\begin{array}{|c|c|c|c|c|c|c|c|}\hline 1 & 9 & 9 & 9 & 9 & 9 & 9 & 1 \\ \hline -3 & 9 & 9 & -11 & 9 & -11 & -3 & 9 \\ \hline -3 & 9 & 9 & -11 & 9 & -3 & -11 & 9 \\ \hline 17 & 9 & 9 & 9 & 17 & 9 & 9 & 9 \\ \hline -3 & 9 & 9 & -3 & 9 & -11 & -11 & 9 \\ \hline 17 & 9 & 17 & 9 & 9 & 9 & 9 & 9 \\ \hline 17 & 17 & 9 & 9 & 9 & 9 & 9 & 9 \\ \hline 125 & 17 & 17 & -3 & 17 & -3 & -3 & 1 \\ \hline \end{array}$$

$$\frac{1}{64}\begin{array}{|c|c|c|c|c|c|c|c|}\hline -5 & 11 & 19 & -1 & 19 & 11 & -1 & 27 \\ \hline 15 & -1 & 11 & -1 & 3 & -1 & -9 & -1 \\ \hline -1 & 3 & -1 & -1 & -1 & 7 & -1 & 11 \\ \hline 7 & 11 & 19 & 11 & 11 & -1 & 3 & 19 \\ \hline 15 & -1 & 3 & -9 & 11 & -1 & -1 & -1 \\ \hline 7 & 11 & 11 & 3 & 19 & -1 & 11 & 19 \\ \hline 19 & -5 & 11 & -1 & 11 & 3 & -1 & 11 \\ \hline 119 & 19 & 7 & 15 & 7 & -1 & 15 & -5 \\ \hline \end{array}$$

$$\frac{1}{64}\begin{array}{|c|c|c|c|c|c|c|c|}\hline -5 & 19 & 11 & 11 & 19 & -1 & -1 & 27 \\ \hline 15 & 11 & -1 & -1 & 3 & -1 & -9 & -1 \\ \hline 15 & 3 & -1 & -1 & 11 & -9 & -1 & -1 \\ \hline 7 & 19 & 11 & -1 & 11 & 11 & 3 & 19 \\ \hline -1 & -1 & 3 & 7 & -1 & -1 & -1 & 11 \\ \hline 19 & 11 & -5 & 3 & 11 & -1 & -1 & 11 \\ \hline 7 & 11 & 11 & -1 & 19 & 3 & 11 & 19 \\ \hline 119 & 7 & 19 & -1 & 7 & 15 & 15 & -5 \\ \hline \end{array}$$

$$\frac{1}{64}\begin{array}{|c|c|c|c|c|c|c|c|}\hline 5 & 5 & 21 & 5 & 5 & 9 & 9 & 13 \\ \hline 1 & 5 & 9 & -7 & 13 & 1 & -7 & 9 \\ \hline 1 & 13 & 9 & -7 & 5 & -7 & 1 & 9 \\ \hline 17 & 21 & 5 & 9 & -3 & 5 & 13 & 5 \\ \hline 9 & 9 & 13 & -7 & 9 & -7 & -7 & 5 \\ \hline 13 & 5 & -3 & 13 & 5 & 9 & 9 & 21 \\ \hline 17 & -3 & 5 & 9 & 21 & 13 & 5 & 5 \\ \hline 89 & 17 & 13 & 9 & 17 & 1 & 1 & 5 \\ \hline \end{array}$$

$$\frac{1}{64}\begin{array}{|c|c|c|c|c|c|c|c|}\hline -5 & 19 & 19 & -1 & 11 & -1 & 11 & 27 \\ \hline -1 & -1 & -1 & -1 & 3 & -1 & 7 & 11 \\ \hline 15 & 3 & 11 & -1 & -1 & -9 & -1 & -1 \\ \hline 19 & 11 & 11 & -1 & -5 & -1 & 3 & 11 \\ \hline 15 & 11 & 3 & -9 & -1 & -1 & -1 & -1 \\ \hline 7 & 19 & 11 & 3 & 11 & 11 & -1 & 19 \\ \hline 7 & 11 & 19 & 11 & 11 & 3 & -1 & 19 \\ \hline 119 & 7 & 7 & 15 & 19 & 15 & -1 & -5 \\ \hline \end{array}$$

$$\frac{1}{64}\begin{array}{|c|c|c|c|c|c|c|c|}\hline 5 & 21 & 5 & 9 & 5 & 5 & 9 & 13 \\ \hline 1 & 9 & 5 & 1 & 13 & -7 & -7 & 9 \\ \hline 9 & 13 & 9 & -7 & 9 & -7 & -7 & 5 \\ \hline 17 & 5 & 21 & 5 & -3 & 9 & 13 & 5 \\ \hline 1 & 9 & 13 & -7 & 5 & -7 & 1 & 9 \\ \hline 17 & 5 & -3 & 13 & 21 & 9 & 5 & 5 \\ \hline 13 & -3 & 5 & 9 & 5 & 13 & 9 & 21 \\ \hline 89 & 13 & 17 & 1 & 17 & 9 & 1 & 5 \\ \hline \end{array}$$

$$\frac{1}{64}\begin{array}{|c|c|c|c|c|c|c|c|}\hline 5 & 5 & 5 & 9 & 21 & 9 & 5 & 13 \\ \hline 9 & 9 & 9 & -7 & 13 & -7 & -7 & 5 \\ \hline 1 & 13 & 5 & 1 & 9 & -7 & -7 & 9 \\ \hline 13 & 5 & 5 & 9 & -3 & 9 & 13 & 21 \\ \hline 1 & 5 & 13 & -7 & 9 & 1 & -7 & 9 \\ \hline 17 & 21 & -3 & 13 & 5 & 5 & 9 & 5 \\ \hline 17 & -3 & 21 & 5 & 5 & 13 & 9 & 5 \\ \hline 89 & 17 & 17 & 1 & 13 & 1 & 9 & 5 \\ \hline \end{array}$$

$$\frac{1}{64}\begin{array}{|c|c|c|c|c|c|c|c|}\hline 7 & 15 & 15 & -1 & 15 & -1 & -1 & 31 \\ \hline 3 & -1 & -1 & 3 & 15 & 3 & -5 & -1 \\ \hline 3 & 15 & -1 & 3 & -1 & -5 & 3 & -1 \\ \hline 7 & 23 & 23 & -1 & -1 & -1 & 15 & 15 \\ \hline 3 & -1 & 15 & -5 & -1 & 3 & 3 & -1 \\ \hline 7 & 23 & -1 & 15 & 23 & -1 & -1 & 15 \\ \hline 7 & -1 & 23 & -1 & 23 & 15 & -1 & 15 \\ \hline 107 & 7 & 7 & 3 & 7 & 3 & 3 & 7 \\ \hline \end{array}$$

Figure 3: Eight representative Γ for three qubits.

The definition of discrete Wigner function for n-qubit is

$$\sum_{\alpha \in \lambda} W_{\alpha} = Tr[\rho Q(\lambda)], \tag{2}$$

where ρ is density matrix of the system and $Q(\lambda)$ is a state related to line λ (a quantum net).

For example, here we calculate discrete Wigner functions for *GHZ* state (Greenberger *et al.* (1989)), *W* state (Coffman *et al.* (2000)) and embedded Bell states related to our eight arbitrary similarity classes. This is similar to Dür *et al.* (2000) characterization of all possible kinds of entanglement of three qubit pure states; Unentangled states, biseparable ones and two different kinds of genuine tripartite entanglement namely the *GHZ* and *W* state.

(a)

$\leftarrow\leftarrow\leftarrow$	$\frac{3}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{3}{32}$
$\rightarrow\leftarrow\leftarrow$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
$\leftarrow\leftarrow\rightarrow$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
$\leftarrow\rightarrow\rightarrow$	$\frac{3}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{3}{32}$
$\leftarrow\rightarrow\leftarrow$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
$\rightarrow\leftarrow\rightarrow$	$\frac{3}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{3}{32}$
$\rightarrow\rightarrow\leftarrow$	$\frac{3}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{3}{32}$
$\rightarrow\rightarrow\rightarrow$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$
	$\uparrow\uparrow\uparrow$	$\uparrow\uparrow\downarrow$	$\uparrow\downarrow\uparrow$	$\downarrow\uparrow\downarrow$	$\downarrow\uparrow\uparrow$	$\downarrow\downarrow\uparrow$	$\uparrow\downarrow\downarrow$	$\downarrow\downarrow\downarrow$

(b)

$\frac{3}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	0
$-\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$	$\frac{1}{24}$	0
$-\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$	0
$-\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$	$\frac{1}{24}$	0
$-\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$	$-\frac{1}{24}$	0
$-\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$	$-\frac{1}{24}$	0
$-\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$	0
$\frac{3}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	0

$\frac{2}{24}$	$\frac{2}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{2}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$
$-\frac{1}{24}$	$\frac{1}{24}$	$\frac{2}{24}$	0	0	$-\frac{1}{24}$	0	0
0	0	$\frac{1}{24}$	$-\frac{1}{24}$	$\frac{1}{24}$	0	$-\frac{1}{24}$	$\frac{1}{24}$
$-\frac{1}{24}$	$\frac{1}{24}$	$\frac{2}{24}$	0	0	$-\frac{1}{24}$	0	0
$-\frac{1}{24}$	$\frac{1}{24}$	0	0	$\frac{2}{24}$	$-\frac{1}{24}$	0	0
$-\frac{1}{24}$	$\frac{1}{24}$	0	0	$\frac{2}{24}$	$-\frac{1}{24}$	0	0
0	0	$\frac{1}{24}$	$-\frac{1}{24}$	$\frac{1}{24}$	0	$-\frac{1}{24}$	$\frac{1}{24}$
$\frac{2}{24}$	$\frac{2}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{2}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$

$\frac{2}{24}$	$\frac{1}{24}$	$\frac{2}{24}$	$\frac{2}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$
$-\frac{1}{24}$	$\frac{2}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$	0	0	0	0
$-\frac{1}{24}$	0	$\frac{1}{24}$	$-\frac{1}{24}$	$\frac{2}{24}$	0	0	0
$-\frac{1}{24}$	$\frac{2}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$	0	0	0	0
0	$\frac{1}{24}$	0	0	$\frac{1}{24}$	$-\frac{1}{24}$	$-\frac{1}{24}$	$\frac{1}{24}$
0	$\frac{1}{24}$	0	0	$\frac{1}{24}$	$-\frac{1}{24}$	$-\frac{1}{24}$	$\frac{1}{24}$
$-\frac{1}{24}$	0	$\frac{1}{24}$	$-\frac{1}{24}$	$\frac{2}{24}$	0	0	0
$\frac{2}{24}$	$\frac{1}{24}$	$\frac{2}{24}$	$\frac{2}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$

$\frac{2}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{2}{24}$	$\frac{1}{24}$	$\frac{2}{24}$	$-\frac{1}{24}$
0	$\frac{1}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$	0	$-\frac{1}{24}$	0	$\frac{1}{24}$
$-\frac{1}{24}$	0	$\frac{2}{24}$	0	$\frac{1}{24}$	0	$-\frac{1}{24}$	0
0	$\frac{1}{24}$	$\frac{1}{24}$	$-\frac{1}{24}$	0	$-\frac{1}{24}$	0	$\frac{1}{24}$
$-\frac{1}{24}$	$\frac{2}{24}$	0	0	$\frac{1}{24}$	0	$-\frac{1}{24}$	0
$-\frac{1}{24}$	$\frac{2}{24}$	0	0	$\frac{1}{24}$	0	$-\frac{1}{24}$	0
$-\frac{1}{24}$	0	$\frac{2}{24}$	0	$\frac{1}{24}$	0	$-\frac{1}{24}$	0
$\frac{2}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{1}{24}$	$\frac{2}{24}$	$\frac{1}{24}$	$\frac{2}{24}$	$-\frac{1}{24}$

Figure 4: (a) Wigner function representation for *GHZ* state (b) Four different Wigner function representations for *W* State.

Next we consider the state $\frac{1}{\sqrt{2}}(|000\rangle + |011\rangle)$ which is the tensor product of state $|0\rangle$ with the Bell state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ (two different Wigner representations of $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ are shown in Figure 5(b) [Cormick and Paz (2006) and Franco and Penna (2006)]). By calculating Wigner function for $\frac{1}{\sqrt{2}}(|000\rangle + |011\rangle)$ (Figure 6(a)), we compare it with Wigner functions of the consistent Bell and $|0\rangle$ state. That the first qubit is not entangled with the other pair of qubits, we can determine some features of our Wigner function from their individual Wigner functions. For example, in Wigner function representation of the Bell state the probability of finding states $|\rightarrow\leftarrow\rangle$ and $|\leftarrow\rightarrow\rangle$, $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ are zero. Corresponding in Wigner function of $\frac{1}{\sqrt{2}}(|000\rangle + |011\rangle)$, the probability of finding any states whose pair of second and third qubits are in each of the above states (like $|\uparrow\downarrow\uparrow\rangle$, $|\uparrow\uparrow\downarrow\rangle$, $|\leftarrow\leftarrow\rightarrow\rangle$, ...) are also zero. Despite these similarities, one can not construct the whole Wigner function by "crossing" Wigner functions of $|0\rangle$ and the Bell state. It is important to note that the freedom of choose of the quantum net to be adopted allows for some ambiguity in writing down the Wigner function (which is also known for the two-qubit case).

(a)

	$\uparrow$	$\downarrow$
$\leftarrow$	$\frac{1}{2}$	0
$\rightarrow$	$\frac{1}{2}$	0

(b)

	$\uparrow\uparrow$	$\uparrow\downarrow$	$\downarrow\uparrow$	$\downarrow\downarrow$
$\leftarrow\leftarrow$	$\frac{1}{4}$	0	0	$\frac{1}{4}$
$\leftarrow\rightarrow$	0	0	0	0
$\rightarrow\leftarrow$	0	0	0	0
$\rightarrow\rightarrow$	$\frac{1}{4}$	0	0	$\frac{1}{4}$

	$\uparrow\uparrow$	$\uparrow\downarrow$	$\downarrow\uparrow$	$\downarrow\downarrow$
$\leftarrow\leftarrow$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\leftarrow\rightarrow$	$\frac{1}{8}$	$-\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{8}$
$\rightarrow\leftarrow$	$\frac{1}{8}$	$-\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{8}$
$\rightarrow\rightarrow$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Figure 5: (a) Wigner function representation for $|0\rangle$ state and (b) two different Wigner function representations for the Bell state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

(a)

$\frac{3}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{3}{32}$	$\frac{1}{32}$
$\frac{3}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{3}{32}$	$\frac{1}{32}$
$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$
$\frac{3}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{3}{32}$	$\frac{1}{32}$
$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$
$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$
$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$
$\frac{3}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{3}{32}$	$\frac{1}{32}$

$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	0	0	0	$\frac{1}{16}$	0
$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	0	0	0	$\frac{1}{16}$	0
$\frac{1}{16}$	$-\frac{1}{16}$	$-\frac{1}{16}$	0	0	0	$\frac{1}{16}$	0
$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	0	0	0	$\frac{1}{16}$	0
$\frac{1}{16}$	$-\frac{1}{16}$	$-\frac{1}{16}$	0	0	0	$\frac{1}{16}$	0
$\frac{1}{16}$	$-\frac{1}{16}$	$-\frac{1}{16}$	0	0	0	$\frac{1}{16}$	0
$\frac{1}{16}$	$-\frac{1}{16}$	$-\frac{1}{16}$	0	0	0	$\frac{1}{16}$	0
$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	0	0	0	$\frac{1}{16}$	0

(b)

$\frac{3}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{3}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$
$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$
$\frac{3}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{3}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$
$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$
$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$
$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$
$\frac{3}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{3}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$
$\frac{3}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{3}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$

$\frac{1}{16}$	0	$\frac{1}{16}$	0	$\frac{1}{16}$	$\frac{1}{16}$	0	0
$\frac{1}{16}$	0	$-\frac{1}{16}$	0	$-\frac{1}{16}$	$\frac{1}{16}$	0	0
$\frac{1}{16}$	0	$\frac{1}{16}$	0	$\frac{1}{16}$	$\frac{1}{16}$	0	0
$\frac{1}{16}$	0	$-\frac{1}{16}$	0	$-\frac{1}{16}$	$\frac{1}{16}$	0	0
$\frac{1}{16}$	0	$-\frac{1}{16}$	0	$-\frac{1}{16}$	$\frac{1}{16}$	0	0
$\frac{1}{16}$	0	$-\frac{1}{16}$	0	$-\frac{1}{16}$	$\frac{1}{16}$	0	0
$\frac{1}{16}$	0	$\frac{1}{16}$	0	$\frac{1}{16}$	$\frac{1}{16}$	0	0
$\frac{1}{16}$	0	$\frac{1}{16}$	0	$\frac{1}{16}$	$\frac{1}{16}$	0	0

(c)

$\frac{3}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{3}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$
$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$
$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$
$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$
$\frac{3}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{3}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$
$\frac{3}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{3}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$
$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$
$\frac{3}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{3}{32}$	$\frac{1}{32}$	$-\frac{1}{32}$	$-\frac{1}{32}$	$\frac{1}{32}$

$\frac{1}{16}$	$\frac{1}{16}$	0	$\frac{1}{16}$	$\frac{1}{16}$	0	0	0
$\frac{1}{16}$	$-\frac{1}{16}$	0	$\frac{1}{16}$	$-\frac{1}{16}$	0	0	0
$\frac{1}{16}$	$-\frac{1}{16}$	0	$\frac{1}{16}$	$-\frac{1}{16}$	0	0	0
$\frac{1}{16}$	$-\frac{1}{16}$	0	$\frac{1}{16}$	$-\frac{1}{16}$	0	0	0
$\frac{1}{16}$	$\frac{1}{16}$	0	$\frac{1}{16}$	$\frac{1}{16}$	0	0	0
$\frac{1}{16}$	$\frac{1}{16}$	0	$\frac{1}{16}$	$\frac{1}{16}$	0	0	0
$\frac{1}{16}$	$-\frac{1}{16}$	0	$\frac{1}{16}$	$-\frac{1}{16}$	0	0	0
$\frac{1}{16}$	$\frac{1}{16}$	0	$\frac{1}{16}$	$\frac{1}{16}$	0	0	0

Figure 6: (a) Wigner function representations for $\frac{1}{\sqrt{2}}(|000\rangle + |011\rangle)$, (b) $\frac{1}{\sqrt{2}}(|000\rangle + |110\rangle)$ and (c) $\frac{1}{\sqrt{2}}(|000\rangle + |101\rangle)$.

All Wigner function representations of three-qubit pure system (apart from the separable states) display some negative values, particularly the embedded Bell states. This is in contrast with the two-qubit case where the parent Bell state may have a Wigner function representation that displays all positive values.

Gibbons *et al.* (2004) suggests a tensor-product construction for phase-space point operators of $N = 4$. We have examined such tensor-product operators correspond to different quantum nets for $N = 8$, but we suspect such construction it is not valid for three-qubit case. In our Maple program we obtain tensor product $A_{(x_1,y_1)} \otimes A_{(x_2,y_2)} \otimes A_{(x_3,y_3)}$ of three phase-space point operators from the set:

$$\left\{ \begin{pmatrix} 1 & \frac{1-i}{2} \\ \frac{1+i}{2} & 0 \end{pmatrix}, \begin{pmatrix} 0 & \frac{1+i}{2} \\ \frac{1-i}{2} & 1 \end{pmatrix}, \begin{pmatrix} 1 & \frac{-1+i}{2} \\ \frac{-1-i}{2} & 0 \end{pmatrix}, \begin{pmatrix} 0 & \frac{-1-i}{2} \\ \frac{-1+i}{2} & 1 \end{pmatrix} \text{ and their complex conjugation} \right\}$$

But the results do not match with phase-space point operators of our different quantum nets.

5. CONCLUSION

By finding appropriate Galois field for three qubits and constructing related discrete phase space, we have enumerated all striations for the discrete phase space for three qubits. Applying translations on three-qubit field bases, we are able to construct the nine mutually unbiased bases given in Table 3. The results are equivalent with that of Tselniker *et al.* (2009) who use Hadamard matrices. The benefit of this method is that it is completely based on phase space.

Assigning appropriate state vector of bases to every line of our phase space, we can calculate Wigner functions for three-qubit system. Here, we have shown the ambiguity of the Wigner function representations by the quantum states arrives from the assignment of such quantum net. We also have suggested that the tensor-product quantum net property which is posed in Gibbons *et al.* (2004) may not be valid for the N-qubit case.

ACKNOWLEDGEMENTS

The present research was supported by the National Fundamental Research Grant Scheme (FRGS) of Malaysia, No. 01-10-07286FR. Also we would like to express my gratitude to Dr. Halimah Mohamed Kamari for her valuable support and encouragement.

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