



UNIVERSITI PUTRA MALAYSIA

**SOME CONDITIONS OF THE SECOND-ORDER SPATIAL
UNILATERAL AUTOREGRESSIVE MOVING AVERAGE MODEL
AND MODEL PARAMETERS ESTIMATION FOR DATA WITH
MISSING OBSERVATIONS**

SAIDATULNISA BINTI ABDULLAH

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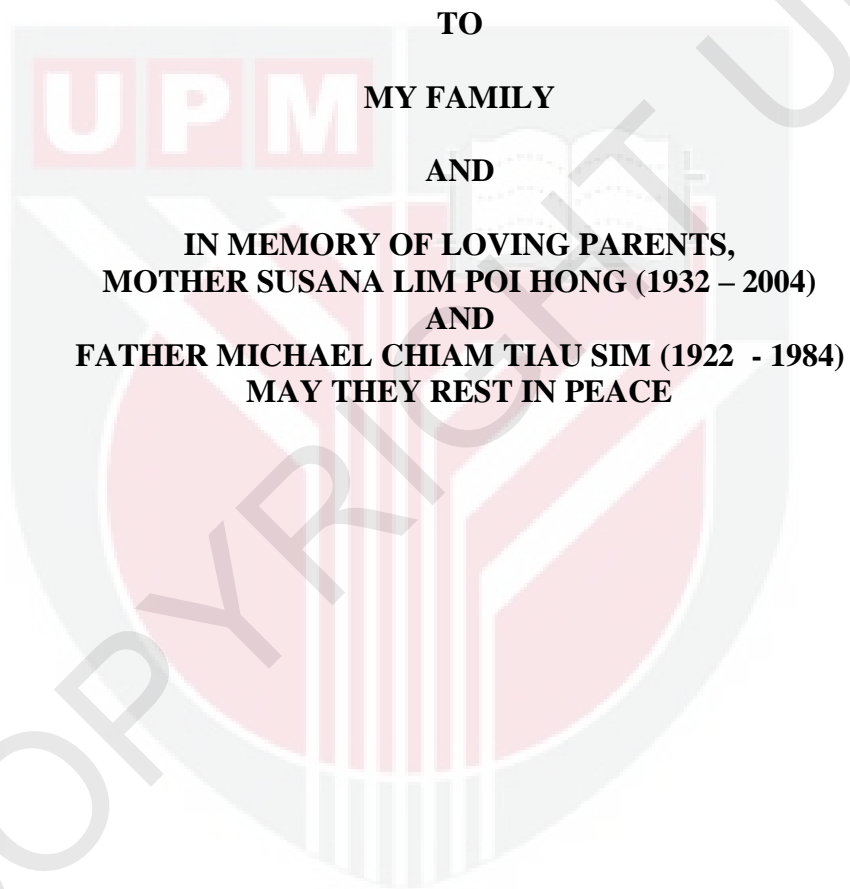
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MISSING OBSERVATIONS**

By

SAIDATULNISA BINTI ABDULLAH

**Thesis Submitted to the School of Graduate Studies,
Universiti Putra Malaysia, in Fulfilment of the Requirements for the Degree of
Doctor of Philosophy**

May 2013



**TO
MY FAMILY
AND
IN MEMORY OF LOVING PARENTS,
MOTHER SUSANA LIM POI HONG (1932 – 2004)
AND
FATHER MICHAEL CHIAM TIAU SIM (1922 - 1984)
MAY THEY REST IN PEACE**

Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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May 2013

Chairman: Mahendran Shitan, PhD

Faculty: Science

Spatial statistics has been given considerable attention in the past four decades. The primary purpose of such studies is for the benefit of modelling, forecasting and interpolation. The models studied however are first-order spatial models. In this thesis, we focus on the second-order spatial unilateral Autoregressive Moving Average model, the ARMA (2,2;2,2) model.

The limitations of statistical models are based on the assumption that the model is stationary. In the early part of this thesis, we provide stationary conditions and their proofs for the spatial model chosen. The simulation results provide illustrations of generated stationary data when the stationary conditions are adhered to. We also provide illustrations when stationary conditions are violated.

For the estimation of model parameters, the correlation structure for the model need to be developed. For the derivation of the explicit correlation structure this thesis will provide some initial terms by focussing on the sector of the fourth quadrant.

Missing observations in a set of data raise problems for the analysis of spatial data sets. Missing data result in reduced sample size, lower statistical power and an increase in bias in the estimates. In this thesis, we introduce a method to handle this problem. This approach first uses available data to estimate the model parameters. A linear combination of backcast and forecast with information of the estimated parameters in the first step are then used to interpolate missing values. The Yule-Walker technique is utilized for the final model parameter estimation. Comparison of the absolute bias and the root mean square errors of the estimated parameters in simulation studies for complete data and incomplete data show significantly that the suggested technique can be suitably used for missing data with larger grid sizes and a lower percentage of data missing. The bias and root mean square error, however, tend to increase with smaller grid size and a higher percentage of data missing.

Finally, some suggestions are given for further research for the properties of the ARMA (2,2;2,2) model especially in the area of the derivation of initial terms to enable model fitting using real data. For parameter estimation in the problem of missing observations in spatial data, techniques using other estimators can be developed and its performance compared with the YW estimator.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**BEBERAPA SYARAT MODEL RERUANG SESISI AUTOREGRESIF
PURATA BERGERAK PERINGKAT KEDUA DAN PENGANGGARAN
PARAMETER MODEL BAGI DATA YANG TIDAK LENGKAP**

Oleh

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Statistik ruang telah mendapat sebahagian besar perhatian dalam empat dekad yang kebelakangan ini. Tujuan utama penyelidikan sebegini adalah untuk memodelkan, meramalkan dan menjalankan interpolasi atas data ruang. Tumpuan kajian adalah terlebih kepada model peringkat pertama sahaja. Dalam tesis ini, kami memberi tumpuan kepada model ruang sesisi autoregresif purata bergerak peringkat kedua, iaitu model ARMA(2,2:2,2).

Pembinaan model daripada data statistik dikekangi berdasarkan anggapan bahawa model adalah pegun. Di bahagian awal tesis, kami menyediakan beberapa syarat kepegunan dan bukti untuk model yang terpilih. Hasil simulasi memberi gambaran bagi data yang dijanakan apabila syarat-syarat kepegunan dipatuhi. Sebagai perbandingan, ilustrasi bagi data yang melanggar syarat-syarat kepegunan juga disediakan.

Bagi penganggaran parameter model, struktur korelasi model perlu dibangunkan. Untuk penerbitan struktur korelasi, tesis ini menyediakan beberapa terma permulaan dengan memberi fokus kepada data dalam sektor sukuan keempat.

Cerapan ruwang yang hilang menimbulkan masalah kepada proses analisis data ruwang. Data yang hilang mengurangkan saiz sampel, menurunkan kuasa statistik dan menambahkan kepincangan parameter yang dianggar. Dalam tesis ini, kami mencadangkan satu kaedah untuk menangani masalah ini. Dalam langkah yang pertama, data yang sedia ada digunakan untuk membuat anggaran awal parameter model. Selepas itu, satu kombinasi anjuran awal dan anjuran akhir, bersama maklumat yang diperolehi daripada anggaran awal parameter model digunakan untuk menentukan data yang hilang. Kemudian, teknik Yule-Walker dipakai untuk membuat penganggaran parameter model yang akhir. Perbandingan pincang mutlak dengan min punca kuasa dua ralat bagi parameter yang dianggar dibuat untuk data yang lengkap dan data yang tidak lengkap. Keputusan simulasi menunjukkan bahawa teknik yang dicadangkan sesuai digunakan untuk saiz grid sampel yang besar serta peratusan data hilang yang kecil. Walau bagaimanapun, nilai kepincangan mutlak dan min punca kuasa dua ralat bagi parameter yang dianggar mempunyai kecenderungan untuk meningkat dengan saiz grid sampel yang kecil dan peratusan data hilang yang tinggi.

Akhir sekali, beberapa cadangan disyorkan untuk kajian lanjutan untuk sifat-sifat lain model ARMA (2,2:2,2) khususnya perkara yang berkaitan dengan terbitan ungkapan permulaan untuk membolehkan pemodelan dibuat dengan menggunakan data benar. Bagi menangani masalah data ruwang yang hilang, pendekatan menggunakan penganggar parameter yang lain boleh dikaji untuk dibandingkan dengan penganggaran parameter YW yang dicadangkan.

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Lastly, my thanks go to my family, especially my husband. With their prayers and Allah's grace, I have been able to complete this work.

I certify that a Thesis Examination Committee has met on the 15 May 2013 to conduct the final examination of Saidatulnisa binti Abdullah on her thesis entitled “Some Conditions of the Second-Order Spatial Unilateral Autoregressive Moving Average Model and Model Parameters Estimation for Data with Missing Observations” in accordance with the Universities and University Colleges Act 197 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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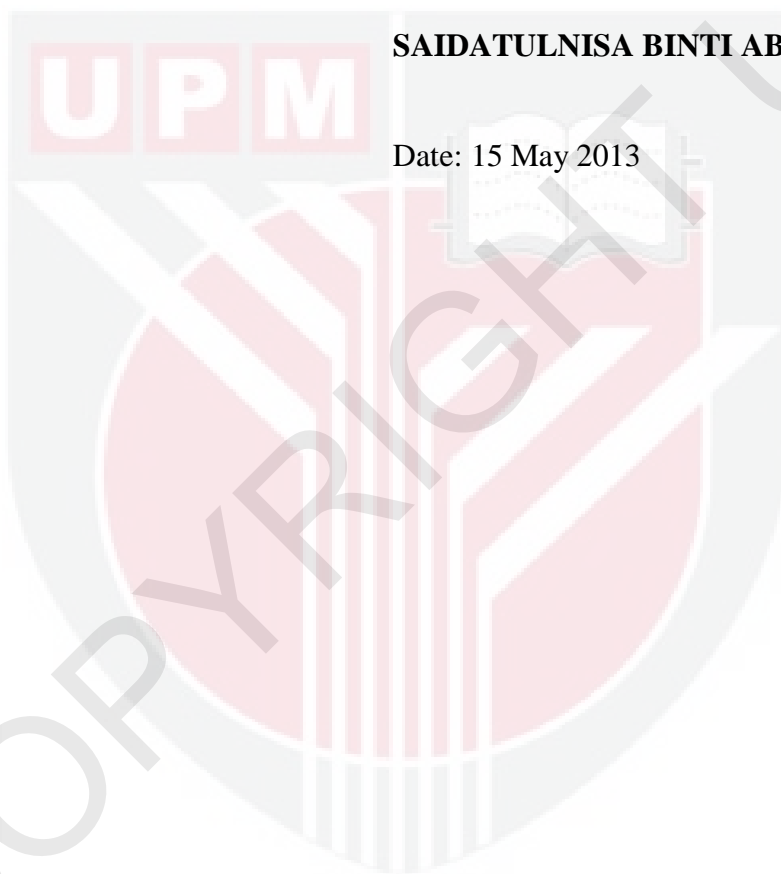
Date:

DECLARATION

I declare that the thesis is my original work except for quotations and clarifications which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at Universiti Putra Malaysia or other institutions.

SAIDATULNISA BINTI ABDULLAH

Date: 15 May 2013



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