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Consensus of Criteria Ranking in Women's Decision-making

[1]Puzziawati Ab. Ghani & [2]Abdul Aziz Jemain
[1]*Center for Statistical Studies,*
Faculty of Information Technology & Quantitative Sciences
Universiti Teknologi MARA
40450 Shah Alam, Selangor Darul Ehsan, Malaysia
[2]*School of Mathematical Sciences, Faculty of Science and Technology*
Universiti Kebangsaan Malaysia
43600 UKM Bangi, Selangor Darul Ehsan, Malaysia

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ABSTRAK

Kajian ini menyelidiki sembilan kriteria yang sering dipertimbangkan oleh wanita bekerja dalam mereka membuat keputusan. Empat kaedah digunakan untuk mengira pemberat bagi setiap kriteria tersebut. Setiap kaedah akan memberikan pemberat yang berbeza. Oleh itu satu kaedah memperoleh kesepakatan pemberat diperlukan. Kertas ini mencadangkan kaedah purata tertib berpemberat. Untuk tujuan itu pengkuantiti kabur dimanfaatkan. Sebagai ilustrasi penggunaan kaedah-kaedah yang dinyatakan, satu set data yang diperoleh daripada 340 kakitangan sokongan dan akademik wanita Universiti Teknologi MARA, Shah Alam digunakan. Data ini diperoleh melalui satu set soal selidik untuk mendapatkan skor kepentingan setiap kriteria. Sembilan kriteria yang digunakan turut disenaraikan.

ABSTRACT

This paper investigates the nine criteria often weighed by working women in their decision-making. Four methods of deriving weight for each criterion are applied. Each method used will produce different weight values. In this paper we are going to suggest a way to reconcile the differences in order to get a consensus. For that purpose, ordered weighted average with fuzzy quantifier will be employed. As an illustration, a set of data collected from 340 women, academic and support staff of Universiti Teknologi MARA in Shah Alam, will be utilised. Ratings of importance of each criterion by these women were obtained through a set of questionnaires. The nine criteria used are also presented in this paper.

Keywords: Decision making, criteria, weight, fuzzy majority, ordered weighted average

INTRODUCTION

In most multiple criteria decision-making problems, it is crucial to evaluate an individual's priority/preference of criteria by deriving weights of criteria, construct the overall ranking of criteria and identify the best/most important criteria. There are many ways of deriving weights in order to rank a set of criteria. Of course, these different methods result in different weight values. This in turn engenders different preference ordering/ranking for the same set of criteria. With many methods available, researchers definitely have many choices of deriving criteria weights and one method may be preferred over others for reasons like easy computations, simplicity, etc. Therefore, different people may have different preferences on the method used based on their own reason/s. As such, more often than not we encounter situations whereby conflicting results arise due to different methods applied to the same set of data.

Hence the issue to be addressed in this paper is: with those different methods of deriving weights, that produce different preferences, how can we reconcile those differences to attain a single/common preference? In other words, how do we go about in reaching a consensus in order to settle these differences to form a common ground?

Since individual preference on the method used to derive criterion weight varies, and we do not have any control over one's preference, therefore, this paper aims at arriving at a way to reconcile the differences and come up with a common preference ordering of criteria or a consensus on criterion priority. To achieve this, the fuzzy majority concept represented by a fuzzy linguistic quantifier by means of the ordered weighted averaging (OWA) operator is utilised to aggregate the preference information and to rank the criteria and select the most prioritised criterion in the selection process. The first section of this paper illustrates four different methods of deriving criterion weight. The subsequent section presents: the method of transforming preference information into fuzzy preference relations and shows how to obtain the collective fuzzy preference relation under fuzzy majority. The next section deals with the selection process in order to get the consensus. The selection process is based on quantifier guided dominance degree and quantifier guided non-dominance degree.

METHODS OF DERIVING WEIGHTS

Deriving weights is aimed at eliciting criteria importance in multiple criteria decision-making. Various weighting techniques have been proposed in order to rank a set of criteria. Different methods of deriving weights result in different weight values that give rise to different preference ordering for the same set of criteria.

Four methods of deriving criteria weights based on (I) the correlation matrix, (II) the coefficient of variation, (III) the concept of entropy, and (IV) the communality values from factor analysis, will be employed in this paper to obtain criteria weights. The details of criteria weights formulation based on the methods used are presented in the appendix. The first two methods have been discussed in Ray (1989). The third method is illustrated in Zeleny (1982) and in Chen and He (1997). Abdul Aziz (2002) suggested the method of deriving criteria weights based on communality values.

Method 1: Weights Based on the Correlation Matrix

Weights based on a correlation matrix are assumed to be proportional to the respective row (column) sums of the absolute values of correlation coefficients.

Assuming there are k criteria, weight for jth criterion based on correlation matrix is formulated as follows:

$$w_j = \frac{r_j}{\sum_{j=1}^{k} r_j} \text{ and } r_j = \sum_{l} \left| r_{jl} \right| \text{ (j = 1,2,...,k)} \tag{1}$$

where r_{jl} is the correlation coefficient between jth and lth criteria that measures the degree of the relationship between criterion j and criterion l.

Method 2: Weights Based on Coefficient of Variation

Weights based on coefficient of variation depend on the assumption that the more important criteria have smaller relative variations compared to the less important ones (Ray 1989). Coefficient of variation for jth criterion, cv_j, is the ratio between its standard deviation and its mean. Criteria weights based on coefficient of variation is given as

$$w_j = \frac{r_j / cv_j}{\sum_{j=1}^{k} r_j / cv_j} \tag{2}$$

Method 3: Weights Based on Entropy

Zeleny (1982) and Chen and He (1997) illustrated the application of weights derived based on the concept of entropy. Entropy measures the size of information on the uncertainty in the decision made on a particular criterion, in which the larger the size, the lesser is the certainty. This measure of entropy for the jth criterion, e_j, is formulated by Shannon and Weaver (1949). (See the appendix for details). Weight based on entropy for jth criterion is as follows:

$$w_j = \frac{\left(1-e_j\right)}{\sum_{j=1}^{k}\left(1-e_j\right)} \tag{3}$$

Method 4: Weights Based on Communalities

Communalities, denoted by h, are estimates of the shared or common variance among the variables/criteria in factor analysis. The ith communality is the sum of squares of the loadings of the ith variable/criterion on the common factors (for details see Johnson and Wichern, 1998). Weights based on communalities for jth criterion, where h_j is the communality for jth criterion, is given by

$$w_j = \frac{h_j^2}{\sum_{j=1}^{k} h_j^2} \tag{4}$$

PREFERENCE TRANSFORMATION

In a multi-criteria decision-making model, individual preferences can be provided in the form of ranking/ordering, utility vectors and so on. Each form of preferences must be transformed into fuzzy preference relations. The concept of fuzzy preference relations has been widely used in various decision-making environments to represent an expert's opinion regarding a set of alternatives or criteria. It is a useful tool in the modeling of decision processes especially in cases where aggregation of experts' preferences into group preferences is needed to reach a consensus. In this paper, we let $E = \{e_1, e_2, \ldots, e_m\}$ denote the set of m number of experts/decision makers.

Transformation of Ordered Vectors to Fuzzy Preference Relations

In this case, criteria are ordered from the best to the worst. An expert/decision maker $e_m (e_m \in E)$ provides his/her preferences on a set of criteria $X = \{x_1, x_2, \ldots, x_k\}$ ($k \geq 2$) as an individual preference ordering or as an ordered vector $O^m = (o^m(1), \ldots, o_m(k))$. The preference orderings can be transformed into fuzzy preference relations between criteria x_i and x_j as follows:

$$p_{ij}^m = \frac{1}{2}\left(1 + \frac{o^m(j)}{k-1} - \frac{o^m(i)}{k-1}\right),\ 1 \leq i \neq j \leq k \tag{5}$$

where $o^m(j)$ represents the ranking position of criterion x_j in O^m, $j = 1, \ldots, k$.

Transformation of Utility Vectors to Fuzzy Preference Relations

In relating between utility values and fuzzy preference relations, it is assumed that an expert/decision maker $e_m (e_m \in E)$ provides his/her preferences on $X = \{x_1, x_2, \ldots, x_k\}$ ($k \geq 2$), by means of a set of utility values $U^m = \{u_i^m, i = 1, \ldots, k\}$. The utility values has to be first transformed into fuzzy preference relations between criteria x_i and x_j according to the following:

$$p_{ij}^m = \frac{\left(u_i^m\right)^2}{\left(u_i^m\right)^2 + \left(u_j^m\right)^2},\ 1 \leq i \neq j \leq k, \tag{6}$$

u_i^m is the utility evaluation of expert m. The idea was suggested in Tanino (1990; 1984) and Chiclana *et al.* (1998).

In this paper preferences are in the form of utility vectors as the weights obtained are considered as utility values. Expert in the context of this paper refers to method of deriving criteria weights. Since there are four methods employed to derive weights, this implies that there are four experts/decision makers .

FUZZY MAJORITY AND THE ORDERED WEIGHTED AVERAGING (OWA) OPERATOR

Preference Aggregation

Once all individual expert's preference relations are obtained and organised in the form of m preference matrices, $P^1, P^2,...,P^m$, the next step is to get a collective fuzzy preference relation matrix. A collective fuzzy preference relation matrix, $\tilde{P}$, is obtained by aggregating all experts' individuals fuzzy preference relations using the OWA operators whose weights are chosen according to the concept of fuzzy majority. Fuzzy majority or sometimes termed as soft majority, which is represented by a fuzzy quantifier is applied in the aggregation operations by means of an OWA operator. The weights of the OWA aggregation operator are computed using the fuzzy quantifier. The membership function of a non-decreasing proportional quantifier is given by

$$Q(r)=\begin{cases} 0 & \text{if } r<a \\ \dfrac{r-a}{b-a} & \text{if } a \le r \le b \\ 1 & \text{if } r>b \end{cases} \quad (7)$$

with $a,b,r \in [0,1]$. Different semantics correspond to different pairs of coefficients of a and b. For example, "at least half" corresponds to $a = 0$, $b = 0.5$ or $(0,0.5)$, and "most" corresponds to $a = 0.3$, $b = 0.8$ or $(0.3,0.8)$, etc.

In order to aggregate ijth fuzzy preference relations of m individuals, p^1_{ij}, $p^2_{ij},...,p^m_{ij}$ ($p^1_{ij}, p^2_{ij},...,p^m_{ij}$ are ijth entries of preference matrices $P^1, P^2,...,P^m$ respectively), to form a collective fuzzy preference relation $\tilde{p}_{ij}$, the OWA operator is used. An OWA operator of dimension m is a function of ϕ and is defined as follows:

$$\phi : [0,1]^m \rightarrow [0,1] \quad (8)$$

In aggregating $p^1_{ij}, p^2_{ij},...,p^m_{ij}$, ϕ is associated with a weight vector $V = [v_1, v_2,..., v_m]$ where $v_l \in [0,1]$, $l = 1,2,...,m$ and $\sum_{l=1}^{m} v_l = 1$. The collective fuzzy preference relation, $\tilde{p}_{ij}$, is obtained by means of ϕ function as follows:

$$\phi(p^1_{ij}, p^2_{ij},...,p^m_{ij}) = \tilde{p}_{ij} = V.\ B^T = \sum_{l=1}^{m} v_l b_l \ \text{ for } 1 \le i \ne j \le k \tag{9}$$

In this case $B = [b_1, b_2,...,b_m]$ and b_l is the lth largest value among the collection of m ijth elements, $p^1_{ij}, p^2_{ij},...,p^m_{ij}$, $l = 1,2,...,m$. $P^l = [p^l_{ij}]_{k \times k}$ is the matrix of fuzzy preference relations between the criteria from expert e_l, $l = 1,2,...,m$ and p^l_{ij} is the ijth element of lth fuzzy preference matrix.

Weights for the aggregation operations (made by means of the OWA operator) are calculated using fuzzy quantifiers as described earlier. The weight vector V, can be obtained by a proportional quantifier Q, as follows:

$$v_l = Q\left(\frac{1}{m}\right) - Q\left(\frac{l-1}{m}\right) \qquad l = 1,...,m \tag{10}$$

where Q is a fuzzy quantifier with a pair of coefficients (a, b) as defined before. The OWA operator, was first introduced by Yager (1988; 1993).

Preference Exploitation

After having obtained the collective preference relations, $\tilde{p}_{ij}$, for all $i,j = 1,...,k(i \ne j)$ the next stage is to order the criteria and select the most important criterion. In order to select the most prioritised criteria acceptable to the group of individuals/experts, two quantifier guided choice degrees of criteria based on the concept of fuzzy majority: a dominance degree and a non-dominance degree, will be used. Both are based on the use of the OWA operator.

The quantifier guided dominance degree, *QGDD*, for criterion i, is used to quantify the dominance that one criterion has over all the others in a fuzzy majority sense and is given by

$$QGDD_i = \phi_Q(\tilde{p}_{ij}, j = 1,...,k,\ j \ne i) \tag{11}$$

While the quantifier guided non-dominance degree for criterion i that represents the degree to which criterion x_i is strictly dominated by criterion x_j is as follows:

$$QGNDD_i = \phi_Q(1 - p^s_{ji},\ j = 1,...,k,\ j \ne i) \ \text{ where, } p^s_{ji} = \max\{\tilde{p}_{ji} - \tilde{p}_{ij}, 0\} \tag{12}$$

THE DATA

Various criteria are often weighed by working women in their decision-making. Intrinsically, these criteria are weighed according to a certain preference/importance. Criteria normally weighed are associated with work and family commitments (Puzziawati *et al.* 2002; 2003). The nine formulated criteria are listed in Table 1. Since various criteria are usually taken into consideration during decision-makings, it is important to ascertain the priorities of these working women. The inputs can be very useful to the organisation where women work.

The data used for analysis are ratings of importance of the nine criteria (refer to Table 1) in every day decision-making of 340 randomly selected women employees (academics and non-academics) of Universiti Teknologi MARA in Shah Alam.

TABLE 1
The nine formulated criteria

Criterion	Criterion description
C1: career	A criterion associated with career development
C2: family	A criterion associated with familial aspect
C3: femininity	A criterion associated with women's feminine aspect such as self-grooming, health and beauty care, etc.
C4: income	A criterion associated with income
C5: social	A criterion associated with social commitments
C6: reproductive	A criterion associated with reproductive aspect (family planning)
C7: extended family	A criterion associated with extended family commitments
C8: benefits	A criterion associated with career benefits
C9: household	A criterion associated with household management

Source: Puzziawati *et al.* (2002)

RESULTS AND DISCUSSION

Criteria Weights

Criteria weights based on the four methods are calculated according to: all respondents (N=340), the academic group (n=190) and the non-academic group (n=150). Criteria weights in this case are treated as utility values. The weights are presented in Table 2.

Fuzzy Preference Relations and the Collective Fuzzy Preference Relation

The fuzzy preference relation matrix for each "expert" was calculated but not shown here. Since there are four methods employed in the analysis, and each method represents an expert, therefore there are four experts altogether. Using the fuzzy majority criterion with fuzzy quantifier "most", with the pair (0.3,0.8), and the corresponding OWA operator with the weighting vector V=[0, 0.4, 0.5, 0.1], the collective fuzzy preference relation for all respondents, academics and non-academics, are presented in Tables 3, 4, and 5 respectively.

TABLE 2
Criteria weights for all respondents, academics and non-academics

Crt	All respondents Weights based on method:				Academics Weights based on method:				Non-academics Weights based on method:			
	1	2	3	4	1	2	3	4	1	2	3	4
C1	0.091	0.104	0.059	0.170	0.082	0.097	0.050	0.106	0.103	0.109	0.077	0.138
C2	0.101	0.158	0.051	0.150	0.103	0.171	0.042	0.128	0.108	0.152	0.066	0.142
C3	0.133	0.138	0.080	0.101	0.142	0.142	0.087	0.107	0.122	0.132	0.067	0.112
C4	0.101	0.068	0.204	0.126	0.098	0.064	0.199	0.116	0.101	0.069	0.215	0.125
C5	0.110	0.080	0.173	0.075	0.110	0.084	0.142	0.048	0.108	0.072	0.227	0.136
C6	0.099	0.068	0.209	0.071	0.096	0.061	0.250	0.168	0.101	0.078	0.135	0.075
C7	0.118	0.102	0.103	0.136	0.122	0.099	0.110	0.107	0.115	0.105	0.092	0.133
C8	0.127	0.149	0.057	0.082	0.126	0.144	0.059	0.078	0.128	0.155	0.053	0.089
C9	0.119	0.133	0.065	0.089	0.122	0.136	0.061	0.142	0.115	0.127	0.069	0.050

TABLE 3
Collective fuzzy preference relation matrix (for all respondents)

C1	C2	C3	C4	C5	C6	C7	C8	C9
-	0.477	0.354	0.490	0.465	0.517	0.415	0.409	0.408
0.485	-	0.439	0.492	0.555	0.588	0.452	0.471	0.481
0.604	0.501	-	0.464	0.573	0.602	0.447	0.550	0.555
0.428	0.420	0.444	-	0.503	0.503	0.427	0.492	0.496
0.440	0.339	0.365	0.453	-	0.525	0.400	0.420	0.418
0.382	0.303	0.328	0.469	0.455	-	0.341	0.379	0.379
0.535	0.483	0.506	0.521	0.541	0.590	-	0.557	0.564
0.525	0.482	0.421	0.401	0.509	0.540	0.371	-	0.485
0.544	0.466	0.436	0.409	0.516	0.548	0.381	0.496	-

TABLE 4
Collective fuzzy preference relation matrix (for academics)

C1	C2	C3	C4	C5	C6	C7	C8	C9
-	0.381	0.277	0.394	0.418	0.314	0.368	0.352	0.343
0.583	-	0.427	0.486	0.564	0.400	0.457	0.468	0.420
0.691	0.508	-	0.516	0.636	0.429	0.519	0.589	0.527
0.538	0.428	0.395	-	0.523	0.430	0.442	0.480	0.375
0.489	0.323	0.297	0.406	-	0.356	0.406	0.336	0.328
0.604	0.497	0.458	0.537	0.553	-	0.504	0.528	0.442
0.581	0.463	0.445	0.496	0.546	0.407	-	0.534	0.416
0.603	0.474	0.382	0.413	0.588	0.347	0.404	-	0.471
0.646	0.547	0.437	0.551	0.581	0.460	0.529	0.496	-

TABLE 5
Collective fuzzy preference relation matrix (for non-academics)

C1	C2	C3	C4	C5	C6	C7	C8	C9
-	0.466	0.475	0.485	0.450	0.544	0.470	0.501	0.487
0.510	-	0.519	0.500	0.466	0.598	0.481	0.533	0.524
0.491	0.456	-	0.468	0.434	0.592	0.454	0.524	0.520
0.451	0.423	0.447	0.468		0.580	0.436	0.474	0.586
0.487	0.458	0.481	0.530		0.608	0.471	0.505	0.611
0.388	0.318	0.345	0.368	0.341		0.376	0.378	0.523
0.512	0.479	0.509	0.506	0.471	0.571	-	0.531	0.547
0.433	0.425	0.442	0.421	0.389	0.552	0.401		0.553
0.457	0.424	0.444	0.304	0.280	0.399	0.392	0.403	-

RANKING OF CRITERIA

Fuzzy quantifier "most" with the pair (0.3,0.8) and the corresponding OWA operator with the weighting vector $W = [0, 0, 0.15, 0.25, 0.25, 0.25, 0.1, 0]$ is applied in the exploitation stage. The quantifier guided dominance degree and the quantifier guided non-dominance degree were calculated for each criterion for all respondents, the academics and the non-academics as shown in Table 6. These values represent the dominance that one criterion has over the "most" criteria according to "most" of the "experts" (methods) and the non-dominance degree in which the criteria is not dominated by "most" of the criteria according to "most" of the methods used.

TABLE 6
Results of the quantifier guided dominance and non-dominance degrees for the criteria (all respondents, academics and non-academics)

Criterion	All respondents		Academics		Non-academics	
	QGDD	*QGNDD*	*QGDD*	*QGNDD*	*QGDD*	*QGNDD*
Career	0.435	0.926	0.355	0.751	0.477	0.983
Family	0.478	0.994	0.453	0.967	0.509	1.000
Femininity	0.534	1.000	0.530	1.000	0.477	0.983
Income	0.458	0.972	0.437	0.933	0.456	0.964
Social	0.412	0.890	0.348	0.781	0.495	0.999
Reproductive	0.365	0.808	0.509	1.000	0.363	0.784
Extended family	0.533	1.000	0.473	0.972	0.509	1.000
Benefits	0.463	0.962	0.431	0.926	0.426	0.916
Household	0.468	0.963	0.522	1.000	0.393	0.850

Criteria order of importance: most important to least important

All respondents: femininity; ext. family; family; household; benefits; income; career; social; reproductive

Academics: femininity; household; reproductive; ext. family; family; income; benefits; career; social

Non-academics: ext. family; family; social; femininity & career; income; benefits; household; reproductive

Results in Table 2 (based on all respondents, the academics and the non-academics respectively) show that weights obtained by different approaches result in different weight values for the same criteria. This gives rise to differences in the ranking of the same set of criteria as discussed earlier. For example, based on the analysis for all respondents, their top priority is as follows: feminine aspects according to method 1, familial aspects according to method 2, reproductive aspects according to method 3 and career development according to method 4.

Based on the results of quantifier guided dominance and non-dominance degrees (Table 6), the consensus is: according to all respondents, the most prioritised criterion is related to feminine aspects (i.e. indulging in women's own self-activities like healthcare, hobbies, self-grooming, etc.), followed by commitments towards the extended family. The third most important criterion is related to familial aspect (i.e. time spent with the husband and children). When considering the academics as a group, they place the feminine aspects as their top priority in their decision-making followed by household management. Their third most important criterion is the reproductive aspects (i.e. family planning), while the non-academics, prioritised highest, the criterion that relates to commitments towards their extended family followed by familial aspect. Their third most important criterion is related to social commitments.

CONCLUSION

In this paper we have presented four different methods of deriving weights. Fuzzy consensus approach based on the concept of fuzzy majority by means of the OWA operator whose weights are calculated by a fuzzy quantifier was adopted to reconcile differences in preference orderings as a result of different methods employed to obtain criteria weights. The application of guided choice degrees has led us to identify the few most prioritised criteria weighed by working women in their decision-making.

Hence, based on the consensus reached, we conclude that for this case study different groups of working women put different priority in their decision-making. While the academics placed top priority to their feminine role, the non-academics put commitments towards extended family as top priority in their decision-making.

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APPENDIX

Detailed formula of criterion weight

w_j : weight of *j*th criterion and $\sum_{j=1}^{k} w_j = 1$ where $w_j \geq 0$.

x_{ij} : rating score of observation i based on *j*th criterion, $i = 1,...,n$ and $j = 1,...,k$.

Method 1: Weights Based on Correlation Matrix

r_{jl}: correlation coefficient between *j*th and *l*th criteria.

$r_{jl} = \dfrac{cov(x_j, x_l)}{\left(\sqrt{s_{jj}}, \sqrt{s_{ll}}\right)}$ where $\mathrm{cov}(x_j, x_l) = s_{jl} = \dfrac{1}{n-1}\sum_{i=1}^{n}\left(x_{ij} - \bar{x}_j\right)\left(x_{il} - \bar{x}_l\right)$ and

$s_j^2 = s_{jj} = \dfrac{1}{n-1}\sum_{i=1}^{n}\left(x_{ij} - \bar{x}_j\right)^2$, the variance, and $\bar{x}_j = \dfrac{1}{n}\sum_{i=1}^{n} x_{ij}$ is the mean.

Method 2: Weights Based on Coefficient of Variation

cv_j : coefficient of variation for *j*th criterion. $cv_j = \dfrac{\sqrt{s_{jj}}}{\bar{x}_j}$.

Method 3: Weights Based on Entropy

e_j : measure of entropy for *j*th criterion. $e_j = -\dfrac{1}{1n(n)}\sum_{i=1}^{n}\dfrac{x_{ij}}{x_j} 1n\left(\dfrac{x_{ij}}{x_j}\right)$ where

$$x_j = \sum_{i=1}^{n} x_{ij}$$