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Modelling of Saltwater Intrusion into a Discharging Well in a Non-Homogeneous Unconfined Aquifer

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ABSTRAK

Kaedah unsur terhingga berdasarkan teknik Galerkin digunakan untuk membentuk penyelesaian bagi pergerakan air masin di dalam satu akuifer ke arah telaga pengepaman. Jenis akuifer adalah tak terkurung, bukan homogen dan isotropik. Persamaan bagi aliran air bumi dan olakan-serakan telah ditukar bentuk kepada dua persamaan kebezaan separa bukan linear untuk menghasilkan nilai-nilai turus piezometrik dan kepekatan air masin pada titik-titik dan masa yang berlainan. Persamaan-persamaan itu telah diselesaikan dengan model Argus-ONE™ SUTRA. Prestasi model berangka telah dibandingkan dengan data yang diperoleh daripada satu model uji kaji di makmal. Perbandingan yang baik telah diperoleh di antara dua model tersebut, dengan perbezaan yang ditunjukkan bagi kepekatan dan turus piezometrik adalah masing-masing 10% dan 11%.

ABSTRACT

Finite element method based on the Galerkin technique was used to formulate the solution for simulating a two-dimensional transient movement of saltwater in a stratified aquifer under pumping conditions. The aquifer system was unconfined, non-homogeneous and isotropic. The groundwater flow and convection-dispersion equations were transformed into two non-linear coupled partial differential equations to yield the values of the corresponding piezometric head and saltwater concentration at various points and times. These two equations were solved by Argus- ONE™ SUTRA model that employs the finite element method. The performance of the numerical model is compared with the data observed from a laboratory experimental model. Good agreement has been achieved between the numerical and experimental models for the concentration and hydraulic head as comparison showed the maximum differences of only 10% and 11% respectively.

Keywords: Non-homogenous aquifer, saltwater intrusion, mathematical modelling, experimental model, validation

INTRODUCTION

Water resource engineers have always been interested in optimising the use of groundwater reservoirs, not only through making the maximum use of the quantity of water available but also by managing the quality of water in the system. Efforts that were done or currently underway include predicting and controlling the movement of a salt water - fresh water interface and mass transport in the flowing groundwater, and predicting quality changes in an aquifer due to changing irrigation patterns and irrigation efficiency. Human activities, such as groundwater abstraction, land reclamation and land drainage have resulted in a drawdown of the groundwater table and piezometric level, and in stratified groundwater reservoirs, a displacement of the saltwater into freshwater zone, which directly influences the quality of water pumped from the well. This leads to the necessity of developing techniques for groundwater utilisation from such reservoirs to meet the desired water quality constraints.

Basic studies have been conducted to explain the pattern of movement and mixing between freshwater and saltwater, and the factors that influence these processes. Many researchers have worked in this field and several of them presented numerical solutions for the flow and convective-dispersion problems. Huyakorn (1987) developed a three-dimensional finite element model for the simulation of saltwater intrusion in single and multiple coastal aquifer systems with either a confined or phreatic top aquifer. Before that, Pickens and Lennox (1976) used the finite element method based on Galerkin technique to formulate the simulation of the two-dimensional transient movement of conservative or non-conservative wastes in a steady state saturated groundwater flow system. Batu (1984) developed a finite element dual mesh to calculate the horizontal and vertical Darcy velocity components in a highly non-homogeneous and anisotropic aquifer of constant porosity under steady flow conditions. The Galerkin method of approximation in conjunction with the finite element was also used by Pinder (1973) as a method of analysis to simulate the movement of groundwater contaminants. For the development of analytical solutions, Gupta and Yapa (1982) used an approach considering both analytical and numerical models for assessing the saltwater encroachment phenomenon in an aquifer in Thailand. Guvanasen and Volker (1982) presented two analytical solutions for a problem of solute transport in transient flow in an unconfined aquifer. Kipp (1973) developed a realistic theoretical solution to the problem of unsteady flow to a single, partially penetrating well of finite radius in an unconfined aquifer.

Rahem (1991) had conducted a very detailed study on selective withdrawal from the density-stratified unconfined groundwater reservoirs. He had employed a finite difference method in solving the governing flow and solute transport equations, as well as preparing sandbox model to simulate the physical problem in order to achieve the verification purposes. However, he used homogeneous porous media in contrast to this study, in which a non-homogeneous and isotropic soil had been used in the experimental set-up.

More recently, Gordon *et al.* (2000) developed an optimisation model for a confined aquifer in which the groundwater flow and Darcy's law are solved by finite element method, while the salinity transport equation is solved by the streamline-upwind Petrov-Galerkin (SUPG) method. Feehley *et al.* (2000) demonstrated that a dual-domain mass transfer approach is more practical for modelling of solute transport in highly heterogeneous aquifers compared to the classical Fickian advection-dispersion model.

NUMERICAL MODELLING

A general form of the equation describing the two-dimensional flow of an incompressible fluid in a non-homogeneous, isotropic aquifer may be derived by combining Darcy's Law with the continuity equation (Rahem 1991). The general groundwater flow equation may be written as:

$$\frac{\partial}{\partial r}\left(\rho\frac{\partial}{\partial r}\right)+\frac{\partial}{\partial z}\left(\rho\frac{\partial h}{\partial z}\right)=\frac{S\rho}{K}\frac{\partial h}{\partial t} \tag{1}$$

where S is the specific storage coefficient; K represents the hydraulic conductivity; h stands for the piezometric head in the aquifer; r and z are the radial and vertical axis respectively; and ρ denotes the water density. Meanwhile, the generalised form of the solute transport equation is:

$$\rho\frac{\partial c}{\partial t}+\frac{\partial}{\partial r}\left(v_r\rho c\right)+\frac{\partial}{\partial z}\left(v_z\rho c\right)=\frac{\partial}{\partial r}\left(D_{rr}\rho\frac{\partial c}{\partial r}\right)+\frac{\partial}{\partial z}\left(D_{zz}\rho\frac{\partial c}{\partial z}\right) \tag{2}$$

where c is the concentration of solute in the aquifer; v_r and v_z represent the seepage velocities in the aquifer in r and z direction; ρ is the water density; and D_{rr} and D_{zz} are coefficients of dispersion in r and z direction respectively. The components of the dispersive coefficient for two-dimensionals are expressed by Bear and Verruijt (1990) as

$$D_{rr} = a_T V : D_{zz} a_L V \tag{3}$$

where a_T is the transverse dispersivity; and a_L is the longitudinal dispersivity; and V is the average velocity of the pore fluid.

The Galerkin technique was used to determine an approximate solution to Equations (1) and (2) under appropriate boundary conditions. A saline intrusion model, SUTRA that employed the finite element method to solve the governing equations was utilised. The SUTRA (which is named from the acronym Saturated Unsaturated Transport) was published by the United States Geological Survey (Voss 1984). The model is two-dimensional and can be applied either aerially or in cross section to establish a salinity profile. The coordinate system may be either Cartesian or radial, which makes it possible to simulate phenomena such as saline up-coning beneath a pumped well. In this study, the software package used is the combination of the USGS-developed code that interfaces SUTRA with Argus ONE™, a commercial software product developed by Argus Interware. The Argus ONE™ (Argus Open Numerical Environment), is a programmable system with geographic-information-system-like (GIS-like) functionality that includes automated gridding and meshing capabilities for linking geo-spatial information with finite element numerical model discretizations. The Graphical-User Interface (GUI) for SUTRA is based on a public-domain Plug-In Extension (PIE) that automates the use of Argus ONE™ to automatically create the appropriate geo-spatial information coverage (information layers) for SUTRA, provide menus and dialogues for inputting geo-spatial information and simulation control parameters for SUTRA, and allow visualisation of SUTRA simulation results.

MATERIALS AND METHODS

Saltwater Source

A solution of sodium chloride (NaCl) was selected to represent the saltwater for modelling the concentration distribution in the aquifer. Saltwater has been found to be easily monitored, safe and easily available. In this investigation, saltwater with a concentration of 0.04 by weight was used in order to produce an approximate density of 1028 kg/m^3, which is equivalent to the concentration of seawater.

Porous Media

For an isotropic and non-homogeneous porous media, it must have a coefficient of uniformity of more than 4. In this study coarse sand was mixed with gravel, which gave a uniformity coefficient of 6 and grain size d_{50} of 1 mm using sieve analysis.

Piezometric Taps

Four piezometric taps were fixed on the floor of the model in a radial line. *Fig. 1(a)* shows the arrangement of the taps. Each tap was connected to a manometer board through a flexible tubing. The water elevation in each tube on the manometer board represents the hydraulic head at that particular tap location.

Conductivity Probes

The concentration of the saltwater tracer was measured using specially constructed probes. The probes had been calibrated by determining the resistance in ohms of different concentrations of saltwater. The probes were installed at locations shown in *Fig. 1(b)*. The probes were identified as Probes 1, 2 and 3, with Probe 1 located nearest to the interface and Probe 3, furthest.

Determination of Parameters

The parameters required in the numerical model viz. longitudinal and lateral apparent dispersivity coefficients, storage coefficient, hydraulic conductivity, porosity and compressibility of soil and water were determined using empirical

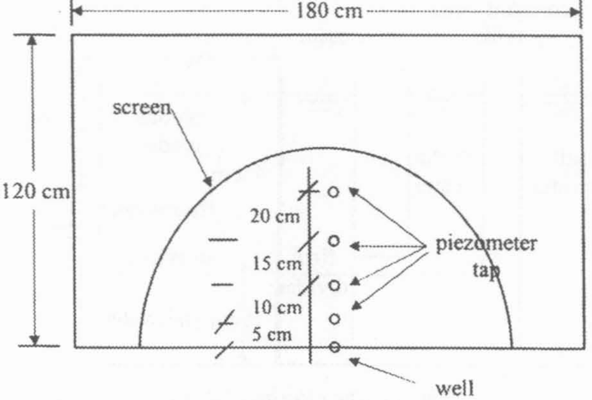

Fig. 1(a): Locations of the piezometer taps (in plan)

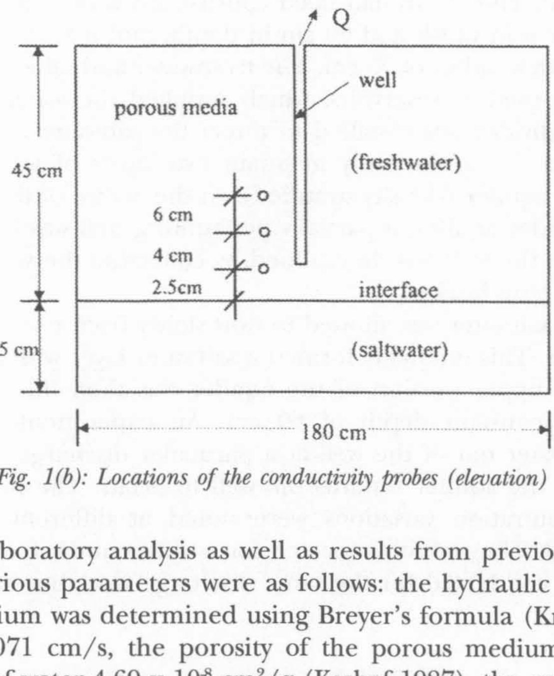

Fig. 1(b): Locations of the conductivity probes (elevation)

formulae and laboratory analysis as well as results from previous studies. The values of the various parameters were as follows: the hydraulic conductivity of the porous medium was determined using Breyer's formula (Kresic 1997) and its value was 0.071 cm/s, the porosity of the porous medium was 0.21, the compressibility of water 4.69 x 10^{-8} cm^2/g (Kashef 1987), the compressibility of coarse-fine sand 0.35 x 10^{-6} cm^2/g (Kashef 1987), the specific storage coefficient 3.52 x $10^{-4}/s^2$, and the longitudinal and transverse dispersivity 0.16 cm and 0.0016 cm respectively (Rahem 1991).

Experimental Procedure

The schematic diagram in *Fig. 2* shows the experimental set-up used in conducting tests on the distributions of hydraulic head and concentration of a conservative tracer (saltwater) in two-dimensional flow towards a partially

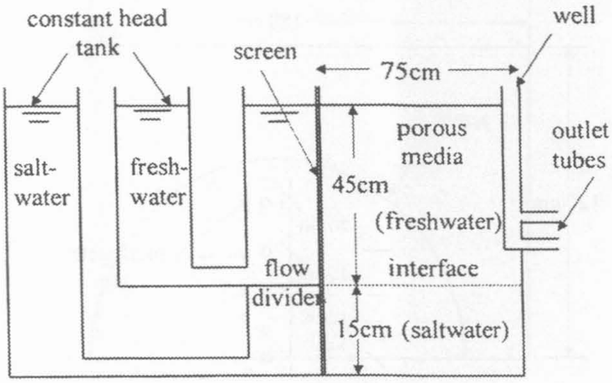

Fig. 2: The experimental layout

penetrating well. The set-up had been constructed with dimensions of 180 cm in length, 120 cm in width and 60 cm in depth, and it formed an aquifer of a half-cylinder with a radius of 75 cm. The freshwater and saltwater constant head chambers were used as reservoirs, which supplied the water into the porous media. A flow divider was installed to direct the saltwater and the freshwater into the aquifer to continuously maintain two layers of water with different densities in the aquifer (density stratified). At the centre of the plane boundary of the half-cylinder aquifer, a partially penetrating well was installed. The rate of flow through the well was determined by collecting the water from the well by using volumetric beaker.

Initially the saltwater was allowed to flow slowly from the saltwater chamber into the aquifer. This gradually formed a saltwater layer with a constant depth of 15 cm. The upper portion of the aquifer was then filled with freshwater gradually to a constant depth of 60 cm. An experimental run started by pumping the water out of the well at a particular discharge rate. This caused a flow through the aquifer towards the well to occur. The pressure head and saltwater concentration variations were noted at different times from the devices installed. The test was repeated for a different discharge rate. Another set of tests was conducted for the same discharge rates but at a different well penetration.

RESULTS AND DISCUSSION

Theoretical model validation was conducted by comparison between the numerical solutions and the experimental test results for two different sets of well penetration (40.0 cm and 35.0 cm) and pumping rates (6.0 cm^3/s and 4.8 cm^3/s). All tests were carried out over a duration of 120 min. Comparison between the concentration ratios (the ratio of the existing concentration to the initial saltwater concentration) of numerical and experimental solution for concentration and piezometric head distributions were shown in *Figs. 3* and *4* respectively. *Fig. 3* shows that the numerical results are almost the same as the experimental results, except in very few cases. Computation has shown that the maximum difference in the concentration between the numerical and experimental results is only 10%. It can be seen from the figure also that at 120 min the saltwater had almost fully replaced the freshwater at the Probe 1 location (2.5 cm above the interface), and that some amount of the saltwater had also reached the Probe 3 point (12.5 cm above the interface). It could be expected that the concentration of saltwater would increase at the latter location as time increased. *Fig. 4* shows two sets of typical results for piezometric head at various distances from the well. It is clear from the figure that the two results are quite comparable, with a maximum difference of about 11%. It can also be seen that at 60 min there was a drawdown of about 0.5 to 2 cm throughout the aquifer for this particular discharge rate (6.0 cm^3/s). This also means that the decrease in the piezometric head towards the well is not significant at this discharge rate.

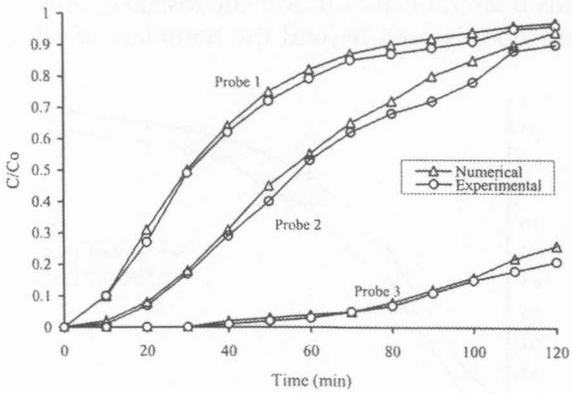

Fig. 3: Comparison between numerical and experimental solutions for concentration ratios at probe locations 1, 2 and 3 for well depth = 40.0 cm and discharge rate = 6.0 cm^3/s

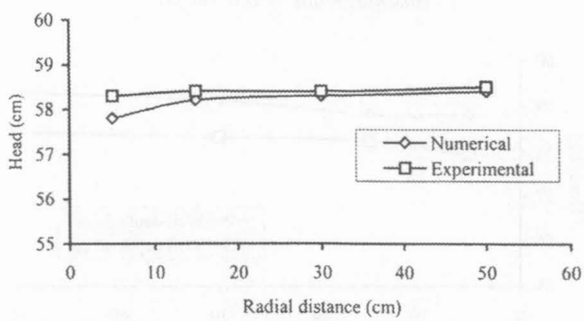

Fig. 4: Comparison between numerical and experimental solutions for piezometric heads at various distances from the well at time = 60 min for well depth = 40.0 cm and discharge rate = 6.0 cm^3/s

The following discussion will use only the numerical results, since it was shown that these results were comparable with the experimental results. For the shallower well, it was found that the saltwater concentrations at a particular location (Probe 1) and at various times were lower when compared to the deeper well (*Fig. 5*). The differences in concentrations resulting from the two well depths are also shown to be quite constant. In the case of the piezometric head profiles, abstraction using the shallower well resulted in higher heads at the corresponding locations, or in other words, smaller drawdown, compared to the deeper well (*Fig. 6*).

The effect of the different discharge rates is shown in *Fig. 7*. As expected, the lower discharge induced less saltwater into the well. Nevertheless, for this particular aquifer system, salt water intrusion has occurred with these discharge rates.

The lowering of the water table due to the constant discharge rate over a certain lapse of time is shown in *Fig. 8*. The amount of reduction in the piezometric heads is almost constant over the distance, showing that the point of no drawdown is located well beyond the boundary of this aquifer system.

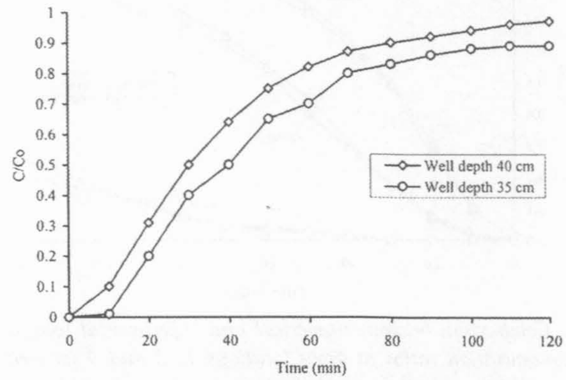

Fig. 5: Variation of concentration ratios at Probe 1 for different well depths (discharge rate = 6.0 cm^3/s)

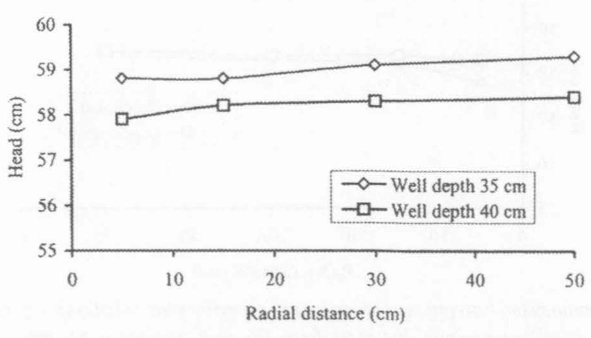

Fig. 6: Piezometric head profiles for different well depths at time = 60 min and discharge rate = 6.0 cm^3/s

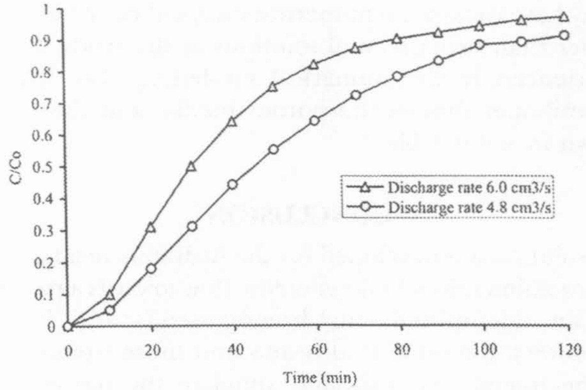

Fig. 7: Variation of concentration ratios at Probe 1 for different discharge rates (well depth = 40 cm)

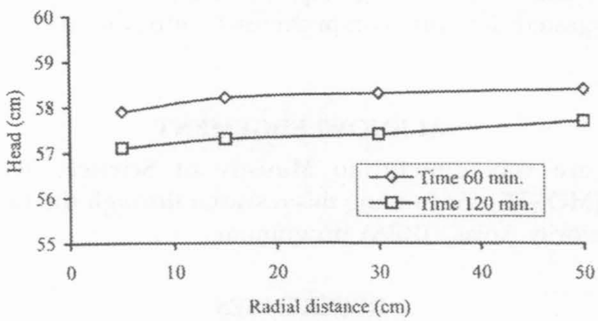

Fig. 8: Piezometric head profiles at different times for discharge rate = 60 cm^3/s and well depth = 40 cm

Although comparison of both solutions had shown the consistency that is needed for the purpose of modelling, it is observed that all these discrepancies need to be investigated further. In this study, for the case of concentration ratio distributions, it was apparent that the values of the numerical solution were greater than experimental solutions, while for the case of pressure head distributions, experimental solutions always had higher values. In reality, the situation may be different because solute reaction processes are neither linear nor equilibrium controlled, and the numerical model may not necessarily represent the true complexities of the reaction. Difficult problems also arise when the concentration of the solute of interest is strongly dependent on the presence of numerous other constituents that exist in the porous media. Here, mineralogical variability may be significant and may affect the rate and of reactions, and yet be ignored in the mathematical modelling instead. At the same time, in many groundwater flow systems, sorption may also cause the retardation of the movement of the contaminants. Sorption refers to the uptake of the dissolved constituents from solution by the porous medium. These phenomena may have caused the numerical values of concentration distribution becoming greater than experimental solutions in this study. Larger movement of solute experienced in the numerical modelling also meant that higher velocity of groundwater flow in the porous media, and this could generate a bigger drawdown in water table.

CONCLUSION

The numerical solution was developed for the hydraulic head and concentration distributions in two-dimensional axi-symmetric flow towards a partially penetrating well through an unconfined, non-homogeneous and isotropic aquifer. Comparisons between the numerical results and those from experimental tests indicate that the model can accurately simulate the movement of pollutant (saltwater) in the saturated zone of a non-homogeneous unconfined aquifer. Deterministic groundwater simulation model can be a valuable tool for analysing aquifer systems and for predicting responses to specific stresses, and its usage is hereby suggested for the comprehensive and intensive hydro-geologic investigations.

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