ISSN: 0128-7680
Pertanika J. Sci. & Technol. 12(1): 137 - 147 (2004)

Stochastic Rainfall Model for Irrigation Projects

Lee Teang Shui & Aminul Haque
Department of Biological and Agricultural Engineering
Universiti Putra Malaysia, 43400 UPM, Serdang, Selangor, Malaysia
E-mail: tslee@eng.upm.edu.my

Received: 19 December 2002

ABSTRAK

Model hujan stokastik adalah berkenaan dengan waktu berlaku dan jumlah ukuran hujan turun. Wujudnya beberapa model hujan berdasarkan skala masa berbeza-beza. Model hujan harian yang telah digunakan dengan luasnya, didapati sesuai diperguna di dalam model-model berkeadaan seimbang air terperinci, pertanian dan persekitaran. Dalam kajian ini, satu model penjanaan hujan stokastik disuaikan untuk Projek Pengairan Besut yang terletak di Terengganu, Malaysia. Model ini menyelaku jujukan kejadian hujan dengan kaedah matrik kebarangkalian alihan, sementara jumlah hujan harian dijanakan dengan menggunakan taburan normal pencong. Data-data hujan daripada enam stesen meteorologi yang terletak dalam Projek Pengairan Besut digunakan dalam model ini. Parameter-parameter model dianggar daripada rekod sejarah hujan. Pengesahan model dengan satu set data berasingan dibuat kemudian. Keputusan yang dihasilkan menunjukkan bahawa model ini boleh diperguna untuk menjanakan data hujan dengan sempurna.

ABSTRACT

Stochastic rainfall models are concerned with the time of occurrence and depth of rainfall. Various rainfall models have been using different time scales. Daily rainfall models have gained wide applicability as being appropriate for use in detailed water balance and agricultural and environmental models. In this study a stochastic daily rainfall generation model was adapted for the Besut Irrigation Scheme located in Terengganu, Malaysia. The model simulates the sequence of rainfall occurrence using the method of transitional probability matrices, while daily rainfall amount was generated using a skewed normal distribution. Rainfall data from six meteorological stations located at the Besut Irrigation Scheme were used for this model. The model parameters were estimated from historical rainfall records. The model validation was then performed with a separate set of data. Results obtained showed that the model could be used to generate rainfall data satisfactorily.

Keywords: Stochastic model, rainfall occurrence, rainfall generation, transitional probability

INTRODUCTION

Stochastic rainfall models are designed as a one-part or two-part model depending on whether time of occurrence and depth are generated simultaneously or separately. For the one-part models, the transition probability matrix, and the modified transition probability are the most popular. In the case of two-part models, the two states Markov chain for simulating the occurrence of rainfall coupled with a statistical distribution for simulating rainfall depth is of interest (Chin 1977; Carey and Haan 1978; Mimikou 1983; Srikanthan and McMahon 1983; Efremides and Tsakiris 1994).

The amount and pattern of rainfall are among the most important weather characteristics and they affect agriculture profoundly. In addition to their direct effects on water balance in soil, they are strongly related to other weather variables such as solar radiation, temperature, and humidity, which are also important factors affecting the growth and development of crops, pests, diseases and weeds. However, rainfall data form an essential input into many climatologic studies for agriculture, wherein considerable research focused on rainfall analysis and modeling (Austine 2001). For instance, in rain-fed agriculture, information on total amount, as well as expected rainfall, is useful in planning agricultural policies. Monthly and seasonal rainfall data are used in determining supplemental irrigation, water requirements, and in engineering studies related to storage analysis and reservoir management.

In recent years, agricultural scientists have shown considerable interest in modeling and simulation of rainfall as new ways of analyzing rainfall data and assessing its impact on agriculture. Among the proposed methods, a combination of Markov chain and a skewed normal distribution is recognized as a simple approach and is demonstrated to be effective in generating daily rainfall for many environments (Hanson *et al.* 1980; Garbutt *et al.* 1981; Stern and Coe 1982; Hanson 1982, 1984; French 1983; Tung 1983; Osborn 1984, 1987; Geng *et al.* 1986; Jimoh and Webster 1996, 1999). In this approach, a Markov chain is used to describe the occurrence of daily rainfall, and skewed normal distribution is applied to fit the amount of rainfall for a rainy day. A first-order Markov chain is generally recognized as a simple and effective description of the rainfall occurrence. This research, while recognizing the difficult task of accurately predicting rainfall, adapted a model for forecasting daily rainfall in the Besut Irrigation Scheme, Terengganu, Malaysia.

Study Area

Irrigation in Malaysia is almost entirely devoted to rice cultivation. Eight designated granaries totaling 217,000 ha are located for rice cultivation in Malaysia. The Besut Irrigation Scheme was completed in 1977 and is one of the eight designated granary areas in Malaysia. The Besut Irrigation Scheme is located at the northeastern corner of Peninsular Malaysia in the state of Terengganu. The project area encompasses 5,164 ha of land with climatic conditions favorable for rice production. The Besut river, one of the two water sources in the scheme, runs northwards towards the South China Sea along the west boundary of the scheme. The Angga river is another water source for the scheme, converges to Besut river towards the south of the scheme area. One important aspect of the scheme is that the production cycle is based primarily on the annual rainfall pattern and distribution. The total mean annual rainfall is about 2900 mm, with extreme rain intensity reaching 400 mm over a 24hr period. Heavier rainfalls (average) occur in October, November, December and January with 280, 590, 550 and 180 mm of rainfall respectively (JICA 1998). Significantly dry periods with low monthly averages are outside the main monsoon season in the months from March to August. During the November-January period, 40% of the total annual rains generally fall. Therefore, rainfall plays a very significant role for rice production in this scheme.

Data

A first-order Markov chain and skewed normal distribution method requires daily weather records for many years in order to estimate the model parameters. Thus the availability of the weather data limits the applicability of the simulation method. Daily rainfall data for six stations in Besut Irrigation Scheme were obtainable from the Data Information Section, Hydrological branch, Department of Irrigation and Drainage, Ampang, Malaysia. The stations were chosen due to their spatial representations as well as availability of adequate data for the study. The information for the six rainfall stations is given in Table 1.

METHODS

A first order Markov chain was used to simulate the occurrence of rainfall. Two states were used in the Markov chain, and they are the wet and dry states. A wet day is defined as one where a trace or larger amount of rainfall is recorded. Dry days, on the contrary, are days that are not wet. The decision to include trace amounts in the wet category arose primarily from solar radiation simulation considerations. Two assumptions made underlying the first-order Markov chain are namely, (1) the probability that the current day is in a particular state (i.e. wet or dry) depends only on the state of the previous day; and (2) for a given season within the year, the stochastic structure of daily rainfall is the same for each day and does not change from year to year. It has been further assumed that these so-called transition probabilities are independent of the particular day within individual months. The probability of a wet day can be calculated directly from the number of wet days by using this equation.

$$PW = NWD \; / \; ND \tag{1}$$

TABLE 1

Location of stations where daily rainfall records were collected for this study

Station	Latitude	Longitude	Period of records
Ibu Bekalan Angga	5°36'00" N	102°30'55" E	1951-1998
Sek Keb Kg Jabi	5°40'45" N	102°33'50" E	1980-1998
Sek Keb Keruk	5°29'00" N	102°29'30" E	1980-1999
Sek Keb Kg Tambila	5°44'25" N	102°36'30" E	1980-1999
Rumah Merinyu Tali air	5°44'15" N	102°30'15" E	1948-1991
Pasir Akar	5°38'25" N	102°30'15" E	1980-1990

where, PW = the probability of a wet day, or % of wet days, in a month
NWD = the number of rainy days in a month
ND = the number of days in a month

The probability of occurrence of daily rainfall consists of two transition probabilities. These are the transition probability of a wet day, given that the previous day was a wet day P (W/W), and the transition probability P (W/D) for the state of a wet day following a given dry day. Therefore from statistical data, the probability of a wet day after a dry day P (W/D) and the probability of a wet day following a wet day P (W/W) can be calculated directly using the following relationship:

$$P\ (W/D) = a + b\ f \tag{2}$$

$$P\ (W/W) = (1\text{-}b) + P\ (W/D) \tag{3}$$

where f is the perennial mean monthly precipitation frequency, being the ratio of the number of perennial monthly rainfall days and number of days of the month, while a, b are regression coefficients.

Input for the model must include monthly probabilities of receiving rainfall. On any given day, the input must include information as to whether the previous day was dry or wet. The probability for the particular day in that month is calculated with either Equation (2) or Equation (3) depending on the known wet-dry condition of the previous day. Then it is input into the random number generation form. The random number generation is obtained from a Visual Basic program written for this purpose. A random uniform number between 0 and 1 is obtained by clicking a button. If the random number is less than or equal to the wet-dry probability entered, rain is predicted to occur for that day and a wet day is expected to follow. On the contrary, when the random number generated is greater than the wet-dry probability, no rain is predicted for that day and a dry day is expected to follow. Since the wet or dry state of the first day can be established, the process can be repeated for the next day and so on throughout the simulation period.

When a rain event is predicted to occur, the rainfall amount to be expected can be generated from a skewed normal daily precipitation distribution (Nicks 1974).

$$R_i = \left(\frac{\left(\left(SND_i - \frac{SCF_k}{6} \right) \left(\frac{SCF_k}{6} \right) + 1 \right)^3 - 1}{SCF_k} \right) RSDV_k + \overline{R}_k \tag{4}$$

where R_i is the amount of rainfall in mm and SND_i is the standard normal deviate for day i respectively, while SCF_k is the skew coefficient, $RSDV_k$ is the standard deviation of daily rainfall, and $\bar{R}_k$ is the mean daily rainfall, for the month *k*. For each month, the total number of wet days and the total sum of rainfall for these days can then be predicted.

RESULTS AND DISCUSSION

Daily rainfall records for the Besut area were used to run the model. The period of rainfall record has permitted the investigation of trends in the annual number of wet days. The time plots of the annual number of wet days at the six stations are presented in *Fig. (1)*. *Fig. (1(a, c, e, f))* shows persistent decline in the annual number of wet days from the 1990s onwards. A simple linear regression analysis was performed for each location separately and for the combined data. Results presented in Table 2 showed that none of the intercepts (a values) is significantly different from zero and none of the slope coefficients (b values) is significantly different from any other slope coefficient among the locations. The combined regression line with a zero intercept and slope 0.75 explains 96% of the total variation that existed among the transitional probabilities across time and space. Monthly transitional probabilities were then calculated with the fractions of wet days, and these are shown in *Fig. 2*. To validate the stochastic rainfall model, which could be used for generating rainfall occurrence and rainfall amount, historical data from one rainfall station, the Angga Station, was selected for evaluation. *Fig. 3* shows the Visual Basic screen where the wet-dry probability calculated is then entered for the month and a random number is generated, after which the condition for the next day is given upon clicking the start button to initiate comparison of numbers. Table 3 shows an example calculation for the case of January 1st to 31st 2001. Table 4 shows a summary of the comparison between the historical and simulated data for frequency of wet days for Angga Station for the years 2000 and 2001. As far as the rainy days are concerned, there was no case in which the generated monthly values were different from the actually observed

TABLE 2

Regression coefficients a and b of regressing the transitional probabilities of a wet day to a dry day for the data of six rainfall stations

Location	a	(s.e)*	b	(s.e)	r^2 **
Ibu Bekalan Angga	0.002	0.006	0.725	0.028	0.980
Sek Keb Kg Jabi	0.008	0.041	0.810	0.029	0.975
Sek Keb Keruk	-0.015	0.012	0.856	0.041	0.970
Sek Keb Kg Tambila	0.021	0.004	0.721	0.035	0.969
Rumah Merinyu Tali air	-0.004	0.015	0.645	0.046	0.965
Pasir Akar	0.006	0.005	0.768	0.015	0.890
Combined	0.003	0.014	0.754	0.032	0.958

* s.e is the standard error ** r^2 is the correlation coefficient

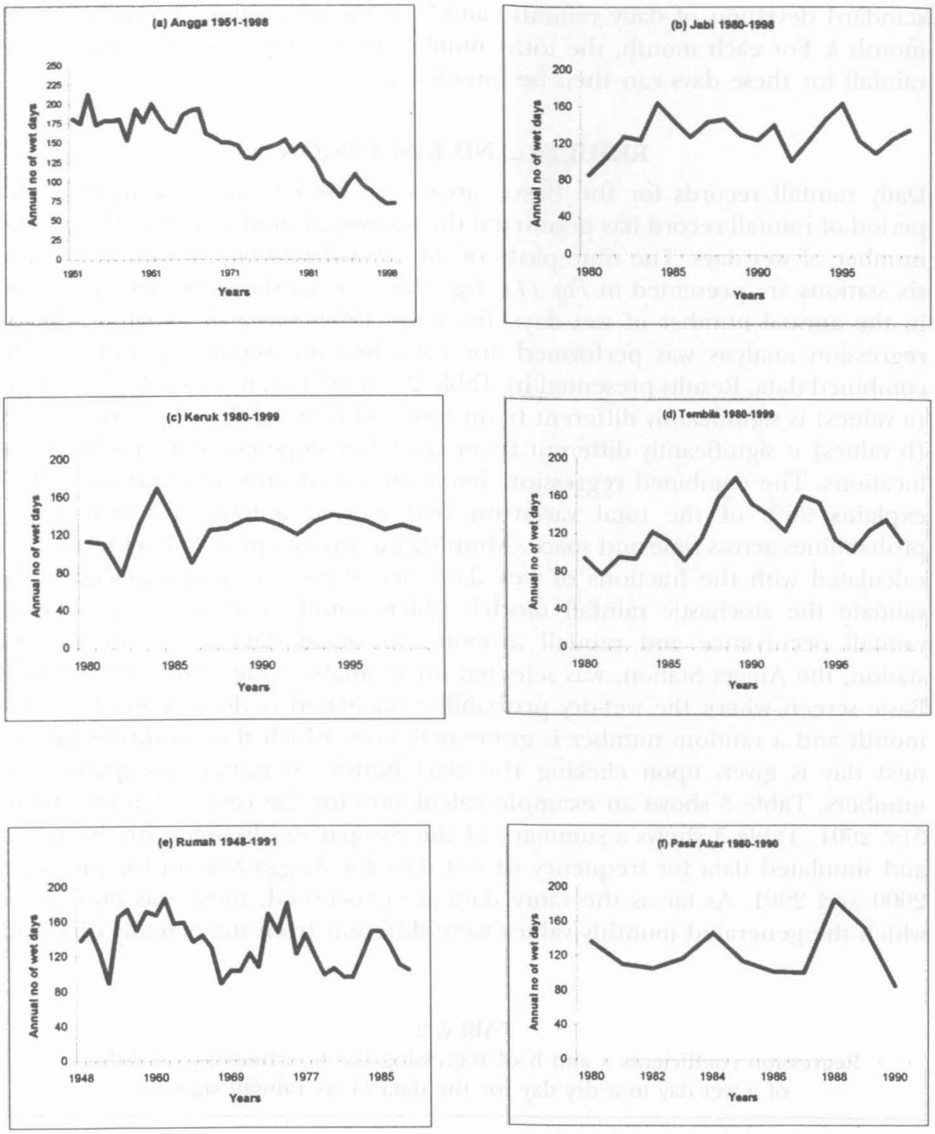

Fig. 1: Time plot of annual number of wet days of six rainfall stations

monthly values by more than two days. In terms of amount of rainfall, simulated results were again very close to the observed values, with a slight overestimation in a few months. The amount overestimated was less than 5% of the observations in all cases. The model thus allows for satisfactory rainfall simulation and can be used for water management of irrigation practices.

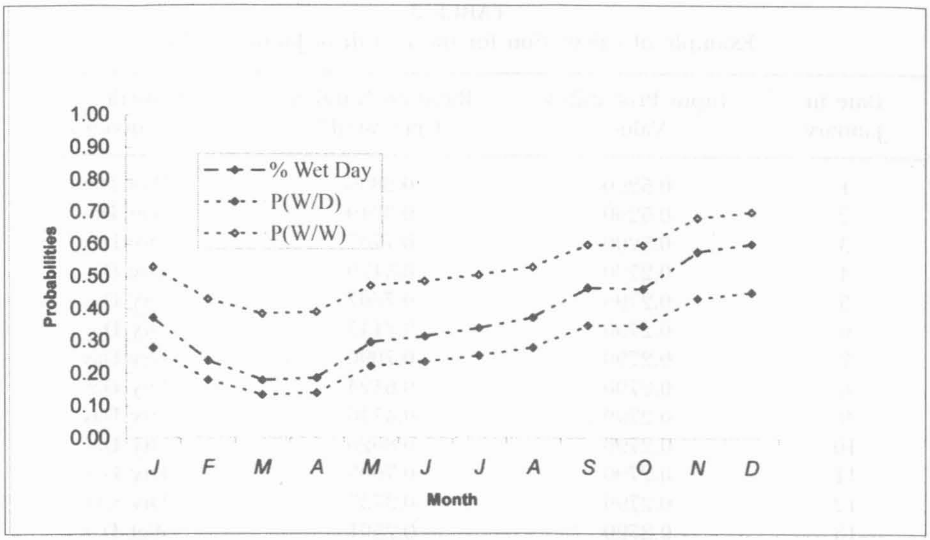

Fig. 2: Transitional probabilities and fractions of wet days for each month

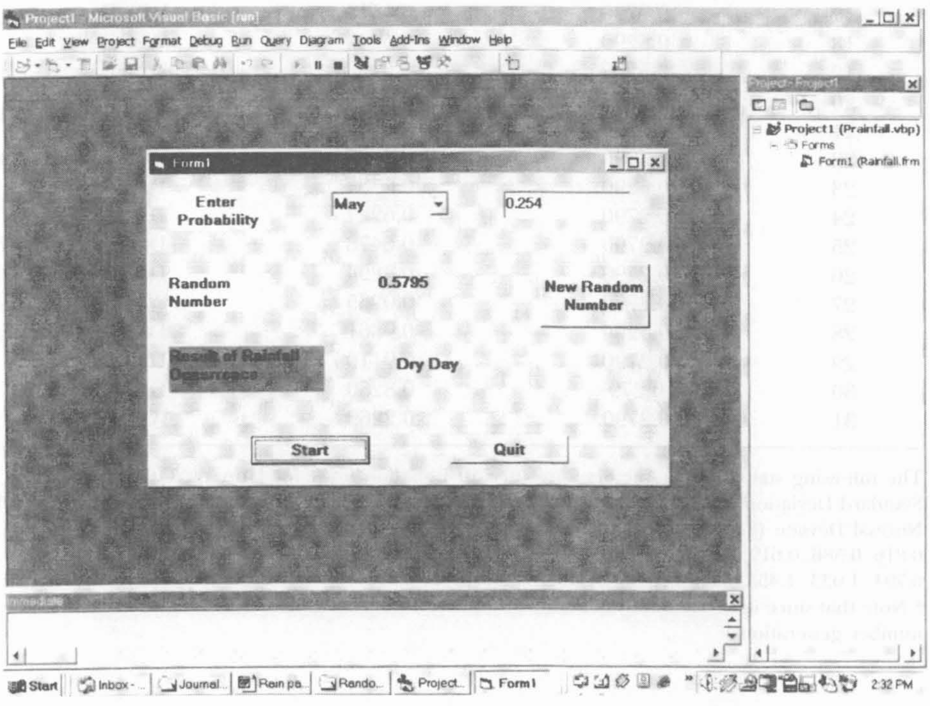

Fig. 3: Visual basic screen showing the random number generation result

TABLE 3
Example of calculation for the month of January 2001

Date in January	Input Probability Value	Random Number Generated*	Prediction of Occurrence
1	0.5290	0.2896	Wet Day
2	0.5290	0.3019	Wet Day
3	0.5290	0.7747	Dry Day
4	0.2790	0.5120	Dry Day
5	0.2790	0.7607	Dry Day
6	0.2790	0.8145	Dry Day
7	0.2790	0.7090	Dry Day
8	0.2790	0.6124	Dry Day
9	0.2790	0.4140	Dry Day
10	0.2790	0.8626	Dry Day
11	0.2790	0.7905	Dry Day
12	0.2790	0.3735	Dry Day
13	0.2790	0.2501	Wet Day
14	0.5290	0.0214	Wet Day
15	0.5290	0.0562	Wet Day
16	0.5290	0.9496	Dry Day
17	0.2790	0.1512	Wet Day
18	0.5290	0.5249	Wet Day
19	0.5290	0.5150	Wet Day
20	0.5290	0.0535	Wet Day
21	0.5290	0.0712	Wet Day
22	0.5290	0.4687	Wet Day
23	0.5290	0.6587	Dry Day
24	0.2790	0.6227	Dry Day
25	0.2790	0.6478	Dry Day
26	0.2790	0.8294	Dry Day
27	0.2790	0.0235	Wet Day
28	0.5290	0.9861	Dry Day
29	0.2790	0.9110	Dry Day
30	0.279	0.8280	Dry Day
31	0.2790	0.2269	Wet Day

The following statistical parameters were used in Equation no.4, for January 2001
Standard Deviation = 31.360, Skew Coefficient = 5.945, Mean Daily Rainfall = 12.074 mm, Standard Normal Deviate (SND) daily (1st - 31st January) = 0.594, 0.673, 0.501, 0.381, 0.375, 0.439, 0.434, 0.816, 0.586, 0.619, 0.632, 0.422, 0.539, 0.740, 0.579, 0.828, 0.790, 0.875, 0.930, 0.551, 1.120, 1.205, 0.704, 1.033, 1.453, 0.858, 1.606, 1.419, 1.080, 0.985, 1.492.
* Note that since it is a randomly generated number, it will change. Numbers shown are a first time number generation.

TABLE 4
Comparison of monthly historical and simulated rainfall values for Angga Station for years 2000 and 2001

Location/Year Angga	Month	Rainfall Amount (mm)		Rainfall Occurrence (days)	
		Historical	Simulated	Historical	Simulated
2001	January	162	166	11	13
	February	85	82	5	4
	March	10	8	2	3
	April	23	17	4	6
	May	55	49	8	7
	June	25	24	2	2
	July	0	0	0	0
	August	30	32	4	4
	September	50	47	7	6
	October	73	75	8	9
	November	85	72	8	7
	December	375	393	16	18
	Total	973 mm	965 mm	75 days	79 days
2000	January	105	97	10	9
	February	45	37	6	5
	March	70	67	6	6
	April	35	29	4	4
	May	27	28	3	4
	June	62	60	6	8
	July	25	27	3	2
	August	95	82	7	6
	September	57	49	7	8
	October	65	52	4	5
	November	90	88	9	10
	December	152	140	11	12
	Total	828 mm	756 mm	76 days	79 days

CONCLUSIONS

A study was carried out to adapt a mathematical model for predicting the probability of rainfall, given a previous day's condition. This prediction is based on a first-order Markov chain process and its accompanying assumptions and wherein the probability of a wet or a dry day's to follow a known previous day condition can be found. This probability, derived from historical data, is then checked against a randomly generated number, whence it is then decided whether it is going to be a wet or dry day. Should rainfall be predicted to follow, then the expected amount of rainfall is evaluated by a method in which its parameters were predetermined from a statistical analysis of past long term historical daily and monthly data. The total predicted number of rainy days for the month, and the total monthly rainfall can be tallied up. In the year 2000, the actual number of days with rainfall recorded was 76 and the total amount of annual rainfall was 828 mm. The number of wet days predicted was 79 with a total annual rainfall of 756 mm. This gave a percentage difference between observed and predicted days of rainfall and amount of rainfall as +4% and -8.6% respectively. Similarly, the total number of wet days and total rainfall was observed to be 75 and 973 mm respectively in 2001. The predicted number of wet days for 2001 is 79 days while the annual rainfall predicted is 965 mm giving a +5.3% more number of wet days and -0.01% less rainfall amount. Hence the model hence can generate satisfactory results.

ACKNOWLEDGEMENTS

The authors wish to express their sincere gratitude to the staff of the Besut Irrigation Scheme, the Drainage and Irrigation Department and the Malaysian Meteorological Service. The authors would also like to thank The Ministry of Science, Technology and the Environment for the funding of the Project IRPA 01-02-04-0422.

REFERENCES

NNAJI, A. O. 2001. Forecasting seasonal rainfall for agricultural decision-making in northern Nigeria. *J. of Agricultural and Forest Meteorology* **107**: 193–205.

CAREY, D. I. and C. T. HAAN. 1978. Markov processes for simulating daily point rainfall. *Journal of Irrigation and Drainage Division ASCE* **104 (IRI)**: 111–125.

CHIN, E. H. 1977. Modelling daily precipitation occurrence process with Markov chain. *Water Resources Research* **13(6)**: 949–956.

EFREMIDES D. and G. TSAKIRIS. 1994. Stochastic modelling of point rainfall in a Mediterranean Island environment. *Water Resources Management* **8**: 171–182.

FRENCH, R. H. 1983. Precipitation in southern Nevada. *J. of Hydraulics Division ASCE* **10** (HY 7): 1023–1036.

GARBUTT, D. J., R. D. STERN, M. D. DENNETT and J. ELSTON. 1981. A comparison of the rainfall climate of eleven places in West Africa using a two-part model for daily rainfall. *Arch. Met. Geophy. Biokl. Ser. B.* **29**: 137–155.

GENG, S., F. W. T. PENNING DE VRIES and I. SUPIT. 1986. A simple method for generating daily rainfall data. *J. Agricultural and Forestry Meteorology* **36**: 363–376.

HANSON, C. L. 1982. Distribution and stochastic generation of annual and monthly precipitation on a mountainous watershed in southwest Idaho. *Water Resources Bulletin AWRA* **18(5)**: 875–883.

HANSON, C. L. 1984. Annual and monthly precipitation generation in Idaho. *Transactions of the ASCE* **27(6)**: 1792–1797.

HANSON, C. L., R. P. MORRIS, R. L. ENGLEMAN, D. C. COON and C. W. JHONSON. 1980. Spatial and seasonal precipitation distribution in southwest Idaho. USDA, ARS Agricultural Research Mannual. ARMW-13.

JICA. 1998. The study on modernization of irrigation water management system in the granary area of Peninsular Malaysia. Draft Final Report, Volume -II, Annexes, March.

JIMOH, O. D. and P. WEBSTER. 1996. Optimum order of Markov chain for daily rainfall in Nigeria. *Journal of Hydrology* **185:** 45–69.

JIMOH, O. D. and P. WEBSTER. 1999. Stochastic modeling of daily rainfall in Nigeria: intra-annual variation of model parameters. *Journal of Hydrology* **222:** 1–17.

MIMIKOU, M. 1983. Daily occurrence modeling with Markov chain of seasonal order. *Hydrology Science Journal* **28(2):** 221–223.

NICKS, A. D. 1974. Stochastic generation of the occurrence, pattern, and location of maximum amount of daily rainfall. In *Proc. Symp. Statistical Hydrology*, p.154–171, Aug.-Sept. 1971, Tucson, AZ.U.S. Dept. Agric., Misc. Publ. No. 1275.

OSBORN, H. B. 1984. Estimating precipitation in mountainous regions. *J. of Hydraulics Div. ASCE* **110(HY 12):** 1859–1863.

OSBORN, H. B. 1987. Closure: estimating precipitation in mountainous regions. *J. of Hydraulics Division ASCE* **113(HY 4):** 549–550.

SRIKANTHAN, R. and T. A. MCMAHON. 1983. Stochastic generation of daily rainfall for Australian stations. *Trans. ASCE*: 754–766.

STERN, R. D. and R. COE. 1982. The use of rainfall models in agricultural planning. *J. of Agric. Meteorology* **26:** 35–50.

TUNG, YEOU-KOUNG. 1983. Point rainfall estimation for a mountainous region. *J. of Hydraulics Division ASCE* **109 (HY 10):** 1386–1393.