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Bootstrap Methods in a Class of Non-Linear Regression Models

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ABSTRAK

Dalam kertas ini, prestasi ralat piawai bootstrap bagi anggaran berpemberat MM (WMM) dibandingkan dengan ralat piawai Monte Carlo dan ralat piawai Berasimptot. Sifat-sifat selang keyakinan bootstrap bagi anggaran berpemberat WMM seperti 'Percentile' (PB), 'Bias-corrected Percentile' (BCP), 'Bias and Accelerated' (BC_a), 'Studentzed Percentile' (SPB) dan 'Symmetric' (SB) telah diperiksa dan dibandingkan. Keputusan kajian menunjukkan bahawa BSE boleh dianggap hampir kepada ASE dan MCSE sehingga 20% titik terpencil. BC_a mempunyai sifat yang menarik dari segi kebarangkalian liputan, kesamaan hujung dan purata panjang selang yang lebih baik jika dibandingkan dengan kaedah lain.

ABSTRACT

In this paper, the performances of the bootstrap standard errors (BSE) of the Weighted MM (WMM) estimates were compared with the Monte Carlo (MCSE) and Asymptotic (ASE) standard errors. The properties of the Percentile (PB), Bias-Corrected Persentile (BCP), Bias and Accelerated (BC_a), Studentized Percentile (SPB) and the Symmetric (SB) bootstrap confidence intervals of the WMM estimates were examined and compared. The results of the study indicate that the BSE is reasonably close to the ASE and MCSE for up to 20% outliers. The BC_a has attractive properties in terms of better coverage probability, equitailness and average interval length compared to the other methods.

Keywords: Outlier, weighted MM, bootstrap sampling

INTRODUCTION

One of the important aspects in statistical inference is to obtain the standard errors of parameter estimates and to construct confidence intervals for the parameters of a model. There exist abundant procedures to provide approximate confidence intervals. In recent years, an increasingly popular method is bootstrapping method which was introduced by Efron (1979). There are considerable papers related to bootstrap methods (DiCiccio and Romano 1988; DiCiccio and Tibshirani 1987; Efron 1981a, 1981b, 1982, 1985, 1987; Efron and Gong 1983; Efron and Tibshirani 1986, 1993; Hall 1986a, 1986b; Huet *et al.* 1990; Loh 1987, Wu 1986).

In this paper, we want to investigate the bootstrap estimates of the standard errors of the Weighted MM (WMM) estimates and compare them with the asymptotic and Monte Carlo standard errors. Five different bootstrap confidence intervals are used for estimating the WMM estimates, namely, the Percentile (PB), Bias-Corrected Percentile (BCP), Bias-Corrected and Accelerated (BC_a) and the Symmetric bootstrap (SB). A 'good' confidence interval is one which posseses a reasonably accurate coverage probability and 'good' equitailness. By equitailed, we mean that a confidence interval for β of level $(1 - 2\alpha)$ is such that the proportion for β lying outside the interval is divided equally between the lower and upper limits of the intervals. In other words, the proportion of β lower than the lower limit of the interval is reasonably equal to α, as is the proportion of β that exceeds the upper limit. A secondary consideration is the confidence interval length.

THE WEIGHTED MM ESTIMATES (WMM)

The proposed technique for the Weighted MM is computed in four stages:
- Compute the Weighted Nonlinear LMS.
- Calculate an M estimate of weighted scale using rho function ρ_0.
- Compute the weighted M estimate using rho function ρ_1.
- Repeat step 2 dan 3 until convergence.

Hampel redescending psi function (Hampel *et al.* 1986), denoted as ρ_H, is used in the analysis. Yohai (1987) revealed that $\rho_0(r)$ and $\rho_1(r)$ can be taken to be $\rho_H(r/k_0)$ and $\rho_H(r/k_1)$, respectively. Stromberg (1993) demonstrated that selecting $k_0 = 0.212$ and $k_1 = .9014$ will guarantee a high breakdown estimate and will result in 95% efficiency under normal errors, respectively.

Considering the general heteroscedastic nonlinear regression

$$y_i = f(x_i,\beta) + \sigma g\{f(x_i,\beta), z_i, \theta\}\varepsilon_i \tag{1}$$

and the residuals,

$$r_i = \frac{y_i - f(x_i,\beta)}{g\{f(x_i,\beta), z_i, \theta\}} \tag{2}$$

and assuming that the variance are proportional to the regressor, the residual r_i in (2) becomes

$$r_i = \frac{y_i - f(x_i,\beta)}{x_i} \tag{3}$$

For model (1), the Weighted Nonlinear LMS, $\hat{\beta}_{WLMS}$ is obtained from

$$\underset{\hat{\beta}_{WLMS}}{\arg\min}\ r^2_{(k)}(\beta_{WLMS}) \tag{4}$$

where

$$r^2_{(i)}(\beta_{WLMS}),\ i = 1, 2, 3, \ldots, n \text{ are the ordered } r^2_{(i)}(\beta_{WLMS})$$

and k is given by Stromberg (1993).

The proposed algorithm for the weighted nonlinear LMS and the Weighted MM estimates are similar to Stromberg (1993), except that y_i and $f(x, \beta)$ are replaced by y_i/x_i and $f(x,\beta)/x_i$, respectively.

The steps in the algorithm of the Weighted Nonlinear LMS are as follows:

- Calculate the initial estimate of WLMS denoted by $\hat{\beta}_{WLMS}$, using GNLLS denoted by $\hat{\beta}$.
- Compute the GNLLS estimate to p randomly selected points, denoted by $\hat{\beta}_{WLS}$.
- If the median squared residual at $\hat{\beta}_{WLMS}$ is less than the median squared residual at $\hat{\beta}$, $\hat{\beta}$ is replaced by $\hat{\beta}_{WLS}$ as the current estimate of $\hat{\beta}_{WLMS}$.
- Steps 2 and 3 are repeated k times, where k is specified by Stromberg (1992, 1993).
- $\hat{\beta}$ is used as a starting value for calculating the LS fit $\hat{\beta}^*_{LS}$, for data points such that $r_i^2\left(\hat{\beta}\right) \le \text{med}_{1 \le i \le n} r_i^2\left(\hat{\beta}\right)$. If $\text{med}_{1 \le i \le n} r_i^2\left(\hat{\beta}^*_{LS}\right) < \text{med}_{1 \le i \le n} r_i^2\left(\hat{\beta}\right)$, then $\hat{\beta}$ is replaced by $\hat{\beta}^*_{LS}$ as the current estimate of $\hat{\beta}_{WLMS}$.
- In order to get an even better estimate, the Nelder-Mead Simplex Algorithm (Nelder and Mead 1965) which is implemented in Press *et al.* (1986) with fractional tolerance 10^{-4}, is used to minimize $\text{med}_{1 \le i \le n} r_i^2\left(\hat{\beta}\right)$ by using $\hat{\beta}$ as the starting value.

The Weighted M-Estimate for Scale

Let $\hat{\beta}_{WLMS}$ be the parameter estimate of the regression function in (1) with a high breakdown point, and the residuals are defined by

$$r_i(\hat{\beta}) = \frac{y_i - f(x_i, \hat{\beta}_{WLMS})}{x_i}, \qquad 1 \le i \le n$$

The weighted M-scale estimate is defined as the value of s which is the solution of

$$\frac{1}{n}\sum \rho_0(r_i / s_n) = b \tag{5}$$

where
b may be obtained from the equation $E_\phi(\rho(r))=b$.

Let ρ_0 in (5) be a real function which satisfies the following assumptions:

- $\rho(0)=0$
- $\rho(-r)=\rho(r)$
- $0 \le u \le v$ implies $\rho(u) \le \rho(v)$ (6)
- ρ is continuous
- Let $a = \sup \rho(r)$, then $0 < a < \infty$
- If $\rho(u)<a$ and $0 \le u \le v$, then $\rho(u) < \rho(v)$

The constant b is such that

$b/a = 0.5$ where $a = \max \rho_0(r)$ (7)

This implies that this scale estimate has a breakdown point equal to 0.5 as verified by Huber (1981).

The Weighted MM Estimate

The weighted MM estimate is found by minimizing

$$S(\beta_{WLMS}) = \sum p_1(r_i(\beta_{WLMS})/S_n) \tag{8}$$

where β_{WLMS} and S_n are defined in (4) and (5) respectively. $\rho_1(0/0)$ is interpreted as 0. ρ_1 is another function which satisfies assumption (6) such that

$p_1(r) \le p_0(r)$ (9)

$\sup \rho_1(r)=\sup \rho_0(r)=a$ (10)

This implies that $S(\beta_{1WMM}) \le S(\beta_{0WMM})$.

The Standard Error

The covariance matrix of the WMM estimates can be approximated by employing Theorem 4.1 of Yohai (1987). The asymptotic variance of the WMM estimate is the diagonal of the covariance matrix:

$$\left[\frac{\left[\frac{1}{(n-p)}\right] S_n^2 \sum_{i=1}^{n}\left[\psi\left(r_i\left(\beta_{WMM}\right)\right)/\hat{S}_n\right]^2}{\left[\frac{1}{n}\sum_{i=1}^{n}\psi'\left(r_i\left(\beta_{WMM}\right)/\hat{S}_n\right)\right]^2} A^{-1}\right]_{\beta=\hat{\beta}_{WMM}} \tag{11}$$

where
the (jk)-th element of A, j, k $\in\{1,2...p\}$, is

$$\sum_{i=1}^{n}\left(\partial\left[h\left(x_i,\beta\right)/x_i\right]/\partial\beta_j\right)\left(\partial\left[\left[h\left(x_i,\beta\right)/x_i\right]/\partial\theta_k\right]\right)$$

and ψ is the derivative of ρ_1 and $\hat{s}_n$ is the scale estimate as defined in (5). However, this estimate possesses several shortcomings when it has a breakdown point equal to 1/n in most regression settings.

THE BOOTSTRAP METHODS

Bootstrap methods can be applied to a nonlinear regression model. Caroll and Ruppert (1988) and Stromberg (1993) used the bootstrap method to compute bootstrap standard errors of the Transform Both Sides (TBS) estimates and the MM estimates, respectively. Huet *et al.* (1990) carried out a simulation study to compare different methods for calculating approximate confidence intervals for parameters in nonlinear regression. The above authors used the resampling method with fixed regressors.

Resampling With Fixed Regressors or Bootstrapping Residuals

- Fit a model to the original sample of observations to get $\hat{\beta}$.
- Construct $\hat{F}$, putting mass 1/n at each observed residuals, $\hat{F}$: masa 1/n at each $\hat{\varepsilon}_i = y_i - f\left(x_i,\hat{\beta}\right), i = 1,2....n$.
- Draw a bootstrap data set, $y_i^* = f\left(x_i,\hat{\beta}\right)+\hat{\varepsilon}$ where $\hat{\varepsilon}$ are i.i.d from $\hat{F}$.
- Compute $\hat{\beta}^*$ for the bootstrap data set.
- Repeat B times the steps 3 and 4, obtaining bootstrap replications $\hat{\beta}^{*1},\hat{\beta}^{*2},...,\hat{\beta}^{*B}$

- Estimate the bootstrap standard errors, by taking square root to the main diagonal of the covariance matrix,

$$\hat{COV} = \frac{1}{B-1}\sum_{b=1}^{B}\left(\hat{\beta}^{*b} - \hat{\beta}^{*.}\right)\left(\hat{\beta}^{*b} - \hat{\beta}^{*.}\right)^{T} \tag{12}$$

where

$$\hat{\beta}^{*.} = \frac{1}{B}\sum_{b=1}^{B}\hat{\beta}^{*b}$$

Bootstrap Confidence Interval

In practice, the estimated standard errors $\hat{\sigma}$, are usually employed to form approximate confidence intervals to a parameter of interest, β. The usual $(1-2\alpha)100\%$ confidence interval for β is, $\hat{\beta} \pm \hat{\sigma} Z_{\alpha/2}$ where $Z_{\alpha/2}$ is the $100.\alpha/2$ percentile point of a standard normal distribution. The validity of this interval depends on the assumption that $\hat{\beta}$ is normally distributed. Otherwise, this approximate confidence interval will not be very accurate. Bootrstrap confidence intervals do not rely upon the usual assumption of normality.

In this section, we will use the bootstrap to calculate better confidence intervals even if the underlying distribution of the estimate is not normal. Some bootstrap methods make substantial rectifications which significantly improve the inferential accuracy of the interval estimate. There are various methods that can be used to construct bootstrap confidence intervals. Huet *et al.* (1990) carried out simulation studies on a nonlinear regression models by using the Studentized Percentile (SPB), the Ordinary Percentile (OP) and the Symmetric (SB) bootstrap confidence intervals. The bias- corrected percentile (BCP) and the bias- corrected and accelerated (BC_a) confidence intervals could be used in the nonlinear regression models as enumerated by Efron (1984) and Hall et al. (1989). We will review five methods of bootstrap confidence intervals, namely, the Percentile (PB), the bias-corrected percentile (BCP), the bias-corrected and accelerated (BC_a), the studentized percentile (SPB), and the Symmetric bootstrap (SB) for the parameter β.

The Bootstrap Standard Errors

The performance of the Weighted MM (WMM) is found to be better than the MM, NLLS and the GNLLS estimates as shown by Midi (1999). In order to examine its asymptotic, Monte Carlo and bootstrap standard errors, simulations studies were carried out using the Ricker; $\left(y_i = \beta_0 x_i \exp\left(-\beta_1 x_i\right) + \varepsilon_i\right)$ and Micahelis-Menten model; $\left(y_i = \dfrac{\beta_1 x_i}{\exp\left(\beta_2\right) + x_i} + \varepsilon_i\right)$. 30 'good' data were generated according to both models, where x_i are uniformly distributed on [0, 10]. For the Michaelis-Menten and the Ricker Models, (β_1, β_2), (β_0, β_1) are set to (10, 0) and (2, 0.04), respectively. The errors ε_i were generated from a normal distribution, $N\left(0, \sigma^2 x_i^2\right), \sigma^2 = \left(.25\right)^2$. We deleted each observation and replaced with an outlier. The outliers were generated by $x_i \sim U[1,2]$, $\varepsilon_i \sim N(0,1)$ and $y_i = 40 + \varepsilon_i$. The performances of the three types of standard errors are presented in Tables 1 and 2 for the Ricker and the Michaelis-Menten models, respectively.

The results in Table 1 for the Ricker model show that the bootstrap standard errors (BSE), the Monte Carlo Standard Errors (MCSE) and the Asymptotic Standard Errors (ASE) are reasonably close to each other for up to 10% outliers. As the percentages of outliers rises, the BSE increases and the increase in BSE of $\hat{\beta}_0$ is remarkably much larger than the ASE and the MCSE. Nevertheless, the BSE of the $\hat{\beta}_1$ increases, but its increment is relatively small. The MCSE and ASE of both $\hat{\beta}_0$ and $\hat{\beta}_1$ increase very little with the increase in outliers and their values are fairly close for up to 40% outliers.

TABLE 1
The Monte Carlo, asymptotic and bootstrap standard errors of the WMM estimates (The Ricker Model)

Outliers %		Monte Carlo Standard Errors	Asymptotic Standard Errors	Bootstrap Standard Errors
0	$\hat{\beta}_0$	.100	.093	.065
	$\hat{\beta}_1$	.009	.010	.008
10	$\hat{\beta}_0$	.104	.091	.294
	$\hat{\beta}_1$	.010	.010	.036
20	$\hat{\beta}_0$	.110	.092	2.460
	$\hat{\beta}_1$	.010	.011	.029
30	$\hat{\beta}_0$	.118	.094	9.910
	$\hat{\beta}_1$	.011	.012	0.113
40	$\hat{\beta}_0$	4.631	.110	24.59
	$\hat{\beta}_1$	.054	.014	.279

TABLE 2
The Monte Carlo, asymptotic and bootstrap standard errors of the WMM estimates (The Michaelis-Menten model)

Outliers %		Monte Carlo Standard Errors	Asymptotic Standard Errors	Bootstrap Standard Errors
0	$\hat{\beta}_0$	.258	.265	.132
	$\hat{\beta}_1$	.066	.052	.027
10	$\hat{\beta}_0$	.268	.277	.150
	$\hat{\beta}_1$	.070	.056	.032
20	$\hat{\beta}_0$	.285	.298	.165
	$\hat{\beta}_1$	.075	.062	.035
30	$\hat{\beta}_0$	.308	.327	2.034
	$\hat{\beta}_1$	.090	.070	3.448
40	$\hat{\beta}_0$	.336	.371	5.457
	$\hat{\beta}_1$	.100	.082	6.419

The results in Table 2 for the Michaelis-Menten model indicate that the BSE is imperceptibly less than the MCSE and ASE for up to 20% outliers. On the other hand, the MCSE is moderately close to the ASE in this situation. It is important to note here that enhancing the proportion of outliers by more than 20% increases the BSE dramatically. This implies that the BSE performs poorly in such a situation. The results of the simulation study also suggest that the reliability of the BSE decreases as the percentages of outliers increases by more than 20%.

A SIMULATION STUDY

In order to investigate the properties of the five types of bootstrap confidence intervals, a series of simulations was conducted, one on a simulated data without outliers and another on a simulated data with 10% outliers. Again, we consider the same simulation procedures as described in section 4 using the Michaelis-Menten and the Ricker models. 200 bootstrap samples were drawn from a sample of size 30 and a bootstrap 95% confidence interval was constructed for each of the five methods. 100 replications of such simulations were executed to determine the percentage of times the true value of the parameter estimates was contained in the interval and the average length was calculated. The same procedure is repeated for the data with 10% outliers. The results of the simulation studies are illustrated in Tables 3 and 4, respectively.

For the Ricker model (see Table 3), it is quite difficult to decide which confidence interval is better or worse than the others. Judging from the coverage probability, equitailness and average interval length, our results are not in favour of the Percentile (PB), Studentized Percentile (SPB) and Symmetric (SB) intervals for estimating $\hat{\beta}_0$ in the case of 'clean' data. However, they showed an improvement in coverage probability for estimating $\hat{\beta}_1$. The performance of the BC_a is slightly better than the PB, SPB and SB intervals, and in close agreement with the BCP method.

TABLE 3
Coverage probabilities and average width of the five types of bootstrap confidence intervals (The Ricker Model)

No Outlier		Coverage	Lower Coverage	Upper Coverage	Ave. Width
	Method				
$\hat{\beta}_0$	PB	93	2	5	0.436
	BCP	95	2	3	0.434
	BC_a	94	2	4	0.460
	SPB	96	0	4	0.522
	SB	93	1	6	0.435
$\hat{\beta}_1$	PB	92	4	4	0.042
	BCP	91	4	5	0.042
	BC_a	92	4	4	0.041
	SPB	95	3	2	0.048
	SB	94	4	2	0.042
10% Outliers					
$\hat{\beta}_0$	PB	94	1	5	0.445
	BCP	94	1	5	0.445
	BC_a	95	1	4	0.451
	SPB	96	1	3	0.502
	SB	95	0	5	0.672
$\hat{\beta}_1$	PB	91	4	5	0.042
	BCP	91	4	5	0.042
	BC_a	95	3	5	0.042
	SPB	95	3	2	0.047
	SB	93	4	3	0.044

TABLE 4
Coverage probabilities and average width of the five types of bootstrap confidence intervals (The Michaelis-Menten Model)

No Outlier	Method	Coverage	Lower Coverage	Upper Coverage	Ave. Width
$\hat{\beta}_0$	PB	96	2	2	1.323
	BCP	95	2	3	1.332
	BC_a	95	2	3	1.351
	SPB	95	1	4	1.616
	SB	96	0	4	1.344
$\hat{\beta}_1$	PB	95	2	3	0.782
	BCP	96	2	2	0.737
	BC_a	96	2	2	0.629
	SPB	93	3	4	0.423
	SB	99	0	1	1.848
10% Outliers					
$\hat{\beta}_0$	PB	95	2	3	1.449
	BCP	95	1	4	1.417
	BC_a	96	1	3	1.470
	SPB	93	2	5	1.669
	SB	96	0	4	1.430
$\hat{\beta}_1$	PB	96	3	1	0.925
	BCP	97	2	1	1.118
	BC_a	95	3	2	0.723
	SPB	91	3	6	0.463
	SB	98	0	2	1.570

For the contaminated data, all the confidence intervals have coverage probabilities fairly close to each other. However, the SPB and the SB display wider average interval lengths than the other three methods. Among the PB, BCP and BC_a, the BC_a confidence interval is appreciably the best method, since it possesses a coverage percentage which is equal to the nominal value and reasonably close to the expected value for $\hat{\beta}_0$ and $\hat{\beta}_1$, respectively.

From Table 4 (the Michaelis-Menten model), where the data is 'cleaned' and 'contaminated', it reveals that the SB method gives erroneous results not only from the point of view of equitailness but also from the point of view of coverage probability. In addition, it possesses an average length which is reasonably larger than the other intervals. The performance of the SPB is also not encouraging in both situations. Its coverage probability was lower than the expected value of 0.95 by about 0.04. For the 'clean' data, the coverages of the

BC_a, BCP, and PB confidence intervals were reasonably close to the expected value of 0.95 for both $\hat{\beta}_0$ and $\hat{\beta}_1$. However, the BC_a is considerably the best method because besides displaying a good coverage probability and equitailness, it has relatively shortest intervals than the PB and BCP methods.

A similar conclusion can be made for the case of contaminated data. As before, the BCa confidence interval gives good results in terms of coverage probability, equitailness and confidence interval length. Its coverage probability is almost equal to the nominal value. The average lengths for all bootstrap methods increase and exhibit consistent pattern with the shortest intervals come from the BC_a. This is followed by the PB, BCP, SPB and the longest being the SB confident interval. The results of the study suggest that the BC_a is the best method to estimate the 95% confidence interval for the WMM estimates. The selection of a good bootstrap method is essential.

Since the BC_a confidence interval possesses a 'good' coverage probability, 'good' equitailness and narrowest average interval length, it can be recommended to be incorporated in the NLLS, GNLLS, MM and WMM procedures in an effort to justify the conclusion of Midi (1999) that the WMM is the most robust method among those considered. Again, we use the same simulation procedures as described earlier and apply the BC_a method to the NLLS, GNLLS, MM and WMM techniques. The results of the simulation study are illustrated in Tables 5 – 6. We would expect that a more robust method would be the one with 'good' coverage probability and 'good' equitailness. Another important property is that the method should have the shortest average confidence length. For the Ricker model (see Table 5) and with the 'clean data', the confidence intervals for the NLLS, GNLLS, MM, and WMM have lower coverage percentages than the nominal value of 0.95. Nevertheless, among these intervals the average lengths of the GNLLS and the WMM are fairly close and turn out to be the smallest.

On the other hand, the confidence intervals for the NLLS and GNLLS give the worst results in the presence of outliers in the data set. Their coverage probability was very small and they displayed very bad equitailness. Besides, their average confidence lengths are prominently large. However, the WMM confidence interval for β_0 gives a coverage probability which is in best agreement with the nominal one and signifies the narrowest average interval length. The coverage probability of the WMM confidence interval for β_1 is sligtly less than the nominal value. The performance of the MM confidence intervals estimates are quite good both in terms of coverage probability and average length, but its accomplishment cannot outperform the WMM method.

For the Michaelis-Menten model (see Table 6) and the data with no outliers, it seems that, on the whole, the GNLLS and WMM estimates perform better than the NLLS and MM estimates. Both the methods adequately provide the expected coverage probabilities and the shortest average lengths, though the $\hat{\beta}_0$ of the GNLLS displays a bad equitailness. The results of the study signify the fact that the NLLS and MM have a lower coverage probability and slightly larger average length than the GNLLS and WMM estimates. The performances of the NLLS and GNLLS are very poor with the presence of outliers. Their coverage probabilities are remarkably much lower than the nominal values and posses average lengths which are much wider than those of the MM and WMM estimates. Nonetheless, the results of the WMM estimates are intuitively appealing. It gives confidence intervals with relatively good coverage probabilities and equitailness. Furthermore, it possesses the smallest average confidence length. On the other hand, the MM estimates yields slightly lower coverage and average lengths than the WMM estimates.

TABLE 5
Coverage probabilities and average width for the BC_a confidence intervals for the NLLS, GNLLS, MM and WMM methods (The Ricker Model)

No Outlier		Coverage	Lower Coverage	Upper Coverage	Ave. Width
	Method				
$\hat{\beta}_0$	NLLS	93	2	5	1.388
	GNLLS	90	0	10	0.434
	MM	91	3	6	0.873
	WMM	94	2	4	0.460
$\hat{\beta}_1$	NLLS	92	6	2	0.268
	GNLLS	91	4	5	0.038
	MM	96	3	1	0.079
	WMM	92	4	4	0.041
10% Outliers					
$\hat{\beta}_0$	NLLS	72	0	28	****
	GNLLS	75	25	0	228.951
	MM	94	3	3	0.876
	WMM	95	1	4	0.451
$\hat{\beta}_1$	NLLS	45	55	0	2.165
	GNLLS	2	98	0	1.653
	MM	95	3	2	0.076
	WMM	92	3	5	0.042

CONCLUSION

The empirical studies suggest that the BSE is fairly close to the ASE and MCSE for up to 20% outliers and its reliability decreases as the percentage of outliers increases by more than 20%. The results also suggest that the BC_a confidence interval stands out to be the best for both situations in which the data are 'clean' and contaminated. The SPB and SB perform poorly in the presence of

TABLE 6
Coverage probabilities and average width for the BC_a confidence intervals for the NLLS, GNLLS, MM and WMM methods (The Michaelis-Menten model)

No Outlier		Coverage	Lower Coverage	Upper Coverage	Ave. Width
	Method				
$\hat{\beta}_0$	NLLS	87	2	11	1.810
	GNLLS	93	1	6	1.098
	MM	95	2	3	2.003
	WMM	95	2	3	1.351
$\hat{\beta}_1$	NLLS	91	4	5	0.919
	GNLLS	95	3	2	0.262
	MM	95	2	3	0.860
	WMM	96	2	2	0.628
10% Outliers					
$\hat{\beta}_0$	NLLS	95	0	5	2.110
	GNLLS	46	54	0	14.001
	MM	93	4	3	2.088
	WMM	96	1	3	1.471
$\hat{\beta}_1$	NLLS	3	0	97	56.860
	GNLLS	77	17	6	12.787
	MM	93	4	3	0.996
	WMM	95	3	2	0.723

outliers. The BC_a confidence intervals associated with the WMM and GNLLS are better than those of the NLLS and MM estimates when there is no contamination in the data. Nonetheless, the accomplishment of the GNLLS's interval deteriorates dramatically with the presence of outliers in the data. The results of the NLLS's interval are also in close agreement with the GNLLS's interval in such a situation with remarkably low coverage probability, poor equitailness and wider average interval lengths.

However, the WMM confidence intervals consistently provide adequate coverage probability or to a lesser extent close to the nominal value, good equitailness and shortest average length. The results of the simulation study agree reasonably well with Midi (1999) that the WMM is the most robust method, followed by the MM, the GNLLS and the NLLS methods in the presence of outliers. These results also confirm the conclusions made by Midi (1999) that the WMM and the GNLLS are equally good in a well behaved data.

It is very important to note here that our results are based on limited studies and these could be improved further by increasing the number of resamplings.

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