

Modelling the Volatility of Currency Exchange Rate Using GARCH Model

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ABSTRAK

Kertas ini mengkaji model GARCH dan modifikasinya dalam menguasai kemruapan kadar pertukaran mata wang. Parameter model tersebut dianggar dengan menggunakan kaedah kebolehdjian maksimum. Prestasi bagi penganggaran dalam sampel didiagnosis dengan menggunakan beberapa statistik kebagusan penyuaian dan kejituan telahan satu langkah ke depan dan luar sampel dinilai dengan menggunakan min ralat kuasa dua. Keputusan kajian menunjukkan kegigihan kemruapan kadar pertukaran mata wang RM/Sterling. Keputusan daripada penganggaran dalam sampel menyokong kebergunaan model GARCH dan model variasi malar pula ditolak, sekurang-kurangnya dalam sampel. Statistik Q dan ujian pendarab Langrange (LM) mencadangkan penggunaan model GARCH yang beringatan panjang menggantikan model ARCH yang beringatan pendek dan berperingkat lebih tinggi. Model GARCH-M pegun berprestasi lebih tinggi daripada model GARCH lain yang digunakan dalam kajian ini, dalam telahan satu langkah ke depan dan luar sampel. Apabila menggunakan model perjalanan rawak sebagai tanda aras, semua model GARCH berprestasi lebih baik daripada model tanda aras ini dalam meramal kemruapan kadar pertukaran mata wang RM/Sterling.

ABSTRACT

This paper attempts to study GARCH models with their modifications, in capturing the volatility of the exchange rates. The parameters of these models are estimated using the maximum likelihood method. The performance of the within-sample estimation is diagnosed using several goodness-of-fit statistics and the accuracy of the out-of-sample and one-step-ahead forecasts is evaluated using mean square error. The results indicate that the volatility of the RM/Sterling exchange rate is persistent. The within sample estimation results support the usefulness of the GARCH models and reject the constant variance model, at least within-sample. The Q-statistic and LM tests suggest that long memory GARCH models should be used instead of the short-term memory and high order ARCH model. The stationary GARCH-M outperforms other GARCH models in out-of-sample and one-step-ahead forecasting. When using random walk model as the naive benchmark, all GARCH models outperform this model in forecasting the volatility of the RM/Sterling exchange rates.

INTRODUCTION

Issues related to foreign exchange rate have always been the interest of researchers in modern financial theory. Exchange rate, which is the price of one currency in terms of another currency, has a great impact on the volume of foreign trade and investment. Its volatility has increased during the last decade and is harmful to economic welfare (Laopodis 1997). The

exchange rate fluctuated according to demand and supply of currencies. The exchange rate volatility will reduce the volume of international trade and the foreign investment.

Modelling and forecasting the exchange rate volatility is a crucial area for research, as it has implications for many issues in the arena of finance and economics. The foreign exchange volatility is an important determinant for pricing

of currency derivative. Currency options and forward contracts constitute approximately half of the U.S. 880bn per day global foreign exchange market (Isard 1995). In view of this, knowledge of currency volatility should assist one to formulate investment and hedging strategies.

The implication of foreign exchange rate volatility for hedging strategies is also a recent issue. These strategies are essential for any investment in a foreign asset, which is a combination of an investment in the performance of the foreign asset and an investment in the performance of the domestic currency relative to the foreign currency. Hence, investing in foreign markets that are exposed to this foreign currency exchange rate risk should hedge for any source of risk that is not compensated in terms of expected returns (Santis *et al.* 1998).

Foreign exchange rate volatility may also impact on global trade patterns that will affect a country's balance of payments position and thus influence the government's national policy-making decisions. For instance, Malaysia fixed the exchange rate at RM3.80/US\$ in September, 1998, due to the economic turmoil and currency crisis in 1997. This turmoil has spread to developed countries such as USA, Hong Kong, Europe and other developing South American countries such as Brazil and Mexico. Due to this currency crisis, various governments have resorted to different national policies so as to mitigate the effect of this crisis.

In international capital budgeting of multinational companies, the knowledge of foreign exchange volatility will help them in estimating the future cash flows of projects and thus the viability of the projects.

Consequently, forecasting the future movement and volatility of the foreign exchange rate is crucially important and of interest to many diverse groups including market participants and decision makers.

Beginning with the seminal works of Mandelbrot (1963a, 1963b, 1967) and Fama (1965), many researchers have found that the stylized characteristics of the foreign currency exchange returns are non-linear temporal dependence and the distribution of exchange rate returns are leptokurtic, such as Friedman and Vandersteel (1982), Bollerslev (1987), Diebold (1988), Hsieh (1988, 1989a, 1989b),

Diebold and Nerlove (1989), Baillie and Bollerslev (1989). Their studies have found that large and small changes in returns are 'clustered' together over time, and that their distribution is bell-shaped, symmetric and fat-tailed.

These features of data are normally thought to be captured by using the Autoregressive Conditional Heteroskedasticity (ARCH) model introduced by Engle (1982) and the Generalised ARCH (GARCH) model developed by Bollerslev (1986), which is an extension of the ARCH model to allow for a more flexible lag structure. The use of ARCH/GARCH models and its extensions and modifications in modeling and forecasting stock market volatility is now very common in finance and economics, such as French *et al.* (1987), Akgiray (1989), Lau *et al.* (1990), Pagan and Schwert (1990), Day and Lewis (1992), Kim and Kon (1994), Franses and Van Dijk (1996) and Choo *et al.* (1999).

On the other hand, the ARCH model was first applied in modeling the currency exchange rate by Hsieh only in 1988. In a study done by Hsieh (1989a) to investigate whether daily changes in five major foreign exchange rates contain any nonlinearities, he found that although the data contain no linear correlation, evidence indicates the presence of substantial nonlinearity in a multiplicative rather than additive form. He further concludes that a generalized ARCH (GARCH) model can explain a large part of the nonlinearities for all five exchange rates.

Since then, applications of these models to currency exchange rates have increased tremendously, such as Hsieh (1989b), Bollerslev, T. (1990), Pesaran and Robinson (1993), Copeland *et al.* (1994), Takezawa (1995), Episcopos and Davies (1995), Brooks (1997), Hopper (1997), Cheung *et al.* (1997), Laopodis (1997), Lobo *et al.* (1998) and Duan *et al.* (1999).

In many of the applications, it was found that a very high-order ARCH model is required to model the changing variance. The alternative and more flexible lag structure is the Generalised ARCH (GARCH) introduced by Bollerslev (1986). Bollerslev *et al.* (1992) indicated that the squared returns of not only exchange rate data, but all speculative price series, typically exhibit autocorrelation in that large and small errors tend to cluster together in contiguous time periods in what has come to be known as volatility clustering. It is also proven that small

lag such as GARCH(1,1) is sufficient to model the variance changing over long sample periods (French *et al.* 1987; Franses and Van Dijk 1996; Choo *et al.* 1999).

Even though the GARCH model can effectively remove the excess kurtosis in returns, it cannot cope with the skewness of the distribution of returns, especially the financial time series which are commonly skewed. Hence, the forecasts and forecast error variances from a GARCH model can be expected to be biased for skewed time series. Recently, a few modifications to the GARCH model have been proposed, which explicitly take into account skewed distributions. One of the alternatives of non-linear models that can cope with skewness is the Exponential GARCH or EGARCH model introduced by Nelson (1990). For stock indices, Nelson's exponential GARCH is proven to be the best model of the conditional heteroskedasticity.

In 1987, Engle *et al.* developed the GARCH-M to formulate the conditional mean as function of the conditional variance as well as an autoregressive function of the past values of the underlying variable. This GARCH in the mean (GARCH-M) model is the natural extension due to the suggestion of the financial theory that an increase in variance (risk proxy) will result in a higher expected return.

Choo *et al.* (1999) studies the performance of GARCH models in forecasting the stock market volatility and they found that i) the hypotheses of constant variance models could be rejected since almost all the parameter estimates of the non-constant variance (GARCH) models are significant at the 5% level; ii) the EGARCH model has no restrictions and constraints on the parameters; iii) the long-memory GARCH model is more suitable than the short-memory and high-order ARCH model in modelling the heteroscedasticity of the financial time series; iv) the GARCH-M is best in fitting the historical data whereas the EGARCH model is best in out-of-sample (one-step-ahead) forecasting; v) the IGARCH is the poorest model in both aspects.

Since Choo *et al.* (1999) have indicated that the GARCH-M model performs well in within-sample estimation and the EGARCH model performs best in out-of-sample forecasting, the combination of both models, EGARCH-M should be able to enhance the performance in both aspects.

In order to know the out-of-sample forecasting performance of EGARCH-M, we compare the performance of EGARCH-M and the other modifications of the GARCH model to the simple random walk forecasting scheme.

The models are presented in the following section. The third section is the background of currency exchange rate data and the methodology used in this study. All the results will be discussed in the fourth section. The conclusion will be in the final section.

MODEL

The conditional distribution of the series of disturbances which follows the GARCH process can be written as

$$\varepsilon_t / \psi_{t-1} \sim N(0, h_t)$$

where ψ_{t-1} denotes all available information at time $t - 1$. The conditional variance h_t is

$$h_t = w + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j}$$

Hence, the GARCH regression model for the series of r_t can be written as

$$\phi_s(B)r_t = \mu + \varepsilon_t, \text{ with } \phi_s(B) = 1 - \phi_1 B - \dots - \phi_s B^s$$

$$\varepsilon_t = \sqrt{h_t} e_t$$

$$e_t \sim N(0, 1)$$

$$h_t = w + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j}$$

where B is the backward shift operator defined by $B^k y_t = y_{t-k}$. The parameter μ reflects a constant term, which in practice is typically estimated to be close or equal to zero. The order of s is usually 0 or small, indicating that there are usually no opportunities to forecast r_t from its own past. In other words, there is always no auto-regressive process in r_t .

1) ARCH

The GARCH(p,q) model is reduced to the ARCH(q) model when $p = 0$ and at least one of the ARCH parameters must be nonzero ($q > 0$).

2) *Stationary GARCH, SG(p,q)*

If the parameters are constrained such that

$$\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1, \text{ they imply the weakly stationary}$$

GARCH (SG(p,q)) model since the mean, variance and autocovariance are finite and constant over time.

3) *Unconstrained GARCH, UG(p,q)*

The parameter of w , α_i and β_j can be unconstrained, thus yielding the unconstrained GARCH (UG(p,q)) model.

4) *Non-negative GARCH, NG(p,q)*

If $p \geq 0$, $q > 0$ and $w > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, yields the non-negative GARCH (NG(p,q)) model.

5) *Integrated GARCH, IG(p,q)*

Sometimes, the multistep forecasts of the variance do not approach the unconditional variance when the model is integrated in variance; that is

$$\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j = 1. \text{ The unconditional variance for}$$

the IGARCH model does not exist. However, it is interesting that the integrated GARCH or IGARCH (IG(p,q)) model can be strongly stationary even though it is not weakly stationary (Nelson 1990a, b).

6) *Exponential GARCH, EG(p,q)*

The exponential GARCH or EGARCH (EG(p,q)) model was proposed by Nelson (1991). Nelson and Cao (1992) argue that the nonnegativity constraints in the linear GARCH model are too restrictive. The GARCH model imposes the nonnegative constraints on the parameters, α_i and β_j , while there is no restriction on these parameters in the EGARCH model. In the EGARCH model, the conditional variance, h_t , is an asymmetric function of lagged disturbances, ε_{t-i} :

$$\ln(h_t) = w + \sum_{i=1}^q \alpha_i g(Z_{t-i}) + \sum_{j=1}^p \beta_j \ln(h_{t-j})$$

where

$$g(Z_t) = \theta Z_t + \gamma [|Z_t| - E|Z_t|]$$

$$Z_t = \varepsilon_t / \sqrt{h_t}$$

The coefficient of the second term in $g(Z_t)$ is set to be 1 ($\gamma = 1$) in this formulation. Note that $E|Z_t| = (2/\pi)^{1/2}$ if $Z_t \sim N(0,1)$.

7) *GARCH-in-Mean, G(p,q)-M*

The GARCH-in-Mean, G(p,q)-M model has the added regressor that is the conditional standard deviation

$$r_t = \mu + \delta \sqrt{h_t} + \varepsilon_t$$

$$\varepsilon_t = \sqrt{h_t} e_t$$

where h_t follows the GARCH process.

8) *Stationary GARCH-in-Mean, SG(p,q)-M*

This model has the added regressor that is the conditional standard deviation

$$r_t = \mu + \delta \sqrt{h_t} + \varepsilon_t$$

$$\varepsilon_t = \sqrt{h_t} e_t$$

where h_t follows the stationary GARCH, SG(p,q) process.

9) *Unconstrained GARCH-in-Mean, UG(p,q)-M*

This model has the added regressor that is the conditional standard deviation

$$r_t = \mu + \delta \sqrt{h_t} + \varepsilon_t$$

$$\varepsilon_t = \sqrt{h_t} e_t$$

where h_t follows the unconstrained GARCH, UG(p,q) process.

10) *Non-negative GARCH-in-Mean, NG(p,q)-M*

This model has the added regressor that is the conditional standard deviation

$$r_t = \mu + \delta \sqrt{h_t} + \varepsilon_t$$

$$\varepsilon_t = \sqrt{h_t} e_t$$

where h_t follows the non-negative GARCH, NG(p,q) process.

11) *Integrated GARCH-in-Mean, IG(p,q)-M*

This model has the added regressor that is the conditional standard deviation

$$r_t = \mu + \delta\sqrt{h_t} + \varepsilon_t$$

$$\varepsilon_t = \sqrt{h_t}e_t$$

where h_t follows the integrated GARCH, IG(p,q) process.

12) Exponential GARCH-in-Mean, EG(p,q)-M

This model has the added regressor that is the conditional standard deviation

$$r_t = \mu + \delta\sqrt{h_t} + \varepsilon_t$$

$$\varepsilon_t = \sqrt{h_t}e_t$$

where h_t follows the exponential GARCH, EG(p,q) process.

Since a small lag of the GARCH model is sufficient to model the long-memory process of changing variance (French *et al.* 1987; Franses and Van Dijk 1996; Choo *et al.* 1999), the performance of GARCH models in forecasting RM-Sterling exchange rate volatility is evaluated by using SG(1,1), UG(1,1), NG(1,1) IG(1,1), EG(1,1), G(1,1)-M, SG(1,1)-M, UG(1,1)-M, NG(1,1)-M, IG(1,1)-M, and EG(1,1)-M.

DATA AND METHODOLOGY

In this study, simple rate of returns is employed to model the currency exchange rate volatility of RM-Sterling. Consider a foreign exchange rate E_p its rate of return r_p is constructed as $r_t = \frac{E_t - E_{t-1}}{E_{t-1}}$. The exchange rate t denotes daily exchange rate observations.

The foreign exchange rate used in this study is focused on the Malaysian Ringgit (RM) to the Pound Sterling. This exchange rate is chosen because in addition to the US dollar, the Pound Sterling is also one of the major currencies traded in the foreign exchange markets. Traditionally and historically, the UK has always

been one of the important trading partners of Malaysia. The data was collected from 2 January 1990 to 13 March 1997, from 1810 observations. The daily closing exchange rates were used as the daily observations. The first 1760 observations are used for parameters estimation and the last 50 observations reserved for forecasting evaluation.

Fig. 1 shows nearly 1810 daily observer cross rates of the Malaysian Ringgit to the Pound Sterling, covering the seven years from 2 January 1990 to 13 March 1997. Some characteristics of the rate of returns, r , are given in Table 1. The means and variances are quite small. The excess kurtosis indicates the necessity of fat-tailed distribution to describe these variables. The skewness of -0.200 indicates that the distribution of rate of returns for RM-Sterling is negatively skewed.

The family of GARCH models is estimated using the maximum likelihood method. This method enables the rate of return and variance processes being estimated jointly. The log-likelihood function is computed from the product of all conditional densities of the prediction errors.

$$l = \sum_{i=1}^n \frac{1}{2} \left[-\ln(2\pi) - \ln(h_t) - \frac{\varepsilon_t^2}{h_t} \right]$$

where $\varepsilon_t = r_t - \mu$ and h_t is the conditional variance. When the GARCH(p,q)-M model is estimated, $\varepsilon_t = r_t - \mu - \delta\sqrt{h_t}$. When there are no regressors (trend or constant, μ the residuals ε_t are denoted as r_t or $r_t - \delta\sqrt{h_t}$. The likelihood function is maximized via the dual quasi-Newton and trust region algorithm. The starting values for the regression parameters μ are obtained from the OLS estimates. When there are autoregressive parameters in the model, the initial values are obtained from the Yule-Walker

TABLE 1
Summary statistics of currency exchange rate data on rate of returns from 2 January 1990 to 13 March 1997

Currency Exchange Rate	n	Mean (x 10-5)	Variance (x 10-5)	Skewness	Excess Kurtosis
RM/Sterling	1809	-3.183	4.076	-0.200	2.370

Source of data: The Federal Reserve, the Central Bank of the United States

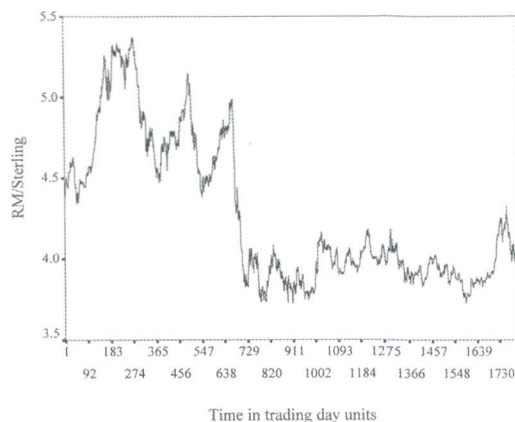


Fig. 1: RM/Sterling, daily from 2 January 1990 to 13 March 1997

estimates. The starting value $1E - 6$ is used for the GARCH process parameters. The variance-covariance matrix is computed using the Hessian matrix. The dual quasi-Newton method approximated the Hessian matrix while the quasi-Newton method gets an approximation of the inverse of Hessian. The trust region method uses the Hessian matrix obtained using numerical differentiation. This algorithm is numerically stable, though computation is expensive.

In order to test for the independence of the indices series, the portmanteau test statistic based on squared residual is used (McLeod and Li 1983). This Q statistic is used to test the non-linear effects, such as GARCH effects, present in the residuals. The GARCH (p,q) process can be considered as an ARMA $(\max(p,q),p)$ process. Therefore, the Q statistic calculated from the squared residuals can be used to identify the order of the GARCH process. The Lagrange multiplier test for ARCH disturbances is proposed by Engle (1982). The test statistic is asymptotically equivalent to the test used by Breusch and Pagan (1979).

The LM and Q statistics are computed from the OLS residuals assuming that disturbance is white noise. The Q and LM statistics have an approximate $(\chi^2_{(q)})$ distribution under the white noise null hypotheses.

Various goodness-of-fit statistics are used to compare the six models in this study. The diagnostics are the mean of square error (MSE), the loglikelihood (Log L), Schwarz's Bayesian information criterion (SBC) by Schwarz (1978) and Akaike's information criterion (AIC) (Judge *et al.* 1985).

The 'true volatility' is measured to evaluate the performance of the six GARCH models in forecasting the volatility in stock returns. As in the studies by Pagan *et al.* (1990) and Day *et al.* (1992), the volatility is measured by

$$v_t = (r_t - \bar{r})^2$$

where \bar{r} is the average return. The measure of the one-step-ahead forecast error is

$$e_{t+1} = v_{t+1} - \hat{h}_{t+1}$$

where \hat{h}_{t+1} is generated using the h_t equations of the GARCH models being studied. The estimated parameters of the GARCH models such as w , α , β , θ and δ are substituted during the generation of \hat{h}_{t+1} . In order to show the performance of GARCH models over a naïve no-change forecast, the forecast errors of the random walk (RW) are calculated as follows:

$$e_{t+1} = v_{t+1} - v_t$$

This is a very important naïve benchmark in the comparison of the forecasts from the GARCH models (Brooks 1997).

RESULTS AND DISCUSSION

Parameter Estimations

The parameter estimates for eleven variations of GARCH models of the rate of returns series are presented in Table 2 (a) and Table 2 (b). These within-sample estimation results enable us to know the possible usefulness of the GARCH

models in modeling the currency exchange rate series.

It can be seen from Table 2(a) that except for μ , all the parameter estimates of the RM/Sterling (w , α and β) are significant at 5% level. However, in Table 2(b), all the two additional parameter estimates (δ and θ) of the EGARCH and all the GARCH models with means are not significant. It appears that for the within-sample estimations, all the family GARCH models perform well in modeling the exchange rate of RM/Sterling.

In general, it can be concluded that almost all α and β (ARCH and GARCH terms) of the RM/Sterling series examined are significant. Hence, the constant variance model can be rejected, at least for the within-sample estimation.

For the linear GARCH models such as SG(1,1), the sum of α and β is close to unity. The properties of $\alpha + \beta = 1$ of IG(1,1) also hold for the series.

Diagnostics Checking

The basic ARCH (q) model is a short memory process in that only the most recent q squared residuals are used to estimate the changing variance. The results for Q statistic and Lagrange Multiplier (LM) test are shown in Table 3. These can help to determine the order of the ARCH process in modeling the RM/Sterling series.

The tests are significant at less than 1% level though order 12. These indicate that the heteroscedasticity terms of the daily RM/Sterling exchange rate series needed to be modeled by a

TABLE 2(a)
Estimation results of rate of returns for the currency exchange rate

Currency Exchange Rate	Model	Parameter estimates			
		δ	t Ratio	θ	t Ratio
RM/Sterling	SG(1,1)				
	UG(1,1)				
	NG(1,1)				
	IG(1,1)				
	EG(1,1)			-0.047	-0.518
	G(1,1)-M	-0.125	-1.305		
	SG(1,1)-M	-0.125	-1.308		
	UG(1,1)-M	-0.125	-1.306		
	NG(1,1)-M	-0.125	-1.306		
	IG(1,1)-M	-0.104	-1.229		
EG(1,1)-M	-0.056	-0.622	-0.093	.	

TABLE 2(b)
Estimation results of rate of returns for the currency exchange rate

Currency Exchange Rate	Model	Parameter estimates							
		μ ($\times 10^{-4}$)	t Ratio	$\omega(\times 10^{-6})$	t Ratio	α	t Ratio	β	t Ratio
RM/Sterling	SG(1,1)	1.6	1.246	0.772	4.749	0.072	9.087	0.910	93.618
	UG(1,1)	1.6	1.243	0.764	4.699	0.072	9.061	0.910	93.697
	NG(1,1)	1.6	1.243	0.764	4.699	0.072	9.06	0.910	93.678
	IG(1,1)	1.42	1.112	0.350	4.842	0.076	10.211	0.924	123.442
	EG(1,1)	1.06	0.787	-291080.0	-2.504	0.162	6.588	0.971	85.153
	G(1,1)-M	8.36	1.559	0.758	4.754	0.071	9.006	0.911	94.63
	SG(1,1)-M	8.38	1.56	0.765	4.802	0.070	9.034	0.911	94.667
	UG(1,1)-M	8.37	1.56	0.756	4.744	0.071	9.005	0.911	94.653
	NG(1,1)-M	8.37	1.56	0.756	4.744	0.071	9.005	0.911	94.646
	IG(1,1)-M	7.11	1.528	0.347	4.904	0.076	10.16	0.924	124.188
	EG(1,1)-M	6.03	.	-294460.0	-2.709	0.163	6.664	0.970	91.004

TABLE 3
Diagnostics for currency exchange rate using Q statistic and Lagrange Multiplier test

Currency Exchange Rate	Diagnostics			
	Q(12)	Prob>Q(12)	LM(12)	Prob>LM(12)
rm/pound	273.447	0.0001	147.373	0.0001

very high order of ARCH model. These results support the use of GARCH model, which allows long memory processes to estimate the current variance of the daily RM/Sterling series instead of the ARCH model.

Goodness of Fit Tests

The result of the goodness-of-fit statistics for the RM/Sterling series is presented in Table 4. Table 5 shows the rankings of various GARCH models.

From Table 5, the ranking of the MSE value indicates that all the family of GARCH in mean models outperform the GARCH models with a slight value of 0.000001. The Log L values however, suggest EG(1,1)-M to be the best model for modeling the volatility of RM/Sterling, followed by UG(1,1)-M, NG(1,1)-M and G(1,1)-M. The SBC values in contrast, ranked indifferently SG(1,1), UG(1,1) and NG(1,1) to be the best model followed by IG(1,1). The AIC values on the other hand, proposed UG(1,1) and NG(1,1) to be the best two models, followed by SG(1,1).

From the goodness-of-fit test, it appears that for within-sample estimations, almost all the GARCH models outperform the GARCH in mean

models in the SBC and AIC test while in the MSE and Log L test, all the GARCH in mean models perform well to model the daily exchange rate compared to their ordinary GARCH model counterparts.

One Step Ahead Forecasting

The good performance in the parameter estimation and goodness-of-fit statistics do not guarantee the good performance in forecasting (Choo *et al.* 1999). The performance of the GARCH models is evaluated through the one-step-ahead forecasting. 50 one-step-ahead forecasts are generated and the mean square error (MSE) is calculated to evaluate the forecasting performance. The results of the forecasting for the GARCH models and the random walk model are shown in Table 6. The rankings of the models based on the performance of the one-step-ahead forecasting are presented in Table 7.

In Table 7, the ranking results of MSE suggest that SG(1,1)-M is the best model for one-step-ahead forecasts, followed by SG(1,1) and G(1,1)-M. It is also noted that, SG(1,1)-M, UG(1,1)-M and NG(1,1)-M clearly outperform

TABLE 4
Goodness-of-fit statistics on rate of returns for the currency exchange rates

Currency Exchange Rate	Model	Goodness-of-Fit Statistics			
		MSE (x10 ⁻⁴)	Log L	SBC	AIC
Rm/pound	SG(1,1)	0.41	6525.371	-13020.9	-13042.7
	UG(1,1)	0.41	6525.414	-13020.9	-13042.8
	NG(1,1)	0.41	6525.414	-13020.9	-13042.8
	IG(1,1)	0.41	6521.151	-13019.9	-13036.3
	EG(1,1)	0.41	6525.729	-13014.1	-13041.5
	G(1,1)-M	0.40	6526.271	-13015.2	-13042.5
	SG(1,1)-M	0.40	6526.232	-13015.1	-13042.5
	UG(1,1)-M	0.40	6526.283	-13015.2	-13042.6
	NG(1,1)-M	0.40	6526.283	-13015.2	-13042.6
	IG(1,1)-M	0.40	6521.992	-13014.1	-13036
	EG(1,1)-M	0.40	6526.674	-13008.5	-13041.3

Modelling the Volatility of Currency Exchange Rate Using GARCH Model

TABLE 5
Rankings of the models averaged across the currency exchange based on the performance of various goodness-of-fit statistics

Model	RM/pound			
	MSE	Log L	SBC	AIC
SG(1,1)	7	9	1	3
UG(1,1)	7	7	1	1
NG(1,1)	7	7	1	1
IG(1,1)	7	11	4	10
EG(1,1)	7	6	9	8
G(1,1)-M	1	4	5	6
SG(1,1)-M	1	5	8	6
UG(1,1)-M	1	2	5	4
NG(1,1)-M	1	2	5	4
IG(1,1)-M	1	10	9	11
EG(1,1)-M	1	1	11	9

TABLE 6
Out-of-sample forecasting performance of various GARCH models and random walk models for the volatility of the currency exchange rates

Model	MSE (x10-9) of one-step-ahead forecast (forecast period = 50)
	RM/pound
SG(1,1)	3.080
UG(1,1)	3.089
NG(1,1)	3.089
IG(1,1)	3.607
EG(1,1)	3.149
G(1,1)-M	3.085
SG(1,1)-M	3.075
UG(1,1)-M	3.087
NG(1,1)-M	3.087
IG(1,1)-M	3.625
EG(1,1)-M	3.150
RW	6.849

TABLE 7
Rankings of the models averaged across the currency exchange rates based on the performance of one-step-ahead forecasting

Model	MSE of one-step-ahead forecast for RM/pound
SG(1,1)	2
UG(1,1)	7
NG(1,1)	6
IG(1,1)	10
EG(1,1)	8
G(1,1)-M	3
SG(1,1)-M	1
UG(1,1)-M	4
NG(1,1)-M	5
IG(1,1)-M	11
EG(1,1)-M	9
RW	12

their ordinary GARCH models counterparts while EG(1,1) and IG(1,1), in contrast, outperform their with mean GARCH counterparts.

In general, almost all the GARCH in mean models outperform the ordinary GARCH models with the exception of EG(1,1) and IG(1,1). However, the family of GARCH models is clearly being proposed instead of their naïve benchmark, the random walk model.

CONCLUSION

Using seven years of daily observed RM/Sterling exchange rate, the performance of GARCH models, including the family of GARCH in mean models to explain the commonly observed characteristics of the unconditional distribution of daily rate of returns series, were examined.

The results indicate that the hypotheses of constant variance model could be rejected, at least within-sample, since almost all the parameter estimates of the ARCH and GARCH models are significant at 5% level.

The Q statistics and the Lagrange Multiplier test reveal that the use of the long memory GARCH model is preferable to the short memory and high-order ARCH model.

The results from various goodness-of-fit statistics are not consistent for RM/Sterling exchange rates. It appears that the SBC and AIC test proposed GARCH models to be the best for within-sample modeling while the MSE and Log L test, suggest the GARCH in mean models to be best to model the heteroscedasticity of daily exchange rates.

The forecasting results show that SG(1,1)-M is the best model for forecasting purpose, followed by SG(1,1) and G(1,1)-M. Almost all the GARCH in mean models outperform the ordinary GARCH models. On the other hand, the family of GARCH models has clearly shown that they perform better than the naïve benchmark, the random walk model.

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