UNIVERSITI PUTRA MALAYSIA

GENERATING MUTUALLY UNBIASED BASES AND DISCRETE WIGNER FUNCTION FOR THREE-QUBIT SYSTEM

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By

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Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia in Fulfilment of the Requirements for the Degree of Master of Science

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Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfillment of the requirement for the degree of Master of Science

GENERATING MUTUALLY UNBIASED BASES AND DISCRETE WIGNER FUNCTION FOR THREE-QUBIT SYSTEM

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May 2011

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In this research, we construct Wigner functions on discrete phase spaces to represent quantum states for the special case of 3-qubit system. For determining this discrete phase space, we label the axes of phase space with finite field (Galois field) having eight elements. Based on this labelling, we developed a program in Matlab software for 3-qubit system which is also extendible for higher dimensions and more number of qubits. Results of this program lead to nine sets of parallel lines which is named as striations. Equivalently we label the horizontal and vertical axes of our phase space by quantum states of 3 qubits. This labelling and the use of suitable translation vectors on our striations produce the nine mutually unbiased bases for the Hilbert space.

For calculating discrete Wigner function, we have determined appropriate quantum net $Q(\lambda)$. There are 8^9 different choices for defining quantum nets but by using some unitary operators we reduce our choices to just eight different choices of quantum nets (eight similarity classes).

We developed another program in Maple, which gives us eight different similarity classes of quantum nets. This program also gives us the phase-space point

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operators A_α related to each class of quantum nets (these A_α operators are used in calculating Wigner function). Based on one of these similarity classes, we calculated the Wigner functions for GHZ and W states and also an embedded Bell state which is not completely entangled. We discussed some of the properties of these Wigner functions.

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Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Master Sains

PENJANAAN ASAS SALING SAKSAMA DAN FUNGSI WIGNER DISKRIT UNTUK SISTEM TIGA-QUBIT

Oleh

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Dalam Penyelidikan ini, dibangunkan fungsi Wigner pada ruang fasa diskrit untuk mewakili keadaan kuantum untuk kes khusus sistem 3-qubit. Bagi menentukan ruang fasa diskrit ini, dilabelkan paksi ruang fasa dengan medan terhingga (medan Galois) yang mempunyai lapan unsur. Berdasarkan penglabelan ini, satu program dalam perisian Matlab telah dibangunkan yang juga dapat dipanjangkan ke dimensi lebih tinggi dan bilangan qubit yang lebih banyak. Keputusan program menghasilkan sembilan set garis selari yang dinamakan jaluran. Secara setara dilabelkan paksi mengufuk dan menegak ruang fasa dengan keadaan kuantum 3 qubit. Penglabelan ini dan penggunaan vektor traslasi yang bersesuaian ke atom jaluran menghasilkan sembilan asas saling saksama bagi ruang Hilbert.

Bagi mengira fungsi Wigner diskrit, kami telah tentukan jaring kuantum $Q(\lambda)$ yang bersesuai. Terdapat 8^9 pilihan berbeza bagi menakrifkan jaring kuantum tetapi dengan menggunakan operator unitari, kami telah mengurangkan pilihan tersebut ke hanya lapan pilihan jaring kuantum yang berbeza (lapan kelas keserupaan).

Kami bangunkan program lain dalam Maple yang memberikan lapan kelas keseru-

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paan jaring kuantum. Program ini juga memberikan operator titik ruang fasa A_α yang berkaitan dengan setiap kelas jaring kuantum (operator A_α ini diguna untuk mengira fungsi Wigner). Berdasarkan salah satu kelas keserupaan, dikirakan fungsi Wigner untuk keadaan GHZ dan W dan juga keadaan bell terbenam yang tidak terbelit selengkapnya. Dibincangkan beberapa ciri fungsi Wigner ini.

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I certify that a Thesis Examination Committee has met on 12 May 2011 to conduct the final examination of Mojtaba Aliakbarzadeh on his thesis entitled "GENERATING MUTUALLY UNBIASED BASES AND DISCRETE WIGNER FUNCTIONS FOR THREE-QUBIT SYSTEM" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Master of Science.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

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