Pertanika 12(3), 413-419 (1989)

An Iterative Explicit Method for Parabolic Problems with Cylindrical Symmetry-Increased Accuracy on Non-Uniform Grid

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Key words : AGE method, cylindrical problem, non-uniform grid.

ABSTRAK

Di dalam makalah ini, kaedah berlelar tidak tersirat kumpulan berselang-seli (TTKS) digunakan untuk menyelesaikan masalah silinder yang melibatkan domain sekata pada sistem grid yang tidak seragam. Tatacara ini menggunakan strategi pecah-belah berperingkat secara berselang-seli pada setiap setengah paras masa. Kaedah ini diterapkan ke atas sistem tiga pepenjuru persamaan beza. Ternyata kaedah ini lebih jitu daripada kaedah TTKS yang sepadan yang digunakan bagi masalah yang sama tetapi pada sistem grid seragam. Walau bagaimanapun, julat kestabilannya terhad.

ABSTRACT

In this paper, the alternating group explicit (AGE) iterative method is applied to cylindrical problems involving regular domains on a non-uniform grid. The procedure uses the fractional splitting strategy which is applied alternately at each half (intermediate) time step on a tridiagonal system of difference equations. The method is shown to be more accurate than the corresponding AGE scheme solved earlier by the authors using an uniform grid system but with a reduced stability range.

1. INTRODUCTION

Consider the following equation in one-space dimension given by,

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} \qquad (1.1)$$

together with the initial-boundary conditions,

$$U(r, 0) = f(r), \; 0 \le r \le 1 \qquad (1.2)$$

$$\frac{\partial U}{\partial r}(0, t) = 0, \; U(1, t) = 0, \; 0 \le t \le T.$$

In Sahimi and Muda (1988), the cylindrical equation (1.1) was approximated by standard finite difference analogues on the usual uniformly-spaced network whose mesh points were $r_i = i\Delta r$, $t_j = j\Delta t$. The truncation error, however, always contains low order derivatives $\frac{\partial U}{\partial r}$ and $\frac{\partial^2 U}{\partial r^2}$. To overcome this, the transformation procedure of Mitchell and Pearce (1963),

$$x = \frac{1}{4} r^2 \qquad (1.3)$$

can be used. The "accuracy difficulty" in the neighbourhood of the axis may be avoided to some extent by considering a rectangular net which is uniformly spaced in the t-direction given by $t_j = j\Delta t$ and unequally spaced in the x-direction indicated by $x_i = i^2\Delta x$. The latter is consistent with equal spacing in the r-direction. The transformation of (1.3) converts (1.1) to

$$\frac{\partial U}{\partial t} = \frac{\partial U}{\partial x} + x\frac{\partial^2 U}{\partial x^2}, \; 0 \le x \le \frac{1}{4}. \qquad (1.4)$$

The higher time derivatives no longer contain low derivatives of U with respect to x. The AGE algorithm can now be applied to the resulting implicit replacement of (1.4).

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2. PROBLEM FORMULATION

Following Mitchell and Pearce (1963) an optimum four point implicit finite difference approximation (the MP approximation) to (1.4) is given by

$$\left[1 + 2\lambda\left(i^2 - \lambda - 1\right)\left[\left(4i^2 - 1\right)\right]^{-1}\right]u_{i,j}$$

$$+\lambda\left(-2i^2 - 2i + 2\lambda + 1\right)\left[4i(2i+1)\right]^{-1}u_{i+1,j}$$

$$+\lambda\left(-2i^2 + 2i + 2\lambda + 1\right)\left[4i(2i-1)\right]^{-1}u_{i-1,j}$$

$$= u_{i,j-1}$$

$$i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \qquad (2.1)$$

while for points on the axis, we have

$$u_{0,j} = \frac{1}{4}\left(4 - 5\lambda + 2\lambda^2\right)u_{0,j-1} + \frac{2}{3}\lambda$$

$$(\lambda - 2)\,u_{1,j-1} + \frac{\lambda}{12}(2\lambda - 1)\,u_{2,j-1}$$

$$\text{for } j = 1, 2, \ldots, n \qquad (2.2)$$

where

$$\Delta x = \frac{1}{4\,(m+1)^2}, \quad \Delta t = \frac{T}{n} \quad \text{and} \quad \lambda = \frac{\Delta t}{\Delta x},$$

the mesh ratio.

The matrix representation of (2.1) thus takes the form

$$\begin{bmatrix} a_1 & b_1 & & & \\ c_2 & a_2 & b_2 & & \\ & c_3 & a_3 & b_3 & \\ & & \ddots & \ddots & \ddots \\ & & c_{m-1} & a_{m-1} & b_{m-1} \\ & & & c_m & a_m \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{m-1} \\ u_m \end{bmatrix}_j = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_{m-1} \\ f_m \end{bmatrix}$$

or

$$A\underset{\sim}{u}_j = \underset{\sim}{f} \qquad (2.3)$$

where

$$a_i = 1 + 2\lambda\left(i^2 - \lambda - 1\right)\left(4i^2 - 1\right)^{-1}, \quad i = 1, 2, \ldots, m$$

$$c_i = \lambda\left(-2i^2 + 2i + 2\lambda + 1\right)\left[4i(2i-1)\right]^{-1}, \quad i = 2, 3, \ldots, m$$

$$b_i = \lambda\left(-2i^2 - 2i + 2\lambda + 1\right)\left[4i(2i+1)\right]^{-1}, \quad i = 1, 2, \ldots, m-1$$

$$f_1 = u_{1,j-1} - \frac{\lambda}{4}(1 + 2\lambda)\,u_{0,j};$$

$$f_i = u_{i,j-1}, \quad i = 2, 3, \ldots, m-1;$$

$$f_m = u_{m,j-1} - \lambda\left(-2m^2 - 2m + 2\lambda + 1\right)\left[4m(2m+1)\right]^{-1}u_{m+1,j}$$

with $u_{o,j}$ and $u_{m+1,j}$ corresponding to the left and right boundary values respectively.

Without loss of generality, let us assume that we have an odd number of internal mesh points. We can then perform the following splitting of A:

$$A = G_1 + G_2$$

$$G_1 = \begin{bmatrix} \frac{1}{2}a_1 & & & & & \\ & \frac{1}{2}a_2 & b_2 & & & \\ & c_3 & \frac{1}{2}a_3 & & & \\ & & & \ddots & & \\ & & & & \frac{1}{2}a_{m-1} & b_{m-1} \\ & & & & c_m & \frac{1}{2}a_m \end{bmatrix}_{(m \times m)}$$

and

$$G_2 = \begin{bmatrix} \frac{1}{2}a_1 & b_1 & & & & & \\ c_2 & \frac{1}{2}a_2 & & & & & \\ & & \frac{1}{2}a_3 & b_3 & & & \\ & & c_4 & \frac{1}{2}a_4 & & & \\ & & & & \ddots & & \\ & & & & & \frac{1}{2}a_{m-2} & b_{m-2} \\ & & & & & c_{m-1} & \frac{1}{2}a_{m-1} \\ & & & & & & & \frac{1}{2}a_m \end{bmatrix}_{(m \times m)}$$

Following Evans and Sahimi (1987), the following iterative AGE convergent scheme was derived,

$$\left(G_1 + \hat{r}\,I\right)\underset{\sim}{u}^{\left(k+\frac{1}{2}\right)} = \left(\hat{r}\,I - G_2\right)\underset{\sim}{u}^{(k)} + \underset{\sim}{f}$$

$$\left(G_2 + \hat{r}\,I\right)\underset{\sim}{u}^{(k+1)} = \left(G_2 - (1 - w)\,\hat{r}\,I\right)\underset{\sim}{u}^{(k)}$$

$$+ (2 - w)\,\hat{r}\,\underset{\sim}{u}^{\left(k+\frac{1}{2}\right)} \qquad (2.4)$$

for any $0 \le w \le 2$ and $\hat{r} > 0$ being a fixed acceleration parameter along each intermediate (half-time) level or iterate. w = 0 leads to the Peaceman-Rachford (PR) scheme and w = 1 gives us the variant due to Douglas and Rachford (DR).

Both stable schemes have truncation errors of the order $T_{PR} = 0((\Delta r)^2 + (\Delta t)^2)$ and $T_{DR} = 0((\Delta r)^2) + \Delta t)$ respectively.

Defining

$$\hat{G}^{(i)} = \alpha_i^{-1} \begin{pmatrix} \frac{1}{2}a_{2i+1} + \hat{r} & -b_{2i} \\ -c_{2i+1} & \frac{1}{2}a_{2i} + \hat{r} \end{pmatrix},$$

$$\hat{\hat{G}}^{(i)} = \hat{\alpha}_i^{-1} \begin{pmatrix} \frac{1}{2}a_{2i+1} + \hat{r} & -b_{2i-1} \\ -c_{2i} & \frac{1}{2}a_{2i-1} + \hat{r} \end{pmatrix},$$

We have

$$(G_1 + \hat{r}I)^{-1} = \begin{bmatrix} 1/\left(\frac{1}{2}a_1 + \hat{r}\right) & & & \bigcirc \\ & \hat{G}^{(1)} & & \\ & & \ddots & \\ \bigcirc & & & \hat{G}^{\frac{1}{2}(m-1)} \end{bmatrix}_{(m \times m)}, \quad (2.5)$$

$$(G_1 + \hat{r}I)^{-1} = \begin{bmatrix} \hat{\hat{G}}^{(1)} & & & & \bigcirc \\ & \hat{\hat{G}}^{(2)} & & & \\ & & \ddots & & \\ \bigcirc & & & \hat{\hat{G}}^{\frac{1}{2}(m-1)} & \\ & & & & 1/\left(\frac{1}{2}a_m + \hat{r}\right) \end{bmatrix}, \quad (2.6)$$

where

$$\left.\begin{aligned} \alpha_i &= \left(\frac{1}{2}a_{2i} + \hat{r}\right)\left(\frac{1}{2}a_{2i+1} + \hat{r}\right) - b_{2i}c_{2i+1}, \\ \text{and } \hat{\alpha}_i &= \left(\frac{1}{2}a_{2i-1} + \hat{r}\right)\left(\frac{1}{2}a_{2i} + \hat{r}\right) - b_{2i-1}c_{2i} \end{aligned}\right| \; i = 1, 2, \ldots, \frac{1}{2}(m-1)$$

3. COMPUTATIONAL ASPECT OF THE AGE SCHEME

Using (2.4) – (2.6), the u-vales at each of the half-iterates can be computed as follows:

(1) at the $\left(k + \frac{1}{2}\right)$th iterate

$$u_i^{\left(k+\frac{1}{2}\right)} = \left(s_1 u_1^{(k)} - b_1 u_2^{(k)} + f_1\right)/\bar{s}_1$$

$$\left.\begin{aligned} u_i^{\left(k+\frac{1}{2}\right)} &= \left(A_i u_{i-1}^{(k)} + B_i u_i^{(k)} + C_i u_{i+1}^{(k)} + D_i u_{i+2}^{(k)} + E_i\right)/\alpha_{i/2} \\ u_{i+1}^{\left(k+\frac{1}{2}\right)} &= \left(\tilde{A}_i u_{i-1}^{(k)} + \tilde{B}_i u_i^{(k)} + \tilde{C}_i u_{i+1}^{(k)} + \tilde{D}_i u_{i+2}^{(k)} + \tilde{E}_i\right)/\alpha_{i/2} \end{aligned}\right] \; i = 2, 4, \ldots, m-1 \qquad (3.1)$$

where

$$A_i = -c_i\bar{s}_{i+1}, \quad \tilde{A}_i = c_i c_{i+1}$$

$$B_i = s_i\bar{s}_{i+1}, \quad \tilde{B}_i = -c_{i+1}s_i$$

$$C_i = -b_i s_{i+1}, \quad \tilde{C}_i = \bar{s}_i s_{i+1}$$

$$D_i = \begin{cases} 0 & \text{if } i = m-1 \\ b_i b_{i+1} & \text{otherwise'} \end{cases}$$

$$\tilde{D}_i = \begin{cases} 0 & \text{if } i = m-1 \\ -b_{i+1}\bar{s}_i & \text{otherwise} \end{cases}$$

$$E_i = f_i\bar{s}_{i+1} - b_i f_{i+1} \quad , \quad \tilde{E}_i = f_{i+1}\bar{s}_i - c_{i+1}f_i$$

with

$$\bar{s}_i = \hat{r} + \frac{1}{2}a_i, \; s_i = \hat{r} - \frac{1}{2}a_i, \; i = 1, 2, \ldots, m.$$

(2) at the (k + 1) th iterate

$$\left.\begin{aligned} u_i^{(k+1)} &= \left(P_i u_i^{(k)} + Q_i u_{i+1}^{(k)} + R_i u_i^{\left(k+\frac{1}{2}\right)} + S_i u_{i+1}^{\left(k+\frac{1}{2}\right)} \right) / \hat{\alpha}_{(i+1)/2} \\ u_{i+1}^{(k+1)} &= \left(\tilde{P}_i u_i^{(k)} + \hat{Q}_i u_{i+1}^{(k)} + \hat{R}_i u_i^{\left(k+\frac{1}{2}\right)} + \hat{S}_i u_{i+1}^{\left(k+\frac{1}{2}\right)} \right) / \hat{\alpha}_{(i+1)/2} \end{aligned}\right] \; i = 1, 3, \ldots, m-2$$

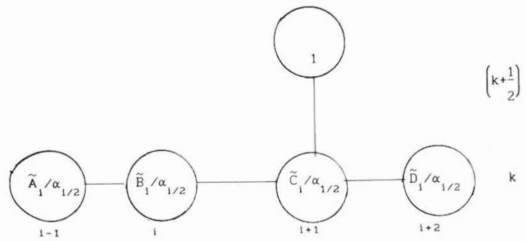
$$u_m^{(k+1)} = \left(q_m u_m^{(k)} + d u_m^{\left(k+\frac{1}{2}\right)} \right) / \bar{s}_m \qquad (3.2)$$

where

$$P_i = \bar{s}_{i+1} q_i - b_i c_{i+1}, \quad \tilde{P}_i = c_{i+1}(\bar{s}_i - q_i)$$

$$Q_i = b_i(\bar{s}_{i+1} - q_{i+1}), \quad \tilde{Q}_i = \bar{s}_i q_{i+1} - c_{i+1} b_i$$

$$R_i = \bar{s}_{i+1} d, \; \tilde{R}_i = -c_{i+1} d, \; S_i = -b_i d, \; \tilde{S}_i = \tilde{s}_i d$$



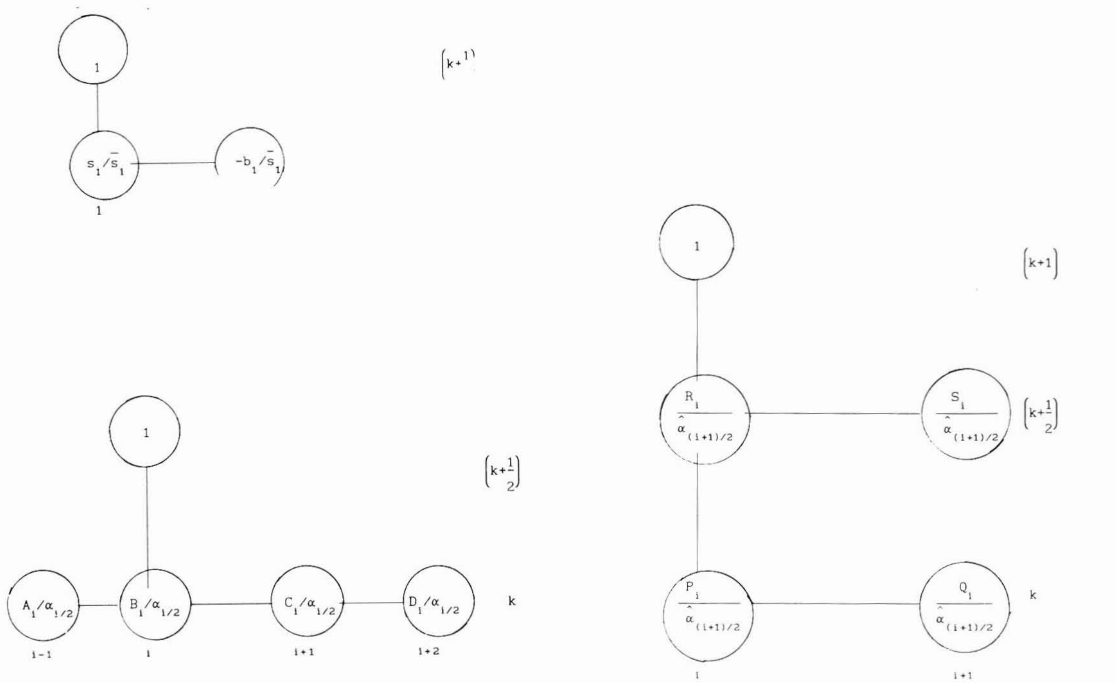


Fig. 1 Computational molecules at level $\left(k + \frac{1}{2}\right)$

TABLE 1
The absolute errors of the numerical solutions to the cylindrical problem
$q = 0.7,\ t = 0.175,\ \Delta x = 0.0025,\ \Delta t = 0.00175,\ \hat{r} = 0.7$

		r	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
Method		x	0.0000	0.0025	0.0100	0.0225	0.0400	0.0625	0.0900	0.1225	0.1600	0.2025	Number of iterations
On Non-Uniform Grid	THOMAS-MP		9.1×10^{-4}	9.2×10^{-4}	9.3×10^{-4}	9.4×10^{-4}	9.5×10^{-4}	9.3×10^{-4}	8.7×10^{-4}	7.6×10^{-4}	5.7×10^{-4}	3.1×10^{-4}	–
	AGE-MP	PR	9.6×10^{-4}	9.9×10^{-4}	9.6×10^{-4}	9.6×10^{-4}	9.6×10^{-4}	9.4×10^{-4}	8.8×10^{-4}	7.6×10^{-4}	5.7×10^{-4}	3.1×10^{-4}	2
		DR	3.5×10^{-3}	3.4×10^{-3}	3.4×10^{-3}	3.1×10^{-3}	2.7×10^{-3}	2.2×10^{-3}	1.7×10^{-3}	1.1×10^{-3}	7.0×10^{-3}	3.0×10^{-4}	6
On Uniform Grid	THOMAS-IMP		3.9×10^{-3}	3.8×10^{-3}	3.6×10^{-3}	3.4×10^{-3}	3.0×10^{-3}	2.6×10^{-3}	2.1×10^{-3}	1.6×10^{-3}	1.1×10^{-3}	5.5×10^{-4}	–
	AGE-IMP	PR	3.9×10^{-3}	3.8×10^{-3}	3.7×10^{-3}	3.4×10^{-3}	3.0×10^{-3}	2.6×10^{-3}	2.1×10^{-3}	1.6×10^{-3}	1.1×10^{-3}	5.5×10^{-4}	3
		DR	8.6×10^{-3}	8.6×10^{-3}	8.2×10^{-3}	7.6×10^{-3}	6.8×10^{-3}	5.9×1^{-3}	4.8×10^{-3}	3.6×10^{-3}	2.4×10^{-3}	1.1×10^{-3}	6
	THOMAS-CN		2.0×10^{-3}	2.0×10^{-3}	1.9×10^{-3}	1.8×10^{-3}	1.6×10^{-3}	1.4×10^{-3}	1.1×10^{-3}	8.9×10^{-4}	6.0×10^{-4}	3.1×10^{-4}	–
	AGE-CN	PR	2.0×10^{-3}	2.0×10^{-3}	1.9×10^{-3}	1.8×10^{-3}	1.6×10^{-3}	1.4×10^{-3}	1.1×10^{-3}	8.9×10^{-4}	6.0×10^{-4}	3.1×10^{-4}	3
		DR	7.0×10^{-3}	6.9×10^{-3}	6.6×10^{-3}	6.1×10^{-3}	5.5×10^{-3}	4.7×10^{-3}	3.8×10^{-3}	2.9×10^{-3}	1.9×10^{-3}	9.6×10^{-4}	6
	THOMAS-DGE		2.8×10^{-3}	2.8×10^{-3}	2.2×10^{-3}	1.7×10^{-3}	1.3×10^{-3}	9.4×10^{-4}	6.4×10^{-4}	4.0×10^{-4}	2.1×10^{-4}	7.6×10^{-5}	–
	AGE-DGE	PR	2.8×10^{-3}	2.8×10^{-3}	2.2×10^{-3}	1.7×10^{-3}	1.3×10^{-3}	9.3×10^{-4}	6.3×10^{-4}	4.0×10^{-4}	2.1×10^{-4}	7.5×10^{-5}	2
		DR	2.0×10^{-3}	1.9×10^{-3}	2.3×10^{-3}	2.5×10^{-3}	2.5×10^{-3}	2.3×10^{-3}	2.0×10^{-3}	1.6×10^{-3}	1.1×10^{-3}	5.7×10^{-4}	6
EXACT SOLUTION			0.36341707	0.35818100	0.34269894	0.31763846	0.28407656	0.24344811	0.19747738	0.14809604	0.09735172	0.04731180	–

TABLE 2

The absolute errors of the numerical solutions to the cylindrical problem

$q = 1.0,\ t = 0.25,\ \Delta x = 0.0025,\ \Delta t = 0.0025,\ r = 0.3$

		r	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
Method		x	0.0000	0.0025	0.0100	0.0225	0.0400	0.0625	0.0900	0.1225	0.1600	0.2025	Number of iterations
On Non-Uniform Grid	THOMAS-MP		1.6×10^{-3}	1.6×10^{-3}	1.3×10^{-3}	1.2×10^{-3}	1.1×10^{-3}	1.0×10^{-3}	9.7×10^{-4}	8.1×10^{-4}	5.9×10^{-4}	3.1×10^{-4}	–
	AGE-MP	PR	3.0×10^{-3}	3.2×10^{-3}	1.8×10^{-3}	1.5×10^{-3}	1.3×10^{-3}	1.2×10^{-3}	1.0×10^{-3}	8.7×10^{-4}	6.3×10^{-4}	3.3×10^{-4}	4
		DR	6.2×10^{-2}	6.9×10^{-2}	1.8×10^{-2}	1.0×10^{-2}	6.4×10^{-3}	4.1×10^{-3}	2.7×10^{-3}	1.7×10^{-3}	1.0×10^{-3}	5.2×10^{-4}	7
On Uniform Grid	THOMAS-IMP		4.3×10^{-3}	4.3×10^{-3}	4.1×10^{-3}	3.8×10^{-3}	3.4×10^{-3}	2.9×10^{-3}	2.4×10^{-3}	1.8×10^{-3}	1.2×10^{-3}	6.0×10^{-4}	–
	AGE-IMP	PR	4.2×10^{-3}	4.2×10^{-3}	4.0×10^{-3}	3.8×10^{-3}	3.4×10^{-3}	2.9×10^{-3}	2.4×10^{-3}	1.8×10^{-3}	1.2×10^{-3}	5.9×10^{-4}	3
		DR	8.1×10^{-3}	8.2×10^{-3}	7.8×10^{-3}	7.3×10^{-3}	6.5×10^{-3}	5.6×10^{-3}	4.6×10^{-3}	3.4×10^{-3}	2.3×10^{-3}	1.1×10^{-3}	6
	THOMAS-CN		1.9×10^{-3}	1.9×10^{-3}	1.8×10^{-3}	1.6×10^{-3}	1.5×10^{-3}	1.3×10^{-3}	1.0×10^{-3}	8.2×10^{-4}	5.5×10^{-4}	2.8×10^{-4}	–
	AGE-CN	PR	1.9×10^{-3}	1.9×10^{-3}	1.8×10^{-3}	1.7×10^{-3}	1.5×10^{-3}	1.3×10^{-3}	1.1×10^{-3}	8.5×10^{-4}	5.7×10^{-4}	2.9×10^{-4}	3
		DR	6.2×10^{-3}	6.2×10^{-3}	5.9×10^{-3}	5.5×10^{-3}	4.9×10^{-3}	4.2×10^{-3}	3.4×10^{-3}	2.6×10^{-3}	1.7×10^{-3}	8.5×10^{-4}	6
	THOMAS-DGE		2.3×10^{-3}	2.3×10^{-3}	1.9×10^{-3}	1.5×10^{-3}	1.2×10^{-3}	9.3×10^{-4}	6.7×10^{-4}	4.5×10^{-4}	2.6×10^{-4}	1.1×10^{-4}	–
	AGE-DGE	PR	2.2×10^{-3}	2.2×10^{-3}	1.8×10^{-3}	1.4×10^{-3}	1.1×10^{-3}	8.7×10^{-4}	6.2×10^{-4}	4.1×10^{-4}	2.4×10^{-4}	9.9×10^{-5}	3
		DR	2.1×10^{-3}	2.1×10^{-3}	2.3×10^{-3}	2.4×10^{-3}	2.3×10^{-3}	2.1×10^{-3}	1.8×10^{-3}	1.4×10^{-3}	9.5×10^{-4}	4.8×10^{-4}	6
EXACT SOLUTION			0.23550905	0.23211586	0.22208286	0.20584265	0.18409317	0.15776428	0.12797338	0.95972263	0.63087877	0.30659974	–

Fig 2 continued

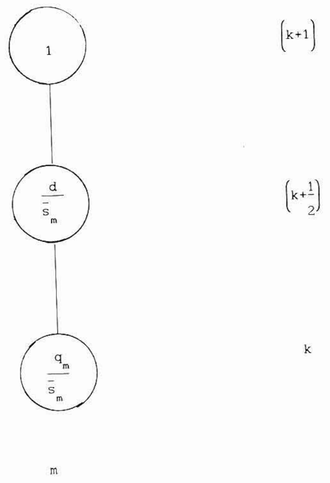


Fig. 2 Computational molecules at level (k + 1)

The computational molecules at each of the half-iterates are shown in *Figs. 1* and *2* :

Solutions are obtained along each time level using (2.2) followed by the application of the AGE algorithm which is executed explicitly utilising equations (3.1) – (3.2) at the $(k + \frac{1}{2})^{th}$ and $(k + 1)^{th}$ iterates in alternate sweeps until a specified convergence criterion is satisfied.

4. NUMERICAL RESULTS

The cylindrical problem (1.1) was considered and the same boundary conditions were used. The initial condition, however, is specifically given by

$$U(r,0) = J_0(\beta r)$$

where $J_0(\beta r)$ is the Bessel function of the first kind of order 0 and β is the first root of $J_0(\beta) = 0$. The exact solution is $U(r,t) = J_0(\beta r) \exp(-\beta^2 t)$ (Mitchell and Pearce (1963)). The convergence criterion employed is $\| \tilde{x}^{(k+1)} - \tilde{x}^{(k)} \|_\infty \leq 10^{-4}$ and the acceleration parameter $\hat{r}$ was chosen to provide the most rapid convergence.

The MP approximation of (2.1) was solved using both the AGE scheme and the Thomas elimination procedure. The solutions were compared with the results obtained from the application of the two schemes on the fully implicit (IMP), the Crank-Nicolson (CN) and the Douglas-equivalent (DGE) approximations to (1.1) (Sahimi and Muda (1988)) for different mesh ratios. Tables 1 and 2 indicate the accuracy of these methods.

It is evident that the AGE-MP(PR) scheme can have comparable accuracy with the most accurate of the standard methods. However, for points near the axis, the AGE-MP scheme is clearly seen to be more superior. It is somewhat restricted by the stability requirement of $\lambda \leq \sqrt{1.5}$. Nevertheless, it has the advantage that being explicit, a parallel algorithm can be developed for computation and although iterative in nature, it requires a small number of iterations for convergence.

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(Received 21 June, 1989)