



UNIVERSITI PUTRA MALAYSIA

**EFFICIENCY MEASUREMENT OF DECISION MAKING UNITS WITH
SERIES AND PARALLEL STRUCTURES BY DATA ENVELOPMENT
ANALYSIS**

ALI ASHRAFI

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ANALYSIS**

By

ALI ASHRAFI

**Thesis Submitted to the School of Graduate Studies, Universiti Putra Malaysia,
in Fulfilment of the Requirements for the Degree of Doctor of Philosophy**

December 2011

DEDICATION

I would like to dedicate my thesis to my dear family, my sweet daughter and my beloved spouse who always inspired and supported me to achieve this goal.



Abstract of thesis presented to the Senate of Universiti Putra Malaysia in fulfilment
of the requirement for the degree of Doctor of Philosophy

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Data Envelopment Analysis (DEA) is a well-known technique for measuring the efficiency of production processes called decision-making units (DMUs). One of the drawbacks of the standard DEA models is the neglect of internal activities within a DMU. In these models, each DMU is treated as a black box in which the efficiency is assessed by considering the initial inputs and final outputs of the DMU.

In this research, the internal structure of DMUs in the DEA model is considered. Knowing about the internal structure of DMUs provides more insights for efficiency evaluation. Because of their basic structures, it is focused on DMUs, whose internal structure is composed of processes connected in series or parallel. Since DEA models in the literature about multi-stage systems have limitations in efficiency measurement, new DEA models are proposed to overcome them.

Current multi-stage DEA models utilize the radial measure of efficiency that results in two limitations: firstly, they assume proportional change of inputs or outputs and secondly, the models disregard the existence of slacks in efficiency measurement. Also, in series multi-stage DEA models, the usual method of adjusting the inputs or outputs by the efficiency scores cannot project the inefficient DMUs onto an efficient frontier. To overcome these drawbacks, non-radial DEA models for series and parallel systems in the slacks-based measures (SBM) framework are proposed. These models are suitable for measuring efficiencies when inputs and outputs change non-proportionally and cover all the inefficiencies including input and output slacks. By this model, the DEA frontier points of inefficient DMUs are determined. Furthermore, multi-stage hybrid DEA models in the presence of radial and non-radial inputs/outputs are introduced which integrate both radial and non-radial assessments in a unified manner.

The existing DEA models for series systems do not provide the efficiency formulation and linearized transformation if the sharing inputs/outputs exist. In this research, a DEA model is introduced for such systems in which the linearity of the DEA model is preserved. It is significant that unlike the existing DEA models, the proposed model is applicable for both constant returns to scale (CRS) and variable returns to scale (VRS) assumptions.

The exact known inputs and outputs are required in the available multi-stage DEA models. However, in the real world, the concern is systems with interval (bounded) data. So, these DEA models do not preserve their linearity. To solve this problem, we

proposed an alternative approach that transforms the non-linear DEA model into an equivalent linear model. Also, the interval DEA models are proposed to measure the lower and upper bounds of the efficiency of each DMU with interval data.

Furthermore, to compute technical, scale and mix efficiencies of series and parallel systems, DEA models are proposed. Consequently, the local returns to scale (RTS) and the sources of inefficiencies in these systems can be determined. Also, assuming the costs and prices of inputs and outputs are known, the overall cost, allocative, revenue and profit efficiencies of series and parallel systems are computed.

Finally, numerical illustrations are provided for each model to explain the proposed models and allow comparison with recent related DEA models.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**PENGUKURAN KECEKAPAN UNIT PEMBUAT KEPUTUSAN
BERSTRUKTUR SIRI DAN SELARI DENGAN ANALISIS
PENYAMPULAN DATA**

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Analisis Penyampulan Data (DEA) ialah teknik yang terkenal bagi mengukur kecekapan proses pengeluaran yang dikenali sebagai unit pembuat keputusan (DMU). Satu daripada kelemahan model DEA standard ialah pengabaian aktiviti dalaman di dalam satu DMU. Dalam model-model ini, setiap DMU dianggap sebagai satu kotak hitam dengan kecekapan dinilai dengan mempertimbangkan nilai awal input dan output DMU.

Dalam penyelidikan ini, struktur dalaman DMU dalam model DFA dipertimbangkan. Mengetahui tentang struktur dalaman DMU menyediakan lebih pemahaman bagi penilaian kecekapan. Oleh kerana struktur asas DMU ini, maka difokuskan kepada DMU yang berstruktur dalaman mengandungi proses yang dihubungkan secara siri dan selari. Oleh kerana dalam kesusasteraan tentang system pelbagai tahap terdapat

pembatasan dalam pengukuran kecekapan, model DEA yang baharu dicadangkan bagi mengatasi pembatasan tersebut.

Model DEA semasa menggunakan ukuran kecekapan radial yang mengakibatkan dua pembatasan: pertamanya, diandaikan pertukaran berkadar bagi input dan output model ini dan keduanya, kewujudan nilai lalai diabaikan dalam pengukuran kecekapan. Juga, dalam model DEA siri pelbagai tahap, kaedah biasa bagi menyesuaikan input dan output dengan skor kecekapan tidak dapat melanjutkan DMU tidak cekap ke atas sempadan kecekapan. Bagi mengatasi kelemahan ini, model DEA tak-radial bagi sistem siri dan selari dalam rangka kerja ukuran berasaskan nilai lalai (SBM) diketengahkan. Model-model ini adalah sesuai bagi mengukur kecekapan apabila pertukaran input dan output tidak-berkadar dan meliputi kesemua ketidakcekapan termasuk nilai lalai input dan output. Dengan model ini, titik-titik sempadan DEA bagi ketidakcekapan DMU ditentukan. Selanjutnya, model DEA hibrid dengan kehadiran input/output radial dan tak-radial diperkenalkan bagi menggabungkan kedua-dua penilaian radial dan tak-radial dalam keadaan yang tersusun.

Model DEA tersedia bagi system siri tidak menyediakan perumusan kecekapan dan penjelamaan linear jika wujudnya pengkongisian input/output. Dalam penyelidikan ini, model DEA diperkenalkan bagi sistem tersebut dengan dikekalkan kelinearan model DEA itu. Perlu dijelaskan bahawa bahawa model yang dicadangkan ini boleh diaplikasikan kepada kedua-dua andaian kembalikan skara malar (CRS) and kembalikan skalar bolehubah (VRS), tidak seperti model-model DEA yang sedia ada.

Input dan output tepat diperlukan dalam model pelbagai-tahap DEA tersedia. Namun, dalam dunia nyata, sistem dengan data berselang (berbatas) diambil perhatian. Jadi, model DEA tidak mengekalkan kelinearannya. Bagi menyelesaikan masalah ini, pendekatan alternatif dicadangkan yang menjelmakan model linear tak-linear kepada aturcara linear yang setara. Juga, model DEA selang dicadangkan bagi mengukur batas bawah dan atas kecekapan bagi setiap DMU dengan data selang.

Selanjutnya, model DEA bagi mengira kecekapan teknikal, skalar dan campuran sistem siri dan selari diketengahkan. Akibatnya, kembalian setempat skalar (RTS) dan sumber ketidakcekan dalam system ini ditentukan. Juga, dengan andaian kos dan harga input dan output diketahui, kecekapan kos keseluruhan, pengagihan, pulangan dan keuntungan sistem siri dan selari bolehh dikira.

Akhir sekali, ilustrasi berangka disediakan bagi setiap model bagi menjelaskan model-model yang dicadangkan dan sebagai perbandingan dengan model DEA semasa yang berkaitan.

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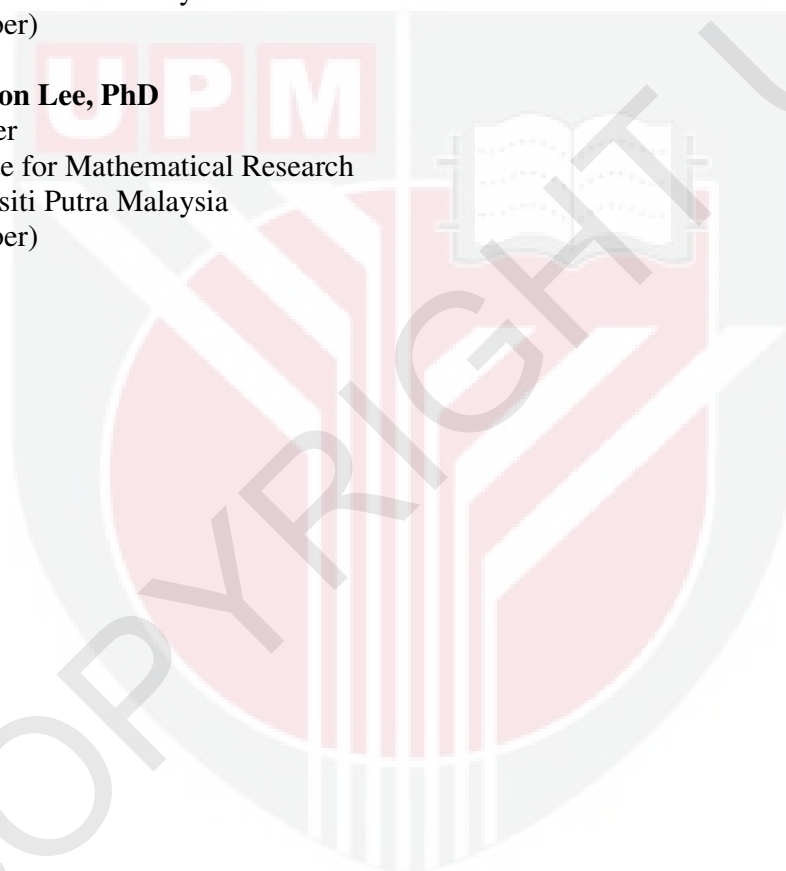
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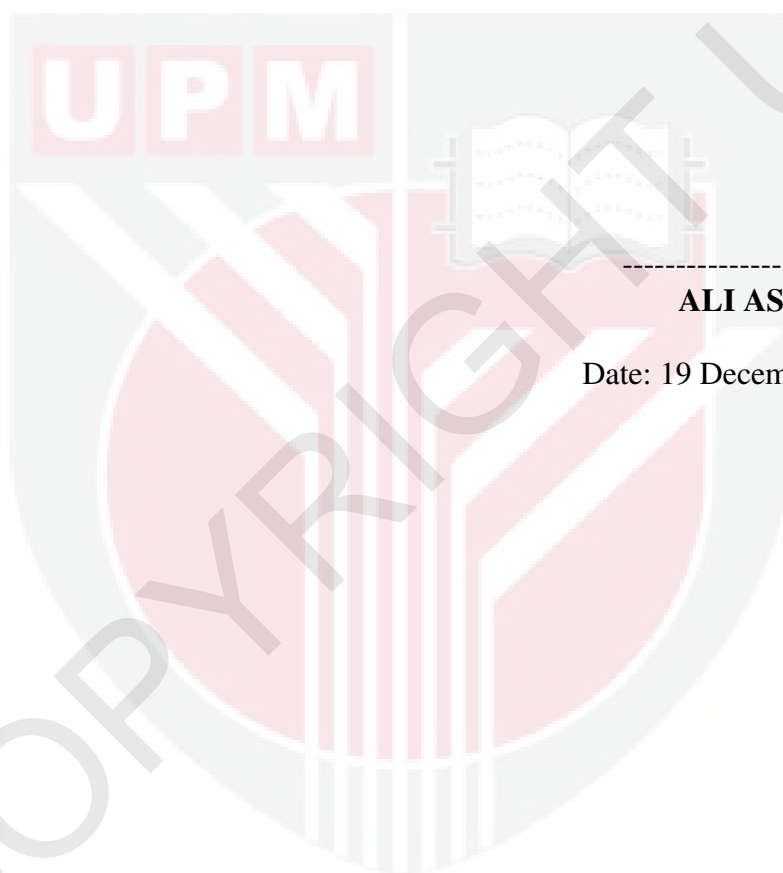
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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.



ALI ASHRAFI

Date: 19 December 2011

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