



**UNIVERSITI PUTRA MALAYSIA**

**NON-PARAMETRIC AND PARAMETRIC ESTIMATIONS OF CURE  
FRACTION USING RIGHT- AND INTERVAL-CENSORED DATA**

**BADER ALJAWDI**

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**NON-PARAMETRIC AND PARAMETRIC ESTIMATIONS OF CURE  
FRACTION USING RIGHT- AND INTERVAL-CENSORED DATA**

**BY**

**BADER ALJAWDI**

**Thesis Submitted to the School of Graduate Studies,  
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in fulfillment of the requirements for the degree of Doctor of Philosophy

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**Chair: Associate Professor Mohd Rizam Abu Bakar, PhD**

**Faculty: Institute for Mathematical Research**

In cancer studies many patients will never experience the event of concern and are hence considered as cured. Therefore, the main interest of the cancer trials is in determining the fraction of cured patients so as to obtain the trends in the survival of cancer patients. Survival models incorporate the cure fraction in the analysis; namely cure rate models, are being widely used in cancer studies, where one of the most common cure models that can be employed in cancer clinical trials is the bounded cumulative hazard (BCH) model. In this thesis, we considered two methods via the expectation maximization (EM) algorithm for cure rate estimation based on the BCH model using the two censoring types common to cancer clinical trials; namely, right and interval censoring.

Then, a series of simulation studies was conducted to evaluate the performance of the proposed estimation approaches.

This research investigated the non-parametric maximum likelihood estimation method for cure rate estimation by considering two common estimators for the survival function: 1) The Kaplan Meier (KM) estimator, which is suitable for the right censoring case; and 2) The Turnbull Estimator, which is suitable for the interval type of data censoring. The parametric maximum likelihood estimation of the cure fraction was also investigated under the same circumstances considering two scenarios: 1) when covariates were excluded from the analysis. In this case, the estimation was developed based on the exponential and Weibull distributions using the right and interval censoring types; and 2) when covariates were incorporated into the analysis through the scale parameter of the exponential distribution only using the same types of data censoring.

The major research findings were as follows: 1) the non-parametric and parametric estimation methods using the right and interval censoring types produced highly efficient cure rate parameters when the censoring rate was decreased to the minimum possible; 2) Non-parametric estimation of the cure fraction using interval censored data based on Turnbull estimator resulted in more precise cure fraction than the

Kaplan Meier estimator considering the interval midpoint to represent the exact life time; 3) The parametric estimation of the cure fraction based on the exponential distribution and right and interval censoring types produced more consistent estimates than the Weibull distribution especially in case of heavy censoring; 4) Parametric estimation of the cure fraction was more efficient when some covariates had been involved in the analysis than when covariates had been excluded; and 5) the non-parametric estimation method is the viable alternative to the parametric one when the data set contains substantial censored observations while in the case of low censoring the parametric method is more attractive.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**PENGANGGARAN TAK BERPARAMETER DAN BERPARAMETER BAGI  
PECAHAN SEMBUH MENGGUNAKAN DATA KANAN DAN SELANG  
TERTAPIS**

Oleh

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Dalam kajian penyakit barah kebanyakan pesakit mungkin tidak mengalami peristiwa yang dikaji maka mereka dianggap sebagai sembuh. Oleh itu minat kajian bagi percubaan penyakit barah adalah untuk menentukan pecahan pesakit yang sembuh untuk memperlihatkan trend mandirian pesakit barah.

Didalam tesis ini, kami membina dua pendekatan analitik melalui algoritma pemaksimuman jangkaan (EM) untuk menganggar kadar sembuh berasaskan model bahaya terbatas melonggok (BCH) menggunakan dua jenis tapisan umum didalam percubaan klinikal penyakit barah iaitu tapisan kanan dan selang. Beberapa siri kajian

simulasi dijalankan untuk menilai kemampuan pendekatan penganggaran yang dicadangkan.

Kami bina kaedah penganggar kebolehjadian maksimum tak berparameter bagi penganggaran kadar sembuh melalui dua penganggar lazim bagi fungsi mandirian: 1) Penganggar Kaplan Meier (KM) yang sesuai untuk kes tapisan kanan, 2) Penganggar Turnbull yang sesuai untuk jenis tapisan selang.

Penganggaran kebolehjadian maksimum berparameter bagi pecahan sembuh turut dibina berdasarkan keadaan yang sama dengan mempertimbangkan dua scenario: 1) bila kovariat tidak disertakan semasa analisis. Dalam hal ini penganggaran dibina berdasarkan taburan eksponen dan Weibull menggunakan tapisan kanan dan tapisan selang dan 2) bila melibatkan kovariat melalui parameter skala taburan eksponen dan Weibull dengan menggunakan tapisan yang sama.

Hasil kajian adalah seperti berikut: 1) kaedah penganggaran tak berparameter dan berparameter menggunakan tapisan kanan dan selang menghasilkan penganggar kadar sembuh yang sangat cekap apabila kadar tapisan diperkecilkan kepada nilai yang sekecil mungkin; 2) Penganggaran tak berparameter bagi pecahan sembuh berasaskan

penganggar Turnbull menghasilkan pecahan sembuh yang lebih jitu berbanding penganggar Kaplan Meier apabila titik tengah selang mewakili masa hayat tepat; 3) Penganggaran berparameter bagi pecahan sembuh berdasarkan taburan eksponen dengan tapisan kanan dan selang menghasilkan penganggar yang konsisten jika dibandingkan dengan taburan Weibull terutamanya bagi kes tapisan yang banyak; 4) Penganggaran berparameter bagi pecahan sembuh lebih cekap apabila beberapa kovariat diambilkira dalam analisis berbanding dengan jika kovariat dikeluarkan; dan 5) Kaedah penganggaran tak berparameter adalah alternatif yang lebih berjaya jika dibandingkan dengan kaedah berparameter apabila data mengandungi cerapan tertapis yang cukup banyak manakala bagi kes bilangan tapisan yang rendah kaedah berparameter adalah lebih menarik.



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I certify that a Thesis Examination Committee has met on 10<sup>th</sup> of August 2011 to conduct the final examination of Bader Aljawdi on his thesis entitled “The non-parametric and parametric estimations of cure fraction using right- and interval-censored data” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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## **DECLARATION**

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at Universiti Putra Malaysia or at any other institution.

The logo of Universiti Putra Malaysia (UPM) is a shield-shaped emblem. It features a red and white geometric design with a central book icon. The letters 'UPM' are prominently displayed in a red box at the top left of the shield.

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**BADER ALJAWDI**

Date: 10 August 2011

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