



UNIVERSITI PUTRA MALAYSIA

**IMPROVED ALGEBRAIC CRYPTANALYSIS OF THE REDUCED-ROUND
ADVANCED ENCRYPTION STANDARD**

DAVOOD REZAEIPOUR

IPM 2011 4

**IMPROVED ALGEBRAIC CRYPTANALYSIS OF THE REDUCED-ROUND
ADVANCED ENCRYPTION STANDARD**

By

DAVOOD REZAEIPOUR

**Thesis Submitted to the School of Graduate Studies, Universiti Putra
Malaysia, in Fulfilment of the Requirements for the Degree of
Doctor of Philosophy**

January 2011

DEDICATION

To

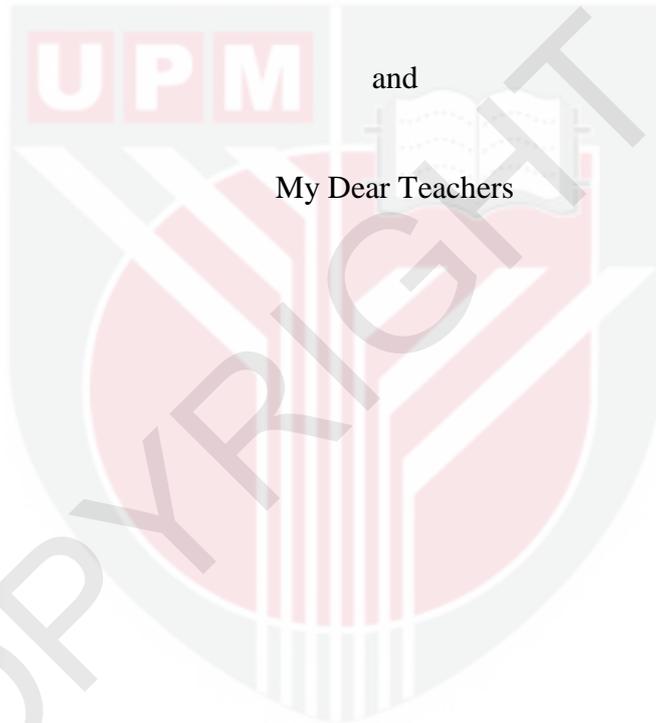
My wife and my children

Manijeh , Mahsa and Sina

For their great patience

and

My Dear Teachers



Abstract of thesis presented to the senate of Universiti Putra Malaysia in fulfilment of the requirement for the degree of Doctor of Philosophy

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Chair: Assoc. Prof. Mohamad Rushdan Md. Said, PhD

Faculty: Institute for Mathematical Research

As we know Cryptology is divided into two parts: “Cryptography” and “Cryptanalysis”. Since block ciphers can be deployed in many different applications, so we focus on Advanced Encryption Standard (AES) which is the successor of Data Encryption Standard (DES).

In cryptography, we propose new block cipher (NBC08) in order to understand the inner structure and other known properties. NBC08 accepts an variable-length key up to 512 bits, which is an improved security/performance tradeoff over existing block ciphers. It cannot be analyzed by known cryptanalytic attacks.

We study AES specifications and also the algebraic structure for AES over Galois Fields $GF(2)$ and $GF(2^8)$. We describe the most common cryptanalytic techniques on block ciphers, such as Differential, Linear and Integral cryptanalysis.

We study the different solving methods for system of equations of AES in both fields, $GF(2)$ and $GF(2^8)$. The process of performing these methods on AES acts as Algebraic attack.

In cryptanalysis, we improve the algebraic cryptanalysis attack on the reduced-round AES. It's called Ground Algebraic attack. The notable property of Ground attack is that less requirements to any information for analyzing AES. Ground Algebraic attack is the first attack on reduced-round AES which can break 4-round and 5-round AES by respectively 2^{56} and $2^{113.5}$ computational complexities. The number of required chosen plaintexts for cryptanalysis 4-round and 5-round AES is 8 and 15, respectively.



Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia
sebagai memenuhi keperluan untuk ijazah Doktor Falsafah

**KRIPTANALISIS ALJABAR YANG DIPERTINGKATKAN KE ATAS
PUSINGAN TERTURUN PENYULITAN PIAWAI LANJUTAN**

Oleh

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Januari 2011

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Populariti saifer blok adalah berdasarkan kepada rekaan-rekaan berjaya (seperti DES dan penggantinya, AES) yang diseragamkan, boleh didapati dengan percuma, dan boleh diatur kedudukannya dalam banyak aplikasi yang berbeza. Dalam tesis ini kami mencadangkan saifer blok baru (NBC08) yang menerima kunci panjang berubah-ubah sehingga 512 bit, yang mana adalah sesuai untuk dilaksanakan dalam pelbagai jenis persekitaran. NBC08 ialah satu sekuriti yang dipertingkatkan dari segi keselamatan/prestasi dibanding dengan saifer blok yang sedia wujud, yang tidak boleh dianalisis dengan mana-mana serangan kriptanalisis.

Kami mengkaji semula spesifikasi AES dan kekuatannya menentang serangan-serangan yang diketahui. Disebabkan oleh jidar keselamatan besar AES menentang kriptanalisis linear dan pembezaan dan penggunaan takrif aljabar mudah, penyelidik-penyelidik cenderung mengeksploitasi ciri-ciri aljabar AES. Kami juga mengkaji struktur aljabar untuk AES ke atas $GF(2)$ dan $GF(2^8)$. Kami menghuraikan teknik-teknik lazim kriptanalisis saifer blok – Kriptanalisis pembezaan, kriptanalisis Linear dan kriptanalisis kamiran.

Sebetulnya, satu pendekatan penting adalah bagi menyatakan operasi penyulitan sebagai satu sistem persamaan-persamaan polinomial. AES boleh digambarkan sebagai sistem persamaan-persamaan kuadratik ke atas $GF(2)$ atau $GF(2^8)$, yang tidak diketahuinya bit kunci dan satu jumlah besar pembolehubah-pembolehubah pertengahan yang terhasil dari operasi penyulitan. Kami boleh menulis penyulitan AES sebagai satu formula algebra tertutup mudah ke atas medan terhingga.

Kami mengkaji kaedah-kaedah penyelesaian berbeza untuk sistem persamaan AES dalam kedua-dua medan $GF(2)$ dan $GF(2^8)$. Sebenarnya, hasil dari menjalankan kaedah-kaedah ini pada AES bertindak sebagai serangan aljabar, tetapi tidak satu-satunya serangan yang mungkin. Kami menghuraikan serangan penyisipan menentang AES yang digunakan daripada ciri-ciri aljabar AES. Kami kemudian memberikan versi AES S-box yang tahan menentang serangan penyisipan.

Sifat terpenting serangan baru ini ialah kurang syarat-syarat untuk apa-apa maklumat untuk mengkaji. Serangan aljabar Ground baru boleh memecahkan 4-pusingan dan 5-pusingan AES oleh masing-masing 2^{56} dan $2^{113.5}$ kerumitan-kerumitan pengiraan. Jumlah teks asal terpilih yang dikehendaki untuk kriptanalisis 4-pusingan dan 5-pusingan AES ialah 8 dan 15, masing-masing.

ACKNOWLEDGEMENTS

First and foremost, all praise to the almighty ALLAH for His blessing and merciful which enables me to complete my study.

This thesis is the result of three years of work which I have been accompanied by some people. I now have the pleasant opportunity to express my sincere appreciation to all of them.

I would like to express my deepest gratitude, appreciation and thanks to Assoc. Prof. Mohamad Rushdan Md. Said, chairman of my supervisory committee for his helpful advices and valuable guidance during this study. I am deeply indebted to him, who has read and re-read many versions of this thesis, and every other research paper I have written to date, provided me with countless hours of his time, and given me sound advice, on matters technical, professional and personal. I would not have been able to continue my study without his supports and encouragements.

I appreciate the helps of my supervisory committee members, Prof. Kamel Ariffin M. Atan and Prof. Mohamed Othman for their supports and assistance.

The last not the least, Special Thanks to my wife Manijeh, my daughter Mahsa and my son Sina for their prayers, encouragements and spiritual supports during my whole life. I dedicate this work to them, with love and gratitude.

I certify that a Thesis Examination Committee has met on 17 January 2011 to conduct the final examination of Davood Rezaeipour on his thesis entitled “**IMPROVED ALGEBRAIC CRYPTANALYSIS OF THE REDUCED-ROUND ADVANCED ENCRYPTION STANDARD**” in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

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DECLARATION

I declare that the thesis is my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously, and is not concurrently, submitted for any other degree at University Putra Malaysia or at any other institution.

DAVOOD REZAEIPOUR

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TABLE OF CONTENTS

	Page
ABSTRACT	iii
ABSTRAK	v
ACKNOWLEDGEMENTS	vii
APPROVAL	viii
DECLARATION	x
LIST OF TABLES	xiii
LIST OF FIGURES	xiv
LIST OF ABBREVIATIONS	xv
CHAPTER	
1 INTRODUCTION	1
1.1. Background of Study	1
1.2. Importance of the Study	25
1.3. Objectives of Research	27
1.4. Organization of Thesis	29
2 LITERATURE REVIEW	32
2.1. Chapter outline	32
2.2. New Block cipher NBC08	32
2.3. AES specification and Mathematical Preliminaries	33
2.4. The interpolation attack against AES	39
2.5. The previous works done	40
2.6. Known Algebraic attacks on AES	44
2.7. Our main work	45
3 CRYPTANALYTIC METHODS AND NEW BLOCK CIPHER	49
3.1. Chapter outline	49
3.2. Cryptanalytic attacks	49
3.3. Methods of cryptanalysis	51
3.4. About the attacks on DES block cipher	52
3.5. Modern block cipher	60
4 THE ESSENTIAL ALGEBRAIC REPRESENTATIONS – AES	71
4.1. Chapter outline	71
4.2. The basic algebraic structure	71
4.3. Algebraic formula for AES over $GF(2)$ and $GF(2^8)$	73
4.3.1. The forming of cipher equations in $GF(2)$	73
4.3.2. The forming of cipher equations in $GF(2^8)$	80
4.3.3. Dual Ciphers	87
4.3.4. The approximations of the algebraic representation	91
5 MULTIVARIATE POLYNOMIAL EQUATIONS AND AES	98
5.1. Chapter outline	98
5.2. The Solving Systems of Multivariate Equations	98
5.2.1. Linearization Algorithm	100

	5.2.2. XL Algorithm	100
	5.2.3. XSL Algorithm	102
5.3.	MQ Attack on AES	103
	5.3.1. XL Attack over GF(2)	103
	5.3.2. XSL Attack over GF(2)	103
	5.3.3. XSL Attack over GF(2 ⁸)	104
5.4.	Interpolation Attack and AES	104
6	IMPROVED ALGEBRAIC CRYPTANALYSIS ON AES	109
6.1.	Chapter outline	109
6.2.	Definition and the plan of attack	109
	6.2.1. Definition of Ground	109
	6.2.2. The plan of Ground attack	110
6.3.	The number of required texts for Ground analysis	111
6.4.	The Ground attack using MixColumn operation	113
6.5.	The Main Ground Attack	117
	6.5.1. Analysis on 1-round AES	118
	6.5.2. Analysis on 2-round AES	119
	6.5.3. Analysis on 3-round AES	120
	6.5.4. Analysis on 4-round AES	121
	6.5.5. Analysis on 5-round AES	123
	6.5.6. The strategy of solving the equations	125
6.6.	The reduce of computational complexity in Ground attack	128
	6.6.1. Using the change in the condition of Ground	128
	6.6.2. Using the estimation of S-box	130
6.7.	Summary	130
7	CONCLUSIONS AND OPEN PROBLEMS	132
7.1.	Concluding remarks	132
7.2.	Open research problems	134
	REFERENCES	137
	BIODATA OF STUDENT	144
	LIST OF PUBLICATIONS	145