



UNIVERSITI PUTRA MALAYSIA

**FUZZINESS IN PERFORMANCE EVALUATION PROBLEMS USING  
DATA ENVELOPMENT ANALYSIS**

**EHSANOLLAH MANSOURIRAD**

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PROBLEMS USING DATA ENVELOPMENT  
ANALYSIS**

**EHSANOLLAH MANSOURIRAD**

**DOCTOR OF PHYLOSOPHY  
UNIVERSITI PUTRA MALAYSIA**

**2010**

**To**

**My parents**

**For their encouragement and supports**

**&**

**My brother and sisters**

**And**

**My dear teachers**



**EVALUATION OF FUZZINESS IN PERFORMANCE PROBLEMS USING  
DATA ENVELOPMENT ANALYSIS**

**By**

**EHSANOLLAH MANSOURIRAD**

**Thesis Submitted to the School of Graduate Studies, Universiti Putra  
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**December 2010**

**Chair: Associate Prof. Mohd Rizam Abu Bakar, PhD**

**Faculty: Institute for Mathematical Research**

Efficiency evaluation is an important part of decision making in many areas particularly in management and manufacturing sectors. Uncertainty and fuzziness of the real world problems have increased utilization of fuzzy sets theory in many research areas and data envelopment analysis is one of them. Utilizing data envelopment analysis to evaluate efficiency scores of decision making units in fuzzy environment requires fuzzy models and mathematical methods for solving fuzzy models with minimum calculation and maximum precision. Since current fuzzy data envelopment analysis models are not able to solve some problems in fuzzy environment, our attempt is to provide fuzzy data envelopment analysis models related to following various problems.

Some problems include uncontrollable data (for manager) that regularly have fuzzy essence. An uncontrollable fuzzy data envelopment analysis model is represented for these types of problems. The advantages of the proposed model are in capability of it in including uncontrollable factors particularly those with fuzzy nature in problems with fuzzy data and controlling factor weights by additional constraints which can avoid the model to become infeasibility. The disadvantage of the method is in using too many restrictions (one restriction for each fuzzy data) which makes the model complicated and expensive to solve.

For cases that interval efficiency scores are helpful, a method for solving fuzzy data envelopment analysis models is represented which interval efficiency scores can be achieved without adding restrictions to the model for each fuzzy data. In comparison with other methods, this method is simple, easy and with no additional constraint for each fuzzy data. In addition a fuzzy weights data envelopment analysis model is proposed to determine effect of data on the efficiency score. The model is informative in problems that the manager needs to know about uncertain effects of factors on efficiency score. The method of solving the model is simple and informative.

By suggesting categorical data envelopment analysis method for problems with uncertain membership in various categories, we can help the decision maker to recognize the efficient decision making units fairly. In comparison with available method for categorical problems, our method is more informative and the traditional

categorical method is a special case of our method. Finally, we provide a solution to comparison of production methods by utilizing fuzzy non-discretionary data envelopment analysis model. The proposed technique is more capable and informative while it includes factors with fuzzy essence that have effect on efficiency of production methods which is a real problem and may be its performance be effected by many fuzzy issues.

Abstrak tesis yang dikemukakan kepada Senat Universiti Putra Malaysia sebagai  
memenuhi keperluan untuk ijazah Doktor Falsafah

**EVALUASI KINERJA KEKABURAN PERMASALAHAN DALAM  
MENGGUNAKAN ANALISIS DATA EFISIENSI**

Oleh

**EHSANOLLAH MANSOURIRAD**

**December 2010**

**Pengerusi : Prof. Madya Mohd Rizam Abu Bakar, PhD**

**Fakulti : Institut Penyelidikan Matematik**

Penilaian keberkesanan merupakan bahagian penting dalam proses membuat keputusan dalam pelbagai bidang, terutama dalam sector pengurusan dan pembuatan. Ketidaktentuan dan kekaburan bagi masalah dunia sebenar telah meningkatkan penggunaan teori set kabur dalam beberapa bidang penyelidikan dan analisis pengumpulan data merupakan salah satu daripadanya. Penggunaan analisis pengumpulan data untuk menilai skor kecekapan terhadap unit membuat keputusan dalam persekitaran kabur memerlukan model kabur dan kaedah matematik bagi menyelesaikan model kabur dengan perkiraan yang minimum tetapi ketepatan maksimum. Memandangkan model analisis pengumpulan data kabur terkini tidak dapat menyelesaikan banyak masalah yang berkaitan dengan

kecekapan penilaian dalam persekitaran kabur, kajian ini merupakan cubaan untuk menyediakan model analisis pengumpulan data kabur yang berkaitan dengan pelbagai masalah.

Sebahagian besar masalah merangkumi data tak terkawal (oleh pengurus) dan lazimnya mempunyai ciri kabur. Dalam kajian ini, model analisis pengumpulan data kabur yang bersifat tak terkawal dikemukakan bagi mengatasi masalah jenis ini. Keuntungan dari model yang dicadangkan dalam kemampuan dalam termasuk faktor tidak terkawal terutama mereka dengan alam kekaburan dalam masalah dengan data kekaburan dan berat faktor kawalan oleh kendala tambahan yang boleh mengelakkan model menjadi ketidaklayakan. Kerugian dari kaedah ini dalam menggunakan sekatan terlalu banyak (satu sekatan untuk setiap data kekaburan) yang membuat model yang rumit dan mahal untuk menyelesaikan.

Bagi kes yang menggunakan skor kecekapan selang, suatu kaedah yang dapat menyelesaikan model analisis pengumpulan data yang mewakili skor kecekapan interval diperoleh tanpa menambah kekangan terhadap model bagi setiap data kabur. Model ini bermaklumat dalam masalah yang pengurus perlu mengetahui tentang kesan pasti faktor pada skor kecekapan. Kaedah penyelesaian model sederhana dan bermaklumat.

Tambahan pula, model analisis pengumpulan data kabur dicadangkan untuk mengenal pasti kesan data ke atas skor kecekapan.

Dengan mencadangkan kaedah analisis pengumpulan data berkategori bagi masalah yang mempunyai ketidaktentuan keahlian dalam pelbagai kategori dapat membantu pembuat keputusan menghargai keberkesanan unit pembuat keputusan secara adil. Dibandingkan dengan kaedah yang sedia untuk masalah kategoris, kaedah kami lebih bermaklumat dan kaedah kategoris tradisional adalah kes khusus dari kaedah kami. Akhirnya, suatu penyelesaian bagi membandingkan kaedah penghasilan dengan menggunakan model analisis pengumpulan data kabur telah dikemukakan. Teknik yang dicadangkan lebih mampu dan bermaklumat sementara itu termasuk faktor dengan esensi fuzzy yang berpengaruh terhadap kecekapan kaedah pengeluaran yang merupakan masalah nyata dan mungkin prestasinya akan dipengaruhi oleh isu fuzzy banyak.

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I certify that an Examination Committee has met on 21-12-2010 to conduct the final examination of Ehsanollah Mansourirad on his **Doctor of Philosophy** thesis entitled "**Evaluation of Fuzziness in Performance Problems Using Data Envelopment Analysis**" in accordance with the Universities and University Colleges Act 1971 and the Constitution of the Universiti Putra Malaysia [P.U.(A) 106] 15 March 1998. The Committee recommends that the student be awarded the Doctor of Philosophy.

Members of the Thesis Examination Committee were as follows:

**Malik Hj Abu Hassan, PhD**

Professor

Faculty of Science

Universiti Putra Malaysia

(Chairman)

**Leong Wah Jun, PhD**

Associate Professor

Faculty of Science

Universiti Putra Malaysia

(Internal Examiner)

**Mansor Bin Monsi, PhD**

Faculty of Science

Universiti Putra Malaysia

(Internal Examiner)

**Josef Jablonsky, PhD**

Professor

Faculty of Informatics and Statistics

University of Economics

Czech Republic

(External Examiner)

---

**BUJANG KIM HUAT, PhD**

Professor and Deputy Dean

School of Graduate Studies

Universiti Putra Malaysia

Date:

This thesis was submitted to the Senate of Universiti Putra Malaysia and has been accepted as fulfilment of the requirement for **Doctor of Philosophy**. The members of the Supervisory Committee were as follows:

**Mohd Rizam Abu Bakar, PhD**

Associate Professor

Faculty of Science

Universiti Putra Malaysia

(Chairman)

**Azmi Jaafar, PhD**

Associate Professor

Faculty of Computer Science and Information Technology

Universiti Putra Malaysia

(Member)

**Lai Soon Lee, PhD**

Lecturer

Faculty of Science

Universiti Putra Malaysia

(Member)

---

**HASANAH MOHD GHAZALI, PhD**

Professor and Dean

School of Graduate Studies

Universiti Putra Malaysia

Date:

## **DECLARATION**

I declare that the thesis is my original work except for quotations and citations, which have been duly acknowledged. I also declare that it has not been previously and is not concurrently submitted for any other degree at Universiti Putra Malaysia or other institution.

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**EHSANOLLAH MANSOURIRAD**

Date: 21 December 2010

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